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PART I, 1940.

The Present Position of Mathematical Statistics

By M. S. Bartlett, D.Sc.

CONTENTS

I. *Scope of mathematical statistics*

In view of the active interest which this Society has always shown in mathematical statistics, I do not suppose that the President meant to be taken too seriously when he reminded us * that explicit reference to this branch of knowledge is missing from the list of Objects to be found in the Society's Bye-laws. At any rate I have not been asked to abandon a long-simmering idea of surveying the present position of mathematical statistics; and recent pleas (Neyman, 1938; Wishart, 1939) for a tidying-up of the more methodological aspects of the subject suggest that discussion directed towards this end should be of value. Perhaps I should stress that classification and systematization are not to be given undue importance. It is always useful for a scientific worker in his more leisured moments to tidy up his apparatus and put it in a reasonable place; but he may have to shift it again when he becomes involved in further work.

In his recent paper on the teaching of statistics, Dr. J. Wishart referred briefly to the position of mathematical statistics, but no

* In the discussion following the paper of Dr. Wishart's referred to (Wishart, 1939).

detailed examination was possible, and I have taken the opportunity given me by my admitted concentration in this paper on the mathematical side of statistics to refer in more detail to some of the points that arise. For convenience my paper has been divided into three sections, although these tend to overlap. The first is intended to give a glimpse of the entire mathematico-statistical wood (critics will doubtless prefer the term "jungle"). The formal problem of mathematical technique is examined in Section II. Finally, although my primary aim has been to comment on mathematical statistics as such, further reference is made in the last section to its applicability, and relation with statistics in general. Sections II and III are both rather lop-sided; this is because I have referred mainly to matters which are still the subject of argument, and have passed by others which may be of equal importance but do not seem to call for special comment. I feel that my self-imposed task has been a presumptuous one, but it may help to show that connecting fields of study which are not ordinarily associated there exists a subject of mathematical statistics whose elasticity of application does not imply an absence of theoretical foundation.

Owing to the entanglement of statistics with so many different and sometimes highly technical sciences, any survey, however incomplete, will occasionally land us far from the usual field of activities of this Society. The annual articles* by Dr. J. O. Irwin (1931 *et seq.*) on recent advances in mathematical statistics have given an idea of the very wide range of subject-matter of published papers dealing fairly directly with statistical theory and methods, although references to literature for which more specialized knowledge is required, such as that of modern physics, have been omitted.

In seeking for some unity underlying this mass of material, I have not attempted to distinguish between *mathematical statistics* and *statistical theory*. I think that a *theory* of statistics must essentially be mathematical in form, and any distinction we might try to make would be rather trivial. On the other hand, by mathematical statistics I do not mean *merely* statistical mathematics; we shall naturally be concerned with the latter a great deal, but it would be misleading to identify any branch of statistics with the mathematical technique appropriate to it.

Statistical theory as applied mathematics.—Yule and Kendall (*Introduction to the Theory of Statistics*, 1937, p. 3) describe the theory of statistics as the exposition of statistical methods; but while I admit that it is unfair to lift a definition from its context, this one hardly appears to me to be adequate, for it would seem possible to expound methods without particular reference to any theory. For

* Now only continued as bibliographies (Irwin, 1938; Hartley, 1939).

example, a bare exposition of the *method* of least squares is hardly sufficient for a *theory* of least squares.*

Fisher begins (*Statistical Methods for Research Workers*, 1938, p. 1), " The science of statistics is essentially a branch of Applied Mathematics and may be regarded as mathematics applied to observational data." Some limitation is presumably implied on the nature of the observational data that can be dealt with, and it is this that requires further comment.

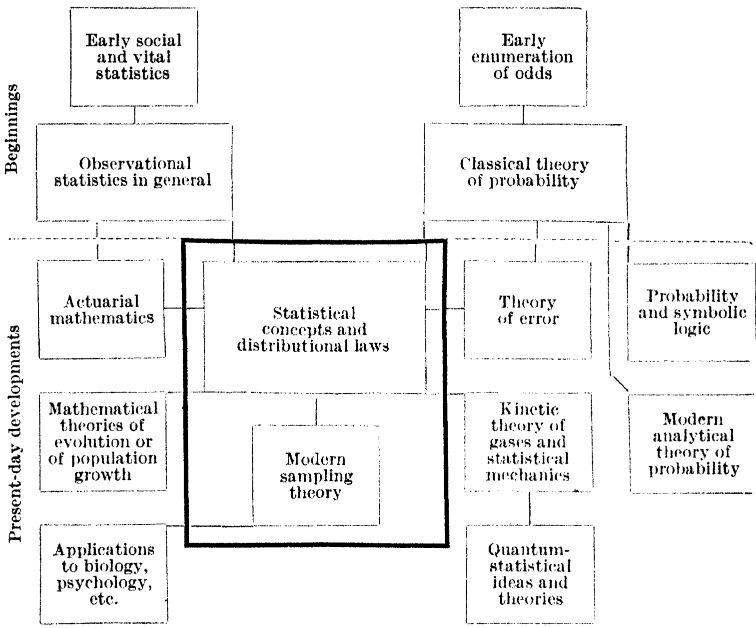
Suppose first of all we consider statistics in relation to three broad theoretical classifications. Firstly, there is the classical theory of probability which developed in France from a study of lotteries and games of chance, and is itself only a few hundred years old. Secondly, there is the theory of the statistical properties of aggregates, represented in the physical sciences by the kinetic theory of gases and its younger more virile brother, statistical mechanics. Statistical data pertaining to biological and other non-physical populations are irrelevant to these theoretical considerations in so far as they are merely matters of fact; but such developments as the mathematical theory of evolution associated with the names of R. A. Fisher, J. B. S. Haldane and Sewall Wright, or the statistical theory of population growth and replacement studied, for example, by A. J. Lotka, may be included. Thirdly, there is the theory of sampling. This partly emanated from the earlier theory of errors, but it owes its importance to the modern application of statistical methods in biology, psychology and other fields where variability is an inevitable major factor in our measurements. This three-fold classification, which could be crudely summarized as (i) theory of probability, (ii) theory of populations, (iii) theory of sampling, has of course no very clear-cut divisions either in theory or in practice. In fact, a single theory which is to stand for *all* the applications I have mentioned might quite consistently be regarded as covered by a general theory of probability. If we do attempt to associate such a general theory with the comprehensive theory of statistics we are trying to define, the essential point to remember is that we should regard it, not from the point of view of pure mathematics or of pure logic, but as emerging from these various applications.

In the schematic picture below I have tried to sketch the connection between different branches of knowledge which are related. I have not omitted the purely empirical studies which really began

* Mr. Yule, who gave this definition in the original edition of his book in 1911, tells me that he would distinguish between a " bare exposition " or " description," and a complete exposition. By an exposition he means " a complete expounding from the foundations upwards. . . ." This explanation largely disposes, of course, of my criticism.

the subject of statistics, as distinct from probability, and to which it owes its name. Modern statistical theory is the offspring of both these parents, and has displayed all the vigour to be expected of such a hybrid.

Mathematical statistics is not explicitly labelled in this scheme, but its main root, from which particular applications may be developed, I would identify with what I have called "statistical concepts and distributional laws," and the offshoot of this, sampling theory.



Some Continental statisticians feel a little aggrieved at the insularity of English statistics, and have difficulty in seeing much distinction between the theory of statistics and, say, the theory of probability developed so extensively by Laplace. We have seen that there is a partial justification for their view. It is interesting to note that the recent conference on "applications of the theory of probability" held in Geneva last summer * would probably, if held in this country or in America, have been called a conference on mathematical

* Convened by the International Institute of Intellectual Cooperation in conjunction with the University of Geneva. The reports read at this conference, ranging over agriculture, astronomy, biology, economics, industry, physics and psychology, gave such an extensive survey of the fields of application of statistical theory that my own discussion in Section III seems rather superfluous, but it should be remembered that I am not so much attempting in this paper to give a detailed survey as to offer comment on particular aspects.

statistics. There is, however, a fundamental difference of approach which prevents our identifying statistical theory with probability. The theory of probability originally developed on the Continent has now become primarily *pure* mathematics—in recent years a highly sophisticated branch of pure mathematics.* The difference between pure and applied mathematics is often one of degree, but briefly we may say that in applied mathematics we employ mathematics as a means to an end, not merely as an end in itself. To quote the words Karl Pearson used in a particular context, "Mathematics are not there for the joy of the analyst, but because they are essential to the solution." In terms of Fisher's definition I would describe statistical theory as a mathematical theory which relates to observational data arising from a physical background of chance; but while it follows that the mathematical theory used must be that of probability, the basis of the subject is a physical theory, not a mathematical technique. It also follows that the mathematical theory is interpreted in statistical terms, for its symbols must, like any other physical concepts, be related to observation. The special nature of this relation for statistics is referred to in Section II.

There are welcome indications that the Continent is taking a more active interest in mathematical statistics, but even in the twentieth century some of the most important developments there have been in the pure theory of probability, in contrast with the advance in this country in statistics. I do not, of course, mean to imply that the development of statistics here has been unaccompanied by contributions to mathematical theory. Many names spring to one's mind. But those responsible would probably be the first to agree that their contributions should not be severed from the background of statistical problems which stimulated the development of such theory. However, I do not want to enter into any detailed historical review, especially as an extremely interesting account of this aspect has been given by Irwin (1935).

It may be felt that the dependence of mathematical statistics on probability is a serious limitation of its scope. To avoid any possible ambiguity here, I will stress that I am using the word probability in its technical statistical sense.† Often interpolation, extrapolation or graduation formulæ have been calculated in popula-

* Von Mises (1939, p. 152) complains rather bitterly of this transition, but it enables me (without, however, accepting his mathematical technique; see Section II) to identify statistical theory with the applied theory he has in mind.

† The confusion which has arisen because we have used the same word as that enjoying a popular but vague ordinary usage has not been confined to the "lesser breeds without the law," and has been particularly damaging to the prestige of statistical theory; but it has always seemed to me largely irrelevant to the latter's proper scientific status. (See also my footnote on p. 12.)

tion or other studies without reference to probability theory, and yet may be reasonably included under mathematical statistics. Thus some elasticity is required, and no one wishes to see the subject enclosed in a strait jacket. But if it is to preserve an underlying unity, the limitation seems on the whole required. It is a valid comment that the limitation implies that mathematical statistics will be of varying usefulness in different fields of statistics, but it would be rash to claim too much for it at the cost of destroying its integrity. The situation is no different from that relating to the application of any other theory which is mathematical in form.

II. *The mathematical framework*

The vast amount of artistic care already devoted by others to the subject of mathematical probability makes me diffident about spending further time on it; I am quite prepared to recognize the model usually put forward for our acceptance. However, it is important to satisfy ourselves first of its soundness, and in my own examination below I have deliberately concentrated on the skeleton of the theory. Perhaps I shall thereby help to dispel the accusation that English statisticians always keep this skeleton hidden in the cupboard.

I do not think that the pure mathematics of statistical probability presents any fundamental difficulty. The pure theory has been brought to an advanced analytical level (see, for example, the tract by Cramer (1937) on *Random Variables and Probability Distributions*), but such refinement is not entirely necessary to the applied mathematician, and I have tried to present the basic principles as simply as possible. (If anyone wishes to take them for granted, he may easily skip the next few paragraphs.)

The mathematical technique of probability numbers and probability distributions.—Mathematically, we set up classes of "objects" such that a probability number $p = 1/a$ is associated with each of the a objects in any aggregate or class A. For b of these objects regarded as a single group or class B, the corresponding probability number is b/a. Individual probability numbers are thus additive, and more generally if B is divided into any two complementary parts B_1 and B_2, we have the addition theorem of probability,

$$\frac{b}{a} = \frac{b_1}{a} + \frac{b_2}{a} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If in a total aggregate N of size n there are two logically independent classifications denoted by B and C, the number associated with the class of objects characterized by both B and C is e/n,

where e is the number in this class. If the number in the class B is b, we have the multiplication theorem of probability,

$$e/n = e/b\, b/n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If $e/b = c/n$, where c is the number in the class B, the probabilities associated with B and C respectively are said to be *independent.* The relation implies that we can represent the total aggregate symbolically by the expression

$$\{bB + (n-b)\bar{B}\}\{cC + (n-c)\bar{C}\}$$
$$= bcBC + (n-b)c\,\bar{B}C + b(n-c)B\bar{C} + (n-b)(n-c)\bar{B}\bar{C} \quad (3)$$

where $\bar{B}$ denotes the class of " not B," and BC denotes the class of B and C, etc. Equation (3) has only to be divided by n^2 for it to be converted into a relation between probability *distributions*, in which we consider simultaneously the different sub-classes into which the total class is divided.* In general, we may specify the distribution for sub-classes A_r of the total class A either by the vector of probability values p_r, or by the symbolic expression

$$p_rA_r = p_1A_1 + p_2A_2 + \ . \ . \ .$$

where the summation convention is used on the left-hand side. The distribution derived from two independent distributions is then either the matrix product (p_rq_s), or the generalisation of equation (3),

$$\{p_rA_r\}\{q_sB_s\} = p_rq_sA_rB_s \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

If in this latter representation the second distribution is a replica of the first, and moreover the terms A_rB_s and B_sA_r are not distinguished, we obtain $(p_rA_r)^2$, or for n similar distributions,

$$(p_rA_r)^n = p_1{}^nA_1{}^n + np_1{}^{n-1}p_2A_1{}^{n-1}A_2 + \ . \ . \ . \quad . \quad (5)$$

which is known as the multinomial distribution. A distribution $p_{rs}A_rB_s \neq p_rq_sA_rB_s$ is a simultaneous distribution which does not resolve into independent factors, but is not different in principle from the distribution p_rA_r.

If the general symbols A_r are replaced by particular symbols α_r corresponding to a characteristic α of the class A, then α is a *random variable* for which the *expectation* is

$$E(\alpha) = p_r\alpha_r \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

One common type of variable α is such that each particular value α_r is a positive integer r. If we define $\beta = z^r$, we have for the variable β,

$$E(\beta) = F(z),\text{ say, } = p_rz^r \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

The function $F(z)$ in this instance specifies the particular distribution just as well as p_rA_r, the component p_r being the coefficient of

* Cf. Soper (1922).

z^r in $F(z)$. $F(z)$ is the *probability-generating function* (or in statistical mechanics, the *partition function*). Among equivalent functions may be mentioned $M(t) = F(e^t)$, which is a generating function for the *moments*, $E(r^s)$. Since these functions are all included in the general expression $p_r A_r$, it may be noted that they will also be multiplied when independent distributions are combined.

It would be possible to go on developing this simple algebra in the orthodox manner, but this should be sufficient to show the basis of the technique. So far all probability numbers are confined to rational fractions, and the number of classes is finite. This would provide a theory adequate in all respects for comparison with observed phenomena, but *as a matter of mathematical convenience* the technique is generalized to allow other kinds of numbers and classes.* This generalization transfers us from algebra to analysis, from summation to integration, from selecting coefficients to taking Fourier transforms, but the basic principles are not affected, and I shall not discuss these convenient refinements of the mathematician, except to remark that the statistician will not need to pursue them further than the stage at which he finds them useful.

Statistical interpretation of a probability number.—All this pure mathematical theory is internally consistent, but we have yet to show that it is consistent with the statistical interpretation we wish to place on it. In practice we find that it is often possible to ascribe probability numbers to certain events, at least with as much justification as we can ascribe mass or temperature to physical systems. Events may be said to be governed in part by the operation of the "laws of chance."

For simple systems it is sometimes possible to risk an *a priori* identification of the possible results of a "trial" with the different elements of an aggregate A in our mathematical theory, but although a probability number might be assigned to an event in this way (for example, the number $\frac{1}{2}$ for the probability of heads with a tossed coin), the validity of this identification always depends ultimately on experiment. The equal chances which the different possible events are allotted are actually idealized concepts assigned by the experimenter to one trial, when he has in mind the results from a large number of possible trials; they are an inference from observed frequencies which provide him with a *theory* of frequency.† If this theory (which we are calling, for better or for worse, probability theory) is at all relevant, it must be conversely permissible from the theory to test the value of a probability from the results

* It is also true that the interpretation of a probability number involves the notion of an infinite class (see p. 10).

† Cf. my note on statistical probability (Bartlett, 1936).

of a large number of independent trials; that is, it is necessary to show theoretically that if we identify the members of our class or aggregate with the possible results of a trial, that the observed proportion of "successes" when the trial is repeated will tend to the corresponding probability number in the class. While the appropriate mathematical theorem is readily established, the nature of the convergence has given rise to some controversy. In order to examine this point, let us consider a simple class of two members, one of which corresponds to the "success" of a trial, with probability $\frac{1}{2}$. For n independent repetitions the probability-generating function for the total number of successes is

$$(\tfrac{1}{2} + \tfrac{1}{2}z)^n$$

or the moment-generating function

$$(\tfrac{1}{2} + \tfrac{1}{2}e^t)^n$$

The well-known binomial moments obtained from this function give*

$$E\left(\frac{r}{n}\right) = \tfrac{1}{2},$$

$$E\left(\frac{r}{n} - \frac{1}{2}\right)^2 = O\left(\frac{1}{n}\right),$$

$$E\left(\frac{r}{n} - \frac{1}{2}\right)^4 = O\left(\frac{1}{n^2}\right).$$

If P be the probability number for the class for which $|r/n - \frac{1}{2}| > \varepsilon$ in the aggregate of all eventualities represented by the distribution $(\frac{1}{2} + \frac{1}{2}z)^n$, then, since the contribution to the last two moments above of any term $|r/n - \frac{1}{2}|$ in this class is diminished if we substitute ε, and the contribution of $|r/n - \frac{1}{2}|$ outside the class is diminished if we substitute zero, we obtain:

$$P\varepsilon^2 \leqslant O\left(\frac{1}{n}\right)$$

$$P\varepsilon^4 \leqslant O\left(\frac{1}{n^2}\right)$$

The first inequality yields Bernoulli's theorem on large numbers. From the second, which is more relevant for our purpose, we find that, when $\varepsilon = \lambda n^{-\frac{1}{4}}$, the probability of $|r/n - \frac{1}{2}| > \lambda n^{-\frac{1}{4}}$ for at least one $n > n_1$, is of order

$$\sum_{n=n_1}^{\infty} n^{\frac{1}{4}}\left(\frac{1}{n^2}\right) = o(1),$$

* Their values are considered for n large. The symbo. $O(x)$ implies "order of magnitude of x," while $o(x)$ implies "smaller order of magnitude than x."

or since $\varepsilon \longrightarrow 0$ with $1/n$, the probability that if r/n were plotted against n, the graph would for $n > n_1$ be enclosed by a band, centre $\frac{1}{2}$, of width converging to 0 as n increases, tends to one as n_1 increases,—in mathematical terms,

$$P(\lim_{n \to \infty} r/n = \tfrac{1}{2}) = 1.$$

This result * raises a point which is often slurred over. We are trying to see whether we can consistently relate the probability numbers in our theory with hypothetical frequencies, for if we cannot do this, we can hardly claim that our theory is relevant. It is not sufficient to say vaguely that frequencies are graduated by probabilities, without discovering the exact relations that our theory predicts. On the other hand, we cannot baldly postulate (as is sometimes done) that relative frequencies do tend to limits in our theory, without becoming involved in contradiction, for all that we have established is that if we identify the elements of an aggregate with events, then $P(\lim_{n \to \infty} r/n = \frac{1}{2}) = 1$, a statement still involving a probability. An alternative technique in which r/n does tend to a limit in the classical mathematical sense has been attempted by von Mises, who has pointed out this difficulty in the usual theory, but even if the mathematical validity of von Mises' theory were established, an assumption about which critics are doubtful, I should not agree that the current theory is unsatisfactory. If we recognize that we are testing the consistency of regarding our probability numbers as the limits of frequency ratios, it is natural to allow the postulate that we shall not be able in practice to distinguish probability 1 from certainty, this equivalence being extended from the finite aggregates we first considered to the infinite secondary aggregates considered in connection with our limits. Logically, we can conceive the possibility that the event *might* occur at every trial, and our theory recognizes this by the formal difference between $P(\lim_{n \to \infty} r/n = \frac{1}{2}) = 1$ and $\lim_{n \to \infty} r/n = \frac{1}{2}$; on the other hand, any difficulty in interpreting the former proposition has disappeared, for on our postulate the two propositions will be observationally indistinguishable. Thus, while it is agreed with von Mises that the interpretation of probability should be a statistical or frequency one, it is denied that the orthodox theory is inadequate to provide this interpretation.

Of course from the practical end the point is trivial, and is in the nature of " hair-splitting." The fact that probability is an

* The derivation given above of this result should be compared with the proof given by Fieller (1936).

idealization related to an infinite number of trials is bound to create some theoretical difficulty, for infinity cannot occur outside mathematics; but no one queries the value of the concept of velocity, an idealization relating to infinitesimals, which are also unknown outside mathematics. In science it is not familiarity which breeds contempt.

It should be noted that as n tends to infinity the *random sample* of n trials tends to that theoretical concept of the statistician known as the *infinite hypothetical population.* The theoretical Law of Large Numbers we have just established shows that it is mathematically justifiable in our theory to regard as equivalent a probability distribution and a very large, or in the limit an infinite, set of sample values. Of course we have only proved this for a very special case, but corresponding theorems exist for more complicated distributions, and enable us to associate ideal populations with the empirical distributions we actually observe.

At this stage I propose to leave the mathematical groundwork, although I have only brought it to the stage when it would begin to be of interest to the statistician, who is quite justified in taking all this theory of statistical probability for granted, and proceeding at once to the further developments with which he is most concerned. It is these developments and applications which have really created the subject of mathematical statistics, but it is just as well for us to be clear about the basis of our theory, so that we do not err in the interpretation of our theoretical results. I hope I need not remind the reader that nothing has been proved about actual frequencies. The usefulness of all these theoretical idealizations (probability numbers, independent events, random samples, populations) can only be a matter of experience in any particular field, and there can be no evidence that our theory is ever more than approximately applicable. Its usefulness largely depends in any instance on how good the approximation is.

Comments on the theory of statistical inference.—There is not much further that I wish to refer to in this section, for the development of mathematical statistics, at least in relation to statistical methods, has become familiar to many readers, and much of it does not call for general comment. But the main body of the theory will include, among other things, some general results made use of by the statistician when concerned with problems of inference, the part of the theory associated with estimation and tests of significance; so that I might emphasize that with the approach adopted here there is no place for such topics as inverse probability, if by this term it is implied that the statistician, in contrast with any other scientist, should be obliged to assess *numerical* " probabilities " of his hypo-

theses.* He will, I hope, prefer to deal with his data in the orthodox statistical manner, making use, for example, of Fisher's theory of estimation (see Fisher, 1935), but always regarding such things as maximum likelihood estimates, sufficient statistics or fiducial intervals in the light of their statistical properties. This viewpoint has been used more generally by Neyman and Pearson (1933) in their original approach to the theory of statistical tests; but while a good deal of recent theoretical research has resulted in the more detailed elucidation of sampling properties, I think the account of this research is misleading if it fails to indicate the relevance of the statistical functions which Fisher found it necessary to consider. I would like to mention an important contribution to small-sample theory which seems yet again to bear out this contention.†

In large samples estimates in general tend to be normally distributed; of this class, the one with minimum variance is known to be the maximum likelihood estimate.‡ In small samples it is less easy to single out an individual estimate as superior to others, though the maximum likelihood equation will provide us with a sufficient statistic if one exists. Aitken and Silverstone begin from the empirical standpoint of the usefulness of an unbiased estimate with minimum variance. While it should be noted that minimum variance is a more arbitrary criterion directly we leave normal

* It was intimated in a footnote towards the end of Section I that the association, with statistical theory, of "probabilities" which are merely someone's degrees of confidence or belief has been more historical than logical, for the concept of statistical probability is quite separate from any mathematical theory of belief (represented in my diagram as "probability and symbolic logic"). It has been claimed that the latter theory includes statistical probability; but we may justifiably refuse to assign any scientific significance to the theory until the beliefs numerically represented in it correspond to something observable. We might retort that if they are to stand for anything more than one particular person's beliefs (and it is contrary to scientific method to record not what is common, but what is peculiar, to one's experiences), the only basis possible is a statistical one. Such an interpretation would, however, deprive the theory of any claim to a fundamental position, for it would itself have to be investigated by statistical methods. F. Y. Edgeworth compared the idea of probability as belief with the economic concept of utility; but similarly it might be argued that one person's indifference curves are of less relevance to an economist concerned with actual events than the representative curves for a group.

The mathematical statistician who refuses to make explicit use of a mathematical theory of belief has sometimes been accused of ignoring the gap between his statistical theory and its application. But statisticians are (or should be) not only aware of the gap, but of its varying width in different circumstances; and they do not consider it very helpful to camouflage the gap they have to jump by filling it with more mathematical symbols, apparently of the same nature as the mathematical ground they are leaving, but on further examination devoid of any solid foundation in fact.

† I am indebted to Dr. A. C. Aitken for permission to refer to this work, which is contained in a Ph.D. thesis by a former student of his, H. Silverstone, and not yet published.

‡ In connection with this result of Fisher's, due credit should be given to the work of Edgeworth (1909), cf. Bowley (1928), p. 26.

distributions, an estimate with these properties admittedly commands respect; and least squares estimates are often thus justified. Examining the problem in general, Silverstone has established that an estimate T of a parameter θ for which $E(T) = \theta$, and $E(T - \theta)^2$ is a minimum only exists if

$$\frac{\partial L}{\partial \theta} = \frac{2}{\lambda}(\theta - T)$$

where L is the logarithm of the likelihood function, and λ is independent of the sample observations. It follows that the solution $\theta = T$ is identical with the maximum likelihood solution; moreover, since $\frac{\partial L}{\partial \theta}$ is a function of T only (apart from θ), T must also be a sufficient statistic. The variance of the estimate is given exactly by the reciprocal of Fisher's "quantity of information," $I = E\left(\frac{\partial L}{\partial \theta}\right)^2$.

III. *Applicability*

We have seen that the practical statistician will hope to apply theory wherever he has reason to believe that a complex of causes is operating to produce an unavoidable element of chance variation in his material. He proceeds to shake his mass of data in a paraphernalia of sieves and similar devices, which remove much of the residue and throw up bigger lumps which he can remove for closer inspection. If his data already consist merely of a few unbreakable fragments, sieving operations will, of course, be a pathetic waste of time; on the other hand, the most painstaking labour may be necessary before he rescues a few grains of gold.

As for the source of data which the statistician can handle by these methods, this may be anywhere, but it is a sobering thought that he is most confident in his apparatus when he has himself put the chance variation into his data. I refer to the principle of randomization, which has so far met with most acceptance in the design of agricultural experiments, in relation to Fisher's analysis of variance technique. A valuable discipline for statistical experimenters in general is to ask themselves whether randomization is at all possible in their own experiments, and if not, how far its absence is likely to introduce unknown disturbing factors. Randomization is not a panacea, but its value may be perceived if we consider the attitude of the "doubting Thomas" who refuses to believe any conclusions which the mathematical statistician makes. If the significance level in relation to some investigated effect is overwhelmingly great, he will have the easy way of escape of asserting that the

theoretical assumptions may be incorrect, and in so far as many unforeseen factors can be affecting the data, his criticism is a valid one. If, however, his criticism is transformed into the assertion that the laws of chance are not even approximately obeyed by specially designed machines for obtaining random numbers, he is placed in more of a dilemma, and may turn back to the belief that the, say, one in a million chance has really turned up. It will, once in a million times, so that he cannot indulge too many beliefs of this kind.

Thus it is well known that in nutrition experiments on the feeding of milk to children the significance of the results has been rendered doubtful, or even completely vitiated, by the absence of appropriate randomization, groups which were claimed to be comparable diverging from each other in spite ostensibly of receiving the same treatment, or, as in the large-scale Lanarkshire experiment, groups receiving different treatments showing the significant initial differences which revealed bias in the selection of the groups.

In card experiments with supposed telepathic or other paranormal subjects, most of us are probably still a little chary about attributing statistically significant deviations to extra-sensory perception. I trust that the possible occurrence of correlation between subject and investigator, due to bad shuffling of the cards or similar causes, is adequately guarded against, but this source of error at least would be automatically eliminated if the cards were deliberately stacked in a pre-determined random order unknown to the subject.

Of course there are very many statistical fields of enquiry where we are obliged to take our material as we find it, and our "doubting Thomas" who refuses to trust us has an easier (but often a necessary) task. Among these fields where chance variation occurs naturally, statistical theory is probably most exactly applicable in physics. As a matter of expediency we may agree to segregate this field for the specialized attention of the physicists, owing to its highly technical character, but mathematical statisticians should not remain oblivious of its *statistical* technique. I have already indicated that the use of partition functions in statistical mechanics can easily be identified with the use of generating functions in statistics. The reaction of the physicists when accused (not by us) of assuming equal *a priori* probabilities has sometimes been to substitute the word *weight* for probability. This is useful in stressing that the probabilities we both use are properties of the systems under discussion; on the other hand, their manœuvre is rather ostrich-like if it leaves them the impression that they are absolved from further explanation, so let me note that the same mathematical framework developed in Section II is at the bottom of their theory as

well as of ours.* I should add that I am referring to the statistical nature of physical aggregates, not to the use of statistical ideas in quantum mechanics. Eventually these two things will fuse, but at present the technique of quantum mechanics is still obscure.†

Turning back to the use of statistical theory in the framing of methods, a use which Yule and Kendall had in mind when they defined statistical theory in these terms, we are still left with several fields where the natural variability is of a sufficiently chance character for statistical theory to be usefully applied. The approach to comparatively less well-surveyed territory will, however, render us cautious. In applications to manufacturing problems in industry, our requirements are only satisfied if the process is mainly under what has been termed "statistical control," apart from the occasional discrepancies which our methods will assist in isolating. But while the value of statistical theory in such fields is a matter of experience, the methods are usually straightforward. In both psychology and economics, however, the statistical methods themselves have been the subject of controversy, and to conclude this section I have added some general comments which briefly indicate the present position of mathematical statistics in these two fields of research.

Mathematical statistics and psychology.—It need hardly be emphasized that the immense value of statistics in biological research, which includes the study of observable physical characteristics in man, will have some counterpart in the statistical study of mental phenomena. This was accepted long ago by psychologists like Spearman, and its recognition is indicated, for example, by the comparatively recent discussion on the use of statistics in psychology, arranged by the Royal Society (see Myers *et al*, 1938).

Observations which may be related to mental variation, such as the number of accidents a person is responsible for in some industry, or an examinee's mark when given a so-called "mental test," will be amenable to analysis by an appropriate statistical method. It is advisable to mention "appropriate," for clearly a method based on normal frequency distributions might not be very appropriate

* The method introduced by Gibbs of regarding the actual system as a random member (or "sample") from an *ensemble* (or "population") of such systems has tended to worry some physicists, but its similarity to the ideas of the statistician will be obvious.

† Mathematically, much use is made of vectors whose components do not, as in Section II, immediately represent probability distributions, but for which the *squares* of the lengths of their components do. This device has no apparent advantage elsewhere in statistical theory; its use by the physicist appears related to the possibility of introducing a further degree of freedom (the phase factor) for each component by making it complex—a step which seems necessary from physical considerations but the precise function of which has largely to be left unexplained.

when applied to a J-shaped histogram of accident frequencies, but this is a technical problem which does not affect the general issue. But while such statistical analysis has often proved directly useful, the psychologist is always ambitious to be able to depart more and more from mere description, and synthesize mental concepts with a connotation wider than the particular facts from which they were derived. Such synthesis is, of course, one of the hall-marks of scientific progress (statistical probability itself is a synthetic concept), but it should be remembered that understanding grows long after a concept is first introduced. Thus temperature is now recognized to be essentially a statistical concept. Mass and energy, at first considered independent, have been related. It will not be surprising, then, if it is found necessary to criticize the interpretation or value assigned to new concepts in psychology.

Psychologists have for some time disagreed among themselves on the value of Spearman's factor theory of ability,* and the statistics associated with this theory have come in for their share of the controversy. The statistician has sometimes felt like the scientist who is blamed for the use to which his discoveries have been put in time of war. As an onlooker, I would suggest that psychologists may have been in too much of a hurry to build up their concepts; the statistics of the factor theory, for example, tended to concentrate for a long time on the proof or disproof of the theory, with the consequent danger of ignoring other lines of approach, further preliminary experimental work planned in relation to the further *analysis* of ability and its manifestations. The point stressed by Godfrey Thomson that a patternless network of components would tend to give statistical agreement with Spearman's theory goes deeper than an empirical testing of the theory and raises the question whether this network is sufficiently uniform statistically to be described in terms of a general intelligence level (or perhaps of a few group factors); this question can hardly be answered completely until mental performance has been subjected to the same type of analysis as is applicable to quantitative measurement in biology, where, for example, disentangling the effects of environment, genetic variation, or homogamy, constitutes a difficult but natural object of enquiry.† Pure psychologists who appreciate the value of statistical methods of investigation and analysis already established in the biological field remember, when considering the probable value of factor technique, the relevance of researches of this kind.

* In this theory general intelligence is represented by a single mathematical factor. A recent discussion among psychologists on factor theories of ability is reported in the *British Journal of Psychology* (Thomson *et al.*, 1939).

† This type of criticism has been made by R. C. Tryon and Bronson Price; see, for example, Price (1937).

Mathematical statistics and economics.—The relation of mathematical statistics to economics is too involved a problem for me to do more than remind the reader of it. With psychology the question was whether the right statistical methods had always been used; with economic research the statistician in his more pessimistic moments may ask himself where there are any " right " statistical methods at all. The nature of his material is such that the gap I referred to between theory and application is as wide as is likely to be found anywhere; the mathematical ground on which the statistician ventures may have its own æsthetic attractions, but if he strays too far away, his contact with the real world of fact may altogether disappear. Incidentally, relevant and unambiguous statistical facts are difficult enough to obtain for the problem of collection itself to be the immediate task which the economic statistician always has to face. Because I am considering the possible use of mathematical statistics, I shall ignore, but not forget, this vital initial problem.

There are obviously " static " problems in economic or social research where the application of sampling theory is an advantage—for example, in assessing the efficiency and validity of actual sampling techniques. Controversy centres more on the " dynamic " problems of economics, in particular on what I shall call * research on the " trade cycle." The business man especially is rather sceptical of the value to him both of abstract theorizing by economists and of any empirical investigations by statisticians going further than simple tabulations. However, the business man is not necessarily the arbiter we should choose of the ultimate value of these researches —on the statistical side the correlations established between marriage or birth-rates and the fluctuations of the trade cycle by investigators like Hooker and Yule are a sufficient indication of their potential value. At the same time, Yule has warned us more than once of the caution necessary in correlating time-series; even if we concentrate on deviations from long-term trends, the cyclical fluctuations remaining are not the independent observations which the mathematical statistician likes to postulate in other fields. Koopmans (1937) has suggested that for economic time-series we should employ regression analysis, allowing any postulate of independence to recede to random residuals which merely disturb the correspondence between one series and another. This reduces our problem to a quantitative assessment of the relation between variables already given us by the economist. Investigations of this kind have been attempted by Tinbergen (1939). Acting in the guise of our " doubting Thomas," Keynes (1939) has set out with gusto the host of

* With due deference to Dr. E. C. Snow (1938).

statistical difficulties that still remain—the validity of the data, the measurability of all the relevant variables, the linearity of the relations, the absence of specified time-lags, the stability of the series. He wonders whether the computation is not so apt to get out of hand that " it becomes like those puzzles for children where you write down your age, multiply, add this and that, subtract something else, and eventually end up with the number of the Beast in Revelation." While recognizing the relevance of much of Keynes's criticism, we may still query the value of economic theories (even those of Keynes) not checked by statistical investigations. The most convincing *a priori* argument can hardly predict the magnitude of the relation of one variable to another. If it does turn out that the relationships to be examined are too involved to permit of adequate statistical analysis of the type used by Tinbergen, the economist will have to be asked for a more explicit framing of his theory. The ideal situation is one where statistical technique assists in the quantitative assessment of the coefficients in a simplified set of equations which contain intrinsically the oscillatory tendencies in the economic system. Whether we shall be able to narrow the admitted gap between this ideal situation and fact by treating our residual chance variation not as independent of the oscillations, but as affecting the course of the oscillations themselves, is a line of approach still in an embryonic stage (see Wold (1938)). We have always to face the possibility that the arbitrary shocks to which the actual economic world is subject are too large for statistical theory to be useful, but the importance of the problem is such that I should say attempts at its solution are more worth while than other activities of more questionable value to which human beings are prone.

If I leave the special problem of time-series and statistical technique, and consider the broader rôle of statistical theory in social and economic science, I shall begin to encroach on the educational ground which Dr. Wishart has already covered. For theory will always have its value as a mental discipline for the statistician, who acquires the habit of setting facts in any field against his random or chance framework. In Edgeworth's words,* " beyond the isolated instances in which the theory of deviations is applied in social statistics with the same strictness and cogency as in physics, there is a wide zone of cases in which the abstract theory is of use as giving us some idea of the value to be attached to statistical results." Because the mathematical framework is of our own construction, that does not make it any the less a valuable gauge.

* Edgeworth (1925), p. 287.

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Discussion on Dr. Bartlett's Paper.

[Proofs of the Paper were sent to Fellows and others known to be interested in the subject, and the comments received, with Dr. Bartlett's reply, are given below.]

Dr. Wishart: In commenting on Dr. Bartlett's paper I would begin by congratulating him on the excellence of his presentation of a difficult subject. By comparison with his choice my own recent task in dealing with the teaching of statistics was an easy one. But there are points of similarity in the two papers in that, in elaborating my theme, I thought it desirable to follow up a section dealing with criticisms and suggestions relating to the teaching of the subject, with one in which the position of mathematical statistics was briefly mentioned. We are now indebted to Dr. Bartlett for debating the subject at length, and his review comes with all the greater authority from one who has in a remarkably short space of time not only worked steadily at clarifying and developing various aspects of the theory of statistics, but also achieved a position of distinction in the subject that few have in the past attained at his age.

I welcome the distinction made between *mathematical statistics* or, if you like, the *theory of statistics*, and *statistical mathematics*. Dr. A. C. Aitken has just published a short book under the last-named title, but while it is a useful, and indeed valuable, exposition of the mathematics underlying the subject of statistics, and is perhaps the first text-book of the kind that can be put confidently into the hands of mathematical students who are being introduced to the subject, thus helping me over one difficulty to which I alluded in my paper, Dr. Aitken himself would be the last to claim that he had dealt adequately with the theory of statistics.

Readers will no doubt give careful attention to Dr. Bartlett's schematic picture, and possibly it will disarm criticism from certain quarters if it is suggested that the black rules serve to emphasise the aspects of the subject that he is dealing with, and that their central position is a matter of artistic presentation rather than a claim that some of the other listed aspects are of a subsidiary character. In one respect, however, there is an inference that statistical concepts and distributional laws are reached through the classical theory of probability and its offshoot, the theory of error, while "probability and symbolic logic," another offshoot of the classical theory, does not reach the same goal. Readers of Dr. Jeffreys' book on the Theory of Probability will know that he derives significance tests from his treatment of probability as the logic of scientific inference, and these may be claimed at least to relate to statistical concepts. As is clear, however, from Dr. Bartlett's exposition of statistical probability, which should be studied with the care evidently devoted to its preparation, he is not prepared to admit that Dr. Jeffreys' approach to significance tests should displace the current methods of the statistician, methods which command a very wide measure of acceptance. Readers of Dr. Jeffreys' book now have an opportunity of studying his objections to current statistical theory in a more convenient way than was

possible when his views were being formulated in a series of papers. But it is also important, in my view, that the present time should see a careful formulation of the viewpoint of the statistician, and this Dr. Bartlett has done very well. For myself I would dispute the statement in the preface of Dr. Jeffreys' book that current statistical theory " limits its scope at the outset in such a way that it cannot state the questions asked, or the answers to them, within the language that it provides for itself, and must either appeal to a feature of ordinary language that it has declared to be meaningless, or else produce arguments within its own language that will not bear inspection." Dr. Jeffreys may claim that his methods are the only ones which proceed directly to the question asked, that of making inferences regarding the properties of a population from knowledge of the sample, whereas the statistician proceeds by way of a series of circumlocutions. But the statistician is not prepared to agree on this point, not because he is unwilling to study alternatives to his own method of approach, but because he disputes the premisses of the opposing viewpoint at its very outset. I trust that by saying this I am not accused of fanning the flames of controversy, but I should like to record my agreement with the views put forward by Dr. Bartlett in this very stimulating section of his paper.

Dr. Bartlett's section on "Applicability" will doubtless be read by Fellows who fear to follow him into the intricacies of his mathematical arguments on the subject of probability, with greater interest than the earlier sections. The references to physicists, psychologists and economists serve to show well that he is not only interested in theory, but also with its outcome in practice. In particular I have been interested in his statement that statistical theory is probably most exactly applicable in physics. It may be that some physicists are not too well aware of the statistical nature of techniques that have been applied in their field, but it is going rather too far to suggest, as Dr. Jeffreys does, that the present attitudes of most physicists and statisticians are diametrically opposed. If they are, then they ought not to be. At any rate, it is to be hoped that Dr. Bartlett's remarks will be studied by physicists, even if there is not a very convenient platform where statisticians and physicists can get together. The remarks on mathematical statistics and economics are likely to be of greater interest to Fellows of the Society. This is a side of the subject which has possibly not received as much attention as it deserves on the part of the mathematician, although a number of authorities have pointed out the difficulties of some of the problems that have to be faced. For this reason, I rather wish that Dr. Bartlett could have gone into greater detail, but he has already given us quite enough to think about. The subject is one which might well engage the exclusive attention of the Society at some future meeting.

Dr. J. O. Irwin: I should like, first, to comment on Section II, which is very important, and then to make a few, more general, remarks. On the conceptual side the definition of probabilities as numbers associated with " ideal objects " is entirely in accordance with the spirit of pure mathematics and should, suitably developed,

enable pure mathematicians to avoid the feeling that there is a lack of rigour in the foundations of the subject. The development, by the theory which proceeds from this definition, of the notion of conceptual frequencies only introduces the further postulate that we may, even in infinite aggregates, ignore for practical purposes the difference between a probability of unity and certainty. The conceptual frequency is the limit to which, in this sense of the theory of probability, " the proportion of successes " tends. So far there is no reference to the world of observation. We now provisionally assume a correspondence between conceptual and perceptual frequencies, just as is done with (say) length in physics. Any observational consequences of a theory involving probabilities can then be confirmed (or not) by examination of the data. The concept of probability is in no special position here, but is used in the same way as other scientific concepts.

The problem of classification of the sciences will always appeal strongly to enquiring minds. When reading, as a boy, J. Arthur Thomson's *Introduction to Science*, I came across Karl Pearson's suggested classification of the sciences. It was the first time I had ever heard his name. For me therefore that association will always have the strongest significance; this, however, is a personal reflection inspired by the diagram on p. 4.

Dr. Bartlett is to be congratulated on the diagram. In a remarkably lucid way, it enables us to keep in mind the ramifications and interrelations of the different branches of our subject. To me, the question of the relation between modern physical ideas and mathematical statistics as we, in our Society, understand the term seems full of fascination. Unfortunately I have not the knowledge of quantum mechanics to pursue the subject. Dr. Bartlett has, and it is to be hoped that when the urgency of war problems is finally passed, he will return to it and clear up the obscurities. I understand there are some forty mathematical students now taking the statistics course at Cambridge, and it is impossible to over-estimate the importance of the developments which may ultimately flow from the fact that a body of men will go into the world, properly trained in their subject, with a clear idea of the connexions between its component parts, before they enter on the inevitable specialization which practical work ultimately demands.

THE PRESIDENT: When speaking on Dr. Wishart's paper (*Journal*, 1931, p. 561) I distinguished three stages in the teaching of statistics —namely, arithmetical, mathematical without probability, application of the concept of probability. Dr. Bartlett confines himself almost exclusively to the third stage, only saying that interpolation and graduation " may be reasonably included under mathematical statistics." In my opinion the second stage is more important than the third in relation to economic and kindred problems. Mr. Yule devotes Part II of his *Introduction* to the Theory of Statistics; Professor R. Frisch's books (*Methods of Measuring Marginal Utility* and *Confluent Analysis*) are wholly or mainly independent of ideas of probability, as is the study by Dr. Tinbergen to which Dr. Bartlett

refers. Mr. Yule develops averages, measures of dispersion and the use of frequency curves generally, and the whole apparatus of partial correlation, in his Part II. Mr. Allen and I did not use probability in our *Family Expenditure.* Many other references could be given.

In descriptive economics the field of observation is not generally regarded as a sample from an infinite world, but as complete in itself. One of the most common mathematical processes used is a so-called regression equation :—

$$y = ax + bt + c + v,$$

where y is an observation supposed to be related to x (another observation) at date t, a, b, c are constants, and v is a residual, by which y differs at any one date from the sum of the first three terms of the equation. a, b, c are chosen empirically so as to give a good fit between the computed and observed values of y. The commonest method is so to choose them that $S(v^2)$ is the least possible. Though this method originated from the normal curve of error, it need not be regarded as depending on it. We can say, for example, that we will choose a plane that passes through the averages of x and y and makes $S(v) = 0 = S(xv) = S(tv)$, equations which do not involve the idea of probability. Or we can use the double median method, or define goodness of fit by any other way we please. Even the correlation coefficient is not necessarily used.

Generally, in summarizing a frequency group by an algebraic expression, no idea of probability is involved, unless the group is regarded as a sample. That the terms mean and standard deviations have replaced the term "probable error" illustrates the distinction.

Dr. Bartlett's paper, as Dr. Wishart's, appears to me to be useful and interesting. But whereas Dr. Wishart's tended to limit statistics to mathematical statistics, the second limits mathematical statistics further—namely, to the field in which probability is applied, or, in other words, to the mathematics of sampling. I do not underrate the importance of this field, in which I have worked for many years; but I do not wish Fellows of the Society to be under the impression that mathematics has no other use, and that before applying it they must master the logical subtleties as well as the analytical difficulties of probability.

Mr. M. G. Kendall : Dr. Bartlett has done a useful service in taking stock of the present position of mathematical statistics. On the whole I agree with his appraisal of the situation; but in the space available to me on the present occasion I will leave the numerous points of agreement and deal only with one or two major questions towards which my approach would differ from his. I hope this will not give my comments too critical an appearance.

Mr. Udny Yule defined the theory of statistics as "the exposition of methods adapted to the elucidation of quantitative data affected by a multiplicity of causes." If we wish to avoid the notion of causality we can amend the definition to refer to "quantitative

data occurring in the counting or measurement of natural phenomena." Since the theory deals essentially with the manipulation and analysis of numerical material, it is all mathematical in one sense, but certain branches have been singled out for the attachment of the adjective "mathematical," not because they are in essence more mathematical than others, but because they use an advanced mathematical technique such as is not usually acquired in a liberal education. I do not regard "mathematical statistics" as in any way an appropriate synonym for "the theory of statistics." The fundamental ideas of the theory do not appear anywhere in mathematics; for example, the idea of statistical dependence stands almost antithetically to the functional dependence of mathematics. Mathematical statistics might be defined as that part of the theory of statistics which depends for the conciseness of its reasoning on mathematical terminology.

My principal difference of viewpoint concerns the place of the theory of probability in statistics. Dr. Bartlett and I differ as to the proper foundations of such a theory, but that is not altogether relevant. Our difference here goes deeper and is ultimately, I think, one of *Weltanschauung*. Dr. Bartlett regards the observational data of statistics as "arising from a physical background of chance," by which I understand him to conceive of any population of statistical units as emanating from a parent population according to various laws including the laws of probability. (The parent may be hypothetical, but that only strengthens my objection.) To such a conception direct probability is, of course, fundamental. My difficulty lies in following Dr. Bartlett to this parent, to the grandparent which presumably is to be imagined as having generated it, and so on up the genealogical tree, getting further from reality at every stage. I myself would regard the data as primary and the laws of probability in Dr. Bartlett's sense (namely the properties of aggregates or sequences of events) as to be established from observation by inductive processes. I hope I have not attributed to Dr. Bartlett views which he does not possess. Much depends on what one means by probability, and perhaps there is not much between us, after all. But I resist the conclusion that mathematical statistics rests on the theory of probability, unless probability is to be much extended beyond its present limits, or unless it is defined as including the measurement of uncertain attitudes of mind; in which case it becomes part of scientific inference, to be used in the theory of statistics, but no more constituting its foundations than mathematics.

It is not clear to me from Dr. Bartlett's paper where he makes room for inductive inference, and his remarks on Likelihood may illustrate my difficulty. There are some irrelevancies to be dismissed here. It is true that the Principle of Maximum Likelihood gives sufficient statistics when they exist; and that it provides estimates with minimum sampling variance. But these are not theoretical justifications of the method, which must stand or fall by other criteria; either, does it give results in the long run which are right more frequently than those given by other methods? or, is it the most reasonable method to adopt? The answer to the first question is, I think, that

nobody knows—I am not even sure that the question has a meaning. The answer to the second is that it is a matter of taste. Apparently, as Dr. Bartlett rejects inverse probability, he must make some new postulate before he can begin to make any inference from sample to population at all. Does he recommend the adoption of the Principle of Maximum Likelihood as a new postulate, or on intuitive grounds, or on utilitarian grounds? And is the principle just for statistical purposes alone, or is it part of Dr. Bartlett's theory of scientific inference?

DR. HAROLD JEFFREYS: Dr. Bartlett has kindly shown me a copy of his paper. With regard to his remarks on degrees of belief, the suggestion that the actual degrees of belief of individuals are unique and capable of numerical assessment does not correspond to my treatment of the theory of probability, nor, I think, to those of Keynes and F. P. Ramsey. I have hitherto used the expression "degree of knowledge"; in my *Theory of Probability* I use "reasonable degree of confidence"—thereby indicating that there are unreasonable ones also. In the most elementary case, if we have no information to decide between alternatives, it is unreasonable to have any more confidence in one than in another, and the probabilities in this sense are equal. The assessments are not a matter of personal belief, but, so far as they differ from those used to express ignorance, the differences are entirely due to the use of observational evidence. If the rules of the theory are followed, anybody will get the same answer given the same evidence. The reason for a formal theory is precisely that without one there is not always general agreement between different persons on the same evidence; an opinion can be maintained in spite of all evidence against it. But differences can be over-emphasized, because people tend to talk more about matters where they differ than where they agree. We do not discuss whether bread is poisonous. There is enough agreement between individuals in everyday and scientific processes of learning to suggest formal rules that will represent the overwhelming majority of actual decisions. An explicit statement of the rules makes it possible to work out the consequences with the minimum number of assumptions. It cannot compel any person to arrange his degrees of belief in the order that the theory indicates; but it can enable other people, and possibly even himself, to see whether on a given occasion he is arguing in the same way as he and other people do on most occasions.

As to whether the rules correspond to actual practice in most cases, it should be enough to point out that they do lead to methods of estimation and significance tests that often agree exactly, and seldom differ seriously, from those in current statistical use. But they are easier to apply and involve fewer postulates—when those used without statement in current statistics are stated.

It is, however, impossible to prove them by experience. Data are not the same as the method of analysis; and a method of analysis applied to any data cannot establish its own validity, for the argument would be circular. They are suggested, not proved, by current

practice. The criteria for the fundamental rules must be quite different from those for an empirical proposition. They are: explicit statement of hypotheses, freedom from self-contradiction, ability to test any empirical hypothesis by reference to data, and impossibility of inferring any empirical proposition from the formal rules alone without the use of empirical data. Apart from these principles I cannot attach any meaning to a statement that the rules are "right." The question is whether they work, and that has been adequately answered in the affirmative. But they can no more be proved than the principle of the syllogism can; nor, of course, can any alternative rules.

Dr. R. C. Geary: The line of demarcation between mathematical statistics (*i.e.* mathematical methods which are used explicitly for statistical purposes) and the mathematical theory of probability is fairly clearly defined. The latter will include, for instance, Cantelli's Uniform Law of Large Numbers (the binomial case of which is given by Dr. Bartlett) and Liapounoff's theorem relating to the conditions for the asymptotic approach to normality of the frequency distribution of the arithmetic mean. These theorems are, however, implicit in some of the most familiar statistical operations. Even the Bienaymé-Tchebycheff theorem and its extensions by Kolmogoroff, Cantelli, Medolaghi, Camp and Fréchet amongst others, must be regarded as part of probability rather than sampling theory, since the inequalities, in stated general conditions regarding the universe, are not sufficiently favourable for statistical application; the problem of deriving inequalities of this type which could be used in practical statistics, with assumptions regarding the universe (which will be unknown in most applications) as unrestrictive as possible, merits the continued attention of mathematical statisticians. Dr. Bartlett does well to stress the importance of pure theory from the point of view of the statistician; even if he does not use it in his practical work, it is part of the faith that is in him.

There will probably be general acceptance of Dr. Bartlett's views on the application of mathematics to economics. Here there is possibly more scope for development than in any other branch of mathematical statistics. The extent of co-operation between the economic theorist, the collecting and compiling agency (usually, but not exclusively, governmental) and the mathematical statistician still leaves much to be desired. Economists have been somewhat prone to theorise *in vacuo*; showing too little disposition to submit their theories to the quantitative test, which, no doubt, partly explains the multiplicity of schools of economic thought. The compiler of statistics has gone his own way collecting such statistics of international trade, production, population, unemployment, prices, wages, and so on, as are required by the public and for administrative purposes, without direct reference to the requirements of economic theory. The gap between economic theory and statistical practice has narrowed in recent years; the League of Nations (through its Committee of Statistical Experts) and the International Statistical Institute have worked unremittingly to this end. As

suitable statistics become available, economic laws will tend to be based more and more on statistics, and, concomitantly, notable developments in mathematical statistics (in particular in the theory of regression) may be anticipated.

It is not fanciful to suggest that the economic laws of the future may take the form of stochastic equations connecting the various statistical series. The closer the series approach the requirements of the economic theorists, the more the effects of what Professor Ragnar Frisch has called the "disturbances" should diminish (and the systematic effects increase) and the more the laws should be functional (and less stochastic) in character. Perhaps the most encouraging feature of Professor Tinbergen's essay in determining linear regressions between various statistical series in time is the improvement in the relations after 1919, compared with the period prior to 1914 when the statistics were generally less suitable for the purpose required.

MR. RICHARD STONE: Dr. Bartlett's exposition is of great interest, especially his section on the application of mathematical statistics to economics. The technique of regression analysis and the formulation of economic relationships by means of sets of equations have, as Dr. Bartlett points out, recently received much attention both in their pure and applied aspects, but the application of this branch of technique seems occasionally to rest on insecure logical foundation.

Although regression analysis is used, economics is concerned with fuctional relationship and not with stochastic association (*i.e.*, where, if the value of one variable is given, the other is not known exactly but may take different values with given probabilities). This can immediately be seen from the simple example of a demand curve. There are not really two demand curves depending on which variable we choose to regard as independent, nor is the (partial) regression obtained by taking the amount demanded as dependent on price the true one. The position is quite different from the familiar case of the heights of fathers and sons.

If this is accepted, then the false appearance of stochastic association may be due to one of two things. Either there are inaccuracies in the data or other errors of a small and random nature due to the interaction of a number of small and mutually conflicting influences acting on the dependent variable. Or else there may be omissions of one or more important determining variables which have a systematic influence on the dependent variable.

Clearly there is nothing much to be done about even the second category of the first factor. No doubt the price of bicycles, and indeed everything else, has some effect on the demand for cheese, just as the gravitational pull of the stars has some effect on the motion of a pendulum on the earth. In neither case, however, do these remote causes amount to anything perceptible at the degree of accuracy in which we are interested, and accordingly they may be ignored.

The second class, on the other hand, is in a different position. All influences which economic theory suggests are likely to exercise a perceptible influence on the dependent variable must be introduced,

with the exception of those which involve multicollinearity in the set. In doing this we shall have to use some such technique as that suggested by Professor Frisch, at any rate in the more involved cases. If we do so, and if our variables are accurately measured, we shall reach the conclusion either that our theory is compatible with the facts or that it is not. Of course if our knowledge of the variables is very inaccurate, it will exercise a disturbing effect and prevent our reaching a final conclusion. This brings up the problem of the use of estimates (with unknown error) rather than more or less exact measurements, a difficulty which has received less attention than it deserves from economic statisticians.

Further, as is implied in the above, the determining variables must be chosen rationally, especially in view of the danger of multi-collinearity. No doubt the seasonality in the hours of sunlight at seaside resorts could be explained by suitably lagged variations in the cost of living. This explanation is so silly that probably no one would feel inclined to put it forward, and it has been introduced simply to show that while an appeal to the facts may be able to knock a theory down, it cannot substantiate it. Whether this would cease to be true if our system of equations was wide enough to encompass the whole of one aspect of the economic system (*e.g.*, the mechanism which generates fluctuations in the national income) remains to be seen. Presumably it would, since otherwise contradictory theories could each offer a perfect explanation of observed phenomena, which is absurd.

Until the analysis has reached a stage at which the determining variables form a rational and, to the degree of accuracy with which we are concerned, a complete set, there seems to be considerable doubt as to the value of the more elaborate forms of regression coefficients which have made their appearance in recent years. If our analysis excludes some important determining variable there does not seem to be much reason for supposing that, say, the geometric mean of the two elementary regressions will be any nearer the true regression than will the elementary regressions themselves. It is true that in the state of our ignorance it may seem the best guess, but our ignorance is so great if the two elementary regressions are at all far apart that we ought to push the analysis further.

If this is at all a true statement of the problem in economic statistics which is at present arousing most interest, it is certainly a far cry from the position in other branches of the subject, as exemplified by the neatness and precision of, say, Professor Fisher's *Design of Experiments*. However, great strides have been made of recent years, especially in the works of Professor Frisch and Dr. Koopmans and in the interesting distinction drawn by Dr. Wold between definite and modifiable units. It is to be hoped that these researches will continue and that other mathematicians will turn their attention to this branch of statistics.

Dr. Bartlett, in reply: Before I reply to those comments that are in the nature of criticisms, I wish to thank all those who have contributed to the discussion.

The main point of my rather fragmentary section on economics was that, since statistical methods form the link between statistical fact and pure economic theory, it is as well to examine their basis. I am sceptical of the permanence of methods with no theoretical foundation, however much I agree with the President that many statisticians may not need to worry over the " logical subtleties " or " analytical difficulties of probability theory." If my motor-car develops engine-trouble, my immediate concern is to find an experienced mechanic, not an expert in thermodynamics.

I do not like Mr. Kendall's amendment of Yule's definition of the theory of statistics; it might apply to all scientific theory of a quantitative character. If I am not to be allowed the convenient approximate identification of mathematical statistics and statistical theory, I should include graduation formulæ of all kinds under mathematical statistics, and still adhere to my view that statistical theory should rest on the foundations I have set out. That does not imply that all statistical data must conform to this theory, or that useful arithmetical operations cannot be made on them. If a cost-of-living index records the actual change in cost of living of a family whose items of expenditure always correspond to those of the index, that is pure arithmetic, but if it is also intended to indicate the cost of living for other families, it raises sampling ideas which have statistical theory as their basis.

My references to inverse probability were rather guarded, for I was not anxious to set in motion a controversial avalanche, but I might explain that I do not regard the methods and formulæ of statistics as more than an aid to our final inductive inferences. There is also no logical objection to a study of the consistency relations that should hold between one's degrees of belief, but I consider it merely confusing in a numerical argument to entangle statistical probabilities or chances with these personal estimates. Dr. Jeffreys notes that in his own development of the theory probabilities are " reasonable degrees of confidence " and do not necessarily refer to the actual beliefs of individuals, but I suspect that in the practical application of his method, Dr. Jeffreys is obliged to substitute degrees of confidence which he himself considers reasonable.

I hope that Mr. Kendall will find the answers to his specific queries in this brief explanation. Maximum likelihood estimates should be judged, as I remarked in my paper, on their statistical properties. I am not sure that I attach so very much importance to his problem of which comes first, fact or theory it is too reminiscent of the hoary problem of the hen and the egg. It is true that theories are first inspired by facts, but it is also true that facts can be classified according to theories. Statistical theory is a theory about certain types of data, but if one surveys its many applications, one may well ask : " What data ? " The only exact answer seems to be : " Data conforming to the theory."

Inter-War Population Changes in Town and Country *

By G. D. A. MacDougall

1. *Introduction*

It is well known that Britain is a nation of town-dwellers. Four-fifths of the population live in administrative areas classified as urban, over one-half in "conurbations" of over 100,000 inhabitants, and two-fifths in the seven "million-cities," or conurbations with over a million inhabitants. In this paper an attempt will be made to discover to what extent, and in what senses, we have become even more "urbanized" since the end of the war of 1914–18. No attempt is made to discuss the effects of the recent evacuation from large towns.

The difficulties before us are of two types. Some concern the meaning of growing or diminishing urbanization, some are statistical. It may perhaps be a useful introduction to the main discussion to give a few illustrations of the problems involved.

There are, first, the problems connected with the question "When is a man living in a town?" There are many possible answers, but for our present purpose we may perhaps use Professor Fawcett's definition of a "conurbation" as "an area occupied by a continuous series of dwellings, factories, and other buildings, harbours and docks, urban parks and playing fields, etc., which are not separated from each other by rural land; though in many cases in this country such an urban area includes enclaves of rural land which is still in agricultural occupation." † Such an area can be fairly clearly identified at any particular time, although it is, of course, continually changing. It very seldom, however, corresponds exactly to any one urban administrative area, or even to any collec-

* This paper is, with minor alterations, a copy of a memorandum prepared for the Royal Commission on the Geographical Distribution of the Industrial Population. It was based partly on evidence that had been submitted to the Commission, not all of which has been published, and I am grateful to the Chairman for his permission to publish it.

† *Geographical Journal*, February 1932, p. 100.

tion of urban administrative areas. A man may be living within an urban administrative area, and yet not within a conurbation, as defined, because the administrative area may include rural land. Similarly a man may be living in a rural administrative area, and yet in a conurbation, possibly a very large one.

Secondly, there is the question " When is a man living in a large town ? " Given our definition of " conurbation," we may say that a town is a large one when it covers a large area or has a large population, and it is clear, for example, that a man may be living in a small administrative area (in terms of population or acreage) and yet in a large conurbation, since conurbations may include many administrative areas.

Thirdly, it follows from what has just been said that a town may grow or decline in two different ways : in population or in area. If the population of a town has grown without spreading over the neighbouring country, the inhabitants have become more closely packed together. If the population of a town has simply spread itself out, without growing in number, so that any increase through the excess of births over deaths or through the incorporation of rural inhabitants has been lost by migration, then the centre of the town is farther from the open country, although the inhabitants are presumably living in less crowded conditions. In practice, over a period of time, changes in the population and in the area of a town may occur in an infinite variety of combinations. These changes open up many possible avenues of investigation. One with which we are particularly concerned is the attempt to measure changes in the population of towns, within changing boundaries determined by inspection of the areas concerned. In practice, we are able to measure population changes within unchanged boundaries, whether they be those of administrative areas or of " conurbations " determined, as far as is practicable, in a scientific manner. No comprehensive data are, however, available by which we can measure population changes within changing boundaries, scientifically determined, although such data could in principle be obtained through laborious investigation. The nearest approach to the kind of changing boundaries we should like to have consists of administrative boundaries which, in the case of urban areas, are commonly extended as the population spreads into the neighbouring country. The extensions are, however, made at irregular intervals, so that the growth of the population of a town between two dates, especially when these are not far apart, may often be as well, or better, measured by using figures for an unchanged area (preferably a more extensive rather than a less extensive one) than by using the administrative areas as they were constituted at each date.

Fourthly, there is the distinction between a man's residence and his place of work. This distinction is roughly the same as that between a night and a day population. We may find, for example, that the population working in a town has increased more quickly than the population living there, one possible reason being that people who previously lived near their work in a town have moved into the country, and now travel in to their work.

We are now in a position to clear up a rather important difficulty. It is quite possible for a country to become more "urbanized" in the sense that a larger proportion of its population is living in large towns than before, without there being any movement of the population from the country and from small towns to large towns. There may even be a movement in the opposite direction. This is simply because, as the population of the country grows, small towns will normally become bigger ones, even if they do not attract any immigrants. The urban population will grow both by the excess of births over deaths and by a process of spreading which will "urbanize" previously rural inhabitants. Thus the towns will grow both in area and in population. Provided the national population is increasing, the proportion living in towns with more than a certain population can, indeed, only be prevented from rising if the population of large towns grows more slowly than that of the rest of the country— that is to say, in general, if there is net migration out of large towns. It will be argued in this paper that what increase there has been in urbanization in this country since the war of 1914–18 has taken the form more of the natural development of towns than of migration into them from the rest of the country.

Finally there are many purely statistical difficulties. Some of the available figures cover the sixteen-year period 1921–37, some the fourteen-year period 1923–37, and some only the intercensal period 1921–31. Some of the figures are for Great Britain, others for England and Wales only. There is the important distinction to be borne in mind between the population usually resident in an area and the population enumerated in the area on the night of the Census. This distinction is particularly important in studying changes during the intercensal period 1921–31. As a final example it may be mentioned that the areas covered by the Ministry of Labour's figures of insured persons in large towns are neither easily defined nor strictly comparable with the areas used in the discussion of changes in total population.

In Part 2 we shall consider the distribution of the whole population according to place of residence, and in Part 3 the working population according to place of work.

2. *Distribution of the whole population according to place of residence*

A. *The growth and spread of towns*

The population statistics furnished to the Royal Commission on the Geographical Distribution of the Industrial Population by the Registrars General contained a table on the " distribution of total population by size of towns ". From this the following information, relating to the post-war period, has been extracted.

TABLE I

Urban and rural distribution of population and distribution by size of towns. Great Britain

	Number of areas			Population of areas (thousands)			Population per 1,000 total population		
	1921	1931	1937	1921	1931	1937	1921	1931	1937
Towns									
Over 250,000 ...	14	15	17	11,129	11,675	11,938	260·4	260·7	259·5
100,000–250,000 ...	36	40	44	5,493	6,083	6,403	128·4	135·8	139·2
50,000–100,000 ...	58	65	80	4,089	4,586	5,611	95·6	102·4	121·9
20,000– 50,000 ...	173	199	239	5,475	6,178	7,506	128·0	137·9	163·1
10,000– 20,000 ...	264	260	237	3,803	3,693	3,416	88·9	82·4	74·3
5,000– 10,000 ...	304	279	224	2,166	1,989	1,611	50·6	44·4	35·0
Under 5,000 ...	595	600	484	1,652	1,628	1,329	38·6	36·3	28·9
Total Towns ...	1,444	1,458	1,325	33,807	35,831	37,814	790·5	799·9	821·9
Rural Areas ...	—	—	—	8,962	8,963	8,193	209·5	200·1	178·1
Total	—	—	—	42,769	44,795	46,008	1,000·0	1,000·0	1,000·0

Notes.—(1) A town is defined as a Borough or Urban District.

(2) For the purpose of this table the Administrative County of London is reckoned as one district. If the County of London (in which the population declined) is excluded, the proportion in towns with over 250,000 inhabitants shows a continuous increase between 1921 and 1937 (1921, 155 per thousand; 1931, 162; 1937, 170).

(3) The figures for 1921 and 1931 are based on Census-enumerated populations, those for 1937 on estimates of resident mid-year populations.

The table is based on administrative areas as constituted at each date. It will be seen that the number of people in each of the first four groups (larger towns) has increased, while the number in each of the remaining four groups (small towns and rural areas) has fallen. The rise in the number in middle-sized towns (20,000–100,000) is particularly noticeable. The number of people in urban administrative areas containing 20,000 or more persons has increased by 19 per cent. between 1921 and 1937, while the number living in other areas has fallen by 12 per cent., and, as a result, the proportion living in the former type of area has increased from 61 per cent. to 68 per cent. It is important, however, not to read too much into the table. The results do not necessarily mean that the number (and the proportion) in what were at some given date small towns and rural areas has declined, while that in larger towns has increased. First, changes of the nature, though not necessarily of the magnitude, indicated in the table might have taken place simply because small

towns had grown into larger ones. Some towns that in 1921 were in one size-group would have moved into the one above even if the population of all areas had expanded at the same proportionate rate and there had been no change in boundaries. Secondly, small towns and rural areas have been in many cases absorbed by large towns through an extension of the boundaries of the latter, and this also would have contributed to the results shown in the table.* As a result of changes of both types, the number of areas is seen to have increased in each of the first four groups, and to have fallen in each of the next three. The number of rural areas has also fallen. The picture of people migrating from the country and from small towns into larger towns, which the table suggests at first sight, may thus be misleading. It is certainly not definitely proved by the table, which is quite consistent with the hypothesis that there was no migration at all.

The table is not, however, without value. It does suggest that we are now even more a nation living in large towns than we were in 1921. A man living in an urban area the population of which has increased since 1921 without change of boundary is more of a town-dweller than he was in 1921, in the sense that he is living in a more crowded area—which may, of course, have its advantages as well as its disadvantages. A man living in an administrative area the population of which has expanded through the uniting of two or more previously separate administrative areas, whether he lives in the centre of a town or on the outskirts, is more of a town-dweller in so far as the extension of boundaries represents a real spreading of one or more of the areas so that they merge into each other. (This will normally be approximately true, although with many qualifications.) He is now living in a more extensive area of a more or less continuous urban nature. It is clear, however, that the figures shown in the table are the net result of changes of so many different types that it is

* There are other partial explanations of a wholly statistical nature. (*a*) The relative increase in the population in larger towns between 1921 and 1931 may be partly explained by the fact that the figures for the two years are based on the populations enumerated at the two Censuses, which were taken at different times of the year (see footnote to page 39). (*b*) The relative increase between 1931 and 1937 in the population in large towns may be partly explained by the fact that the 1937 figures are based on estimates of mid-year resident populations which tend to be bigger than Census-enumerated populations in the case of large towns and smaller in the case of small towns and rural areas. (Table 9A of the General Tables of the 1931 *Census of England and Wales* shows that the enumerated population was smaller than the resident population in Greater London and the County Boroughs outside Greater London, and greater in the other urban areas and rural districts outside Greater London. Table 16 of Volume II of the *Report of the* 1931 *Census of Scotland* shows that the enumerated population was smaller than the resident population in the Large Burghs and greater in the rest of the country.)

impossible to obtain more than a tentative and very general impression.

The table, as already stated, certainly does not prove that there was a movement of population from the country and from small

TABLE II

Population changes in urban and rural areas and in towns of various sizes

England and Wales, 1921–31

Enumerated population of area, 1931 (1)	No. of areas (2)	Enumerated population, 1931 (thousands) (3)	Enumerated population of same areas, 1921 (thousands) (4)	Mean percentage of increase (+) or decrease (−) of population, 1921–31 (5)	Same as (5) but excluding urban areas within Greater London † (6)
Urban Areas					
Over 1,000,000 *	2	5,399	5,407	−0·1	+8·7 ‡
500,000 and under 1,000,000 ...	3	2,134	2,053	+4·0	+4·0
250,000 ,, 500,000 ...	8	2,614	2,528	+3·4	+4·2
150,000 ,, 250,000 ...	13	2,590	2,474	+4·7	+2·3
100,000 ,, 150,000 ...	25	3,150	2,965	+6·3	+3·3
75,000 ,, 100,000 ...	21	1,832	1,667	+9·9	+2·6
50,000 ,, 75,000 ...	41	2,523	2,344	+7·6	+4·3
40,000 ,, 50,000 ...	34	1,498	1,370	+9·3	+2·8
30,000 ,, 40,000 ...	54	1,835	1,741	+5·4	+3·1
20,000 ,, 30,000 ...	95	2,324	2,121	+9·6	+5·4
15,000 ,, 20,000 ...	90	1,566	1,435	+9·2	+7·2
10,000 ,, 15,000 ...	143	1,761	1,685	+4·5	+4·0
5,000 ,, 10,000 ...	232	1,658	1,605	+3·3	+3·1
4,000 ,, 5,000 ...	89	395	391	+1·0	+1·0
3,000 ,, 4,000 ...	92	320	320	−0·0	−0·0
2,000 ,, 3,000 ...	93	233	240	−2·6	−2·9
Under 2,000	83	116	119	−2·7	−2·7
Total Urban Areas	1,120	31,948	30,463	+4·9	+3·8
Rural Areas	638	8,000	7,424	+7·8	+7·8
England and Wales	1,758	39,948	37,887	+5·5	+5·5

Compiled from *Census of England and Wales*, 1931, *Preliminary Report*, Tables E, G, and VI. The figures, being those of the preliminary report, differ slightly from the final figures, and, being those of enumerated populations, are subject to the difficulty (*a*) discussed in the footnote to page 39.

* Administrative County of London here reckoned as one district.

† In the compilation of this column, urban districts which are partly inside and partly outside the Greater London boundary have been treated as being wholly within the boundary. The population of all the urban areas of Greater London taken together increased by 8·1 per cent.

‡ Birmingham only.

towns into larger towns. Table II suggests, on the contrary, that such a movement did not take place. In this table, which is for England and Wales only, urban areas are classified according to the size of their populations in 1931. The populations *of the same areas* in 1921 are also shown, together with the corresponding

rates of increase in each class. Similar details are shown for rural areas as a whole. In this way the effects of changes in administrative boundaries are eliminated, while the table also differs from the previous one in that particular areas do not appear in one group in 1921 and in another in 1931.

Column 5 shows in the first place that in the areas that remained "rural" in an administrative sense in 1931 the population had grown since 1921 actually more quickly than in the areas that were urban in 1931. "The recently noticeable tendency for new industries to take advantage of the relatively cheaper sites in the country for their works and factories may account in part for this, but it is probably due also to urban development in peripheral areas which have not yet been brought within the sphere of urban administration" (*Census of England and Wales*, 1931, *Preliminary Report*, p. xvi). It seems, therefore, that there was no migration from rural areas as constituted in 1931 into urban areas as constituted in 1931. There might, however, have been migration from rural areas far from towns into those rural areas on the fringes of towns that were mentioned in the quotation. This would constitute a real movement from country to town. The Registrar General for England and Wales, has, however, expressed the opinion that "the character of the most recent advance [in urbanization] . . . as indeed of the whole of the advance during the present century is essentially different from that of periods prior to 1901 when the labour demands of newly developing industries were drawing masses of people from the countryside to the towns. The comparatively small increases in urbanization now being registered represent little more than the natural development of the urban areas themselves. . . . Organized town planning and the more generous scale of housing now demanded by the population have destroyed the rural character of many areas adjacent to towns; but apart from the incorporation of the few old inhabitants of the rural areas annexed, the population movement involved has been largely the spreading or the decentralization of the towns themselves" (*Census of England and Wales*, 1931, *Preliminary Report*, p. xv). It will be noted that the relative decline of the rural areas shown in Table I is almost certainly fully accounted for by the incorporation of rural into urban administrative areas.

In the second place, column 5 shows that the population of towns with between 5,000 and 20,000 inhabitants increased, whereas, according to the previous table, the population living in towns of those sizes fell. The very small towns declined, and the middle-sized towns showed the greatest expansion. Many of the middle-sized towns that expanded most rapidly are, however, in Greater London, and if these are excluded, as is done in column 6, the rates

of increase in the various classes appear to be merely irregular. There is no clear tendency for the populations of large towns to increase more quickly than those of small towns, if we exclude towns with very small populations.

Further evidence * for the whole of Great Britain shows that, over the longer period 1921–37, there has been only a very slight increase in the proportion of the population living in Greater London and in the 46 largest urban administrative areas outside Greater London (those with populations of over 100,000 in 1937); and this slight increase can probably be wholly explained by extensions of administrative boundaries. Apart from Greater London and a number of seaside and midland towns, a net gain by migration has definitely been the exception rather than the rule.

We have seen that the proportion of the population in the larger urban administrative areas has increased, while the proportion in the smaller urban and rural administrative areas has fallen. On the other hand, we have seen that, if we take unchanging boundaries, the larger urban administrative areas have in general increased no more quickly than the smaller urban and rural administrative areas. These facts may seem to suggest that towns have merely grown and spread outwards, while there has been on balance little migration from the smaller towns and from the country to the larger towns. It may, however, be legitimately argued that our " smaller towns " are often not separate units surrounded by open country, but rather parts of large conurbations, and that such large conurbations may well have contained those administrative areas, both urban and rural and of all sizes, the populations of which increased most rapidly. (We have already seen that many of the middle-sized " towns " that expanded most rapidly are in fact parts of one great conurbation, that of London.) If this has been so, the population of those large conurbations will have increased more rapidly than that of the rest of the country and attracted immigrants, so that the proportion of the population in them will have become larger. This possibility is neither proved nor disproved by the figures so far examined. It must be considered as a separate problem.

On examination it appears that we need not greatly modify our provisional conclusions. There has indeed been considerable net migration into the Greater London conurbation. Between 1921 and 1931 this amounted to nearly a quarter of a million, while between 1931 and 1937 there was a further immigration of a quarter of a million. In the other large conurbations, however, the position was in general the reverse. Between 1921 and 1931 each of the

* Contained in the statistics furnished to the Commission by the Registrars General.

fifteen largest conurbations * outside Greater London (those with populations in excess of 250,000 in 1931) lost population by migration, while the same is true of the majority of the smaller conurbations.

TABLE III

Growth of the largest conurbations 1921–31

Conurbation *	Mid-year resident population		
	1931 (thousands)	Increase (+) or decrease (−) 1921–31, as per cent. of 1921 population	Gain (+) or loss (−) by migration, 1921–31, as per cent. of 1921 population
London	8,238	+9·3	+3·2
Manchester	2,398	+0·5	−2·9
Birmingham	1,893	+6·7	−1·8
West Yorkshire	1,434	+0·9	−1·5
Merseyside	1,288	+4·2	−4·6
Glasgow †	1,253	+1·6	‡
Tyneside	1,105	+0·8	−8·5
Sheffield	513	−1·3	−7·1
Edinburgh	443	+3·1	−0·1
Bristol	424	+4·6	−0·6
Stoke-on-Trent	340	+2·1	−6·9
Nottingham	340	+5·3	−1·2
Hull	326	+6·7	−2·2
Teesmouth	297	+8·8	−2·4
Portsmouth	296	+1·8	−5·6
Leicester	260	+2·8	−2·4
16 *Conurbations*	20,849	+5·2	‡
Rest of Great Britain ...	23,982	+4·5	‡
Great Britain	44,831	+4·8	−1·3

Calculated from *Census of England and Wales*, 1931, General Tables, Table 9B, and from statistics furnished by the Registrars General of England and Wales and of Scotland.

* As defined by Professor Fawcett, *Geographical Journal*, February 1932.

† The Glasgow conurbation for which figures are given is that defined by the Registrar General for Scotland in a memorandum prepared for the Commission. This memorandum contained estimates of the *resident* population of the conurbation, while particulars of enumerated population only are available for the conurbation as defined by Professor Fawcett. The populations contained in the two areas, however, differ by only about 5 per cent., that defined by Professor Fawcett containing the larger population.

‡ No figures for migration are available for the Glasgow conurbation, but there was an estimated net loss by migration of 73,000 (6·9 per cent.) from the City of Glasgow, which contains nearly nine-tenths of the population of the whole conurbation (*Census of Scotland*, 1931, Vol. II, Table 2). The rate of net migration out of the 16 conurbations was about 1·0 or 1·1 per cent., and out of the Rest of Great Britain about 1·4 per cent.

The loss by migration between 1921 and 1931 of the 15 largest conurbations outside Greater London amounted to about 450,000 (the 6 " million cities " outside Greater London alone lost about one

* As defined by Professor Fawcett in the *Geographical Journal*, February 1932.

third of a million), while Greater London, as we have seen, gained less than a quarter of a million. The 16 largest conurbations, including Greater London, thus lost nearly a quarter of a million by migration, or about 1 per cent. of their 1921 population. In the whole of Great Britain, however, the rate of loss by migration was rather higher, and the rate of natural increase rather lower, so that the population of the large conurbations increased rather more rapidly than that of the country as a whole (5·2 per cent. as against 4·8 per cent.). The divergence becomes a little greater if we compare the 16 conurbations with the rest of the country, in which the population increased by 4·5 per cent.* Moreover, if rather larger areas had been taken for the conurbations, it is possible that they would have shown a somewhat higher rate of increase. (This is certainly true of the London, Birmingham and Tyneside conurbations, for which statistics were prepared for the Commission by the Registrar General for England and Wales, taking (*a*) areas defined by Professor Fawcett, and (*b*) those areas together with all contiguous urban areas.) Nevertheless it can hardly be said that the relative growth of the conurbations was very important, and there is no reason to believe that it would appear to be more important if the whole inter-war period were taken. Moreover, if the London conurbation is excluded, we find that the remaining conurbations with more than a quarter of a million inhabitants expanded by only about 2·7 per cent.†

It may be, of course, that the population of the areas just outside the conurbations expanded rapidly, but such areas were not, according to Professor Fawcett, part of the conurbations in 1931. If this has

* According to the figures given by Professor Fawcett (*Geographical Journal*, February 1932), the population of the 16 largest conurbations increased between 1921 and 1931 by 6·2 per cent. as against 4·7 per cent. in the whole of Great Britain, and 3·5 per cent. in Great Britain excluding the 16 conurbations. The divergence thus appears to be much more marked. It was necessary, however, at the date of writing, to use the Preliminary Census Report, which does not give figures for resident populations. Professor Fawcett points out that 'the 1921 Census was taken on June 19–20 and that of 1931 on April 26–27, so that the population of many holiday resorts was unduly swollen in 1921,' and that of some industrial areas was correspondingly reduced. Thus the figures tend to exaggerate the real increase in such industrial areas, and to diminish that in holiday resorts." It is this fact that explains the discrepancies between the results obtained by Professor Fawcett and those reached in the text, which are based on mid-year resident populations.

† In the 21 conurbations with between 100,000 and 250,000 inhabitants the rate of increase was about 5·2 per cent., but it may be noted that if we exclude three seaside resorts, Blackpool, Bournemouth and Southend-on-Sea, each of which expanded by about 30 per cent., the rate of increase in the remaining 18 conurbations is reduced to only about 1·9 per cent. In the 38 smaller conurbations with populations of between 50,000 and 100,000 the rate of increase was 2·8 per cent. The population of all the conurbations with over 50,000 taken together increased by 5 per cent., and that of the rest of the country, which contains 40 per cent. of the population, by 4·6 per cent. Thus, if we exclude London and the three seaside resorts, we find that outside all the conurbations the population expanded more quickly than in any of the three groups of conurbations (50,000–100,000; 100,000–250,000; over 250,000).

happened there may have been a significant migration towards, if not into, the large conurbations. No statistics relating to this question are readily available. Also it is very likely that, by taking unchanged boundaries, we are under-stating the increase in the population living in conurbations, in so far as the urbanization, through the spreading of towns, of people previously living in the country is not counted. We need not, however, modify the general conclusion that there has been no great measure of migration into the large conurbations as a whole. It is also unlikely that, even after allowing for the incorporation of rural inhabitants, the 15 large conurbations outside London would show an increase as great as that in the rest of the country (excluding Greater London).

We may now briefly examine the problem on a regional, rather than on a national basis. Our results so far certainly suggest that in the regions that lost population by net emigration the large towns lost in general as much as the smaller towns and rural areas, and that in the regions that gained by net immigration the smaller towns and rural areas gained as much as the larger towns.

One source of information consists of population estimates for 46 large towns outside Greater London, furnished to the Commission by the Registrars General. The 46 towns are those urban administrative areas (mostly County Boroughs) that had resident populations of more than 100,000 persons in 1937. 35 of those towns extended their boundaries appreciably between 1921 and 1937, and in none of the others was there any appreciable net reduction in area. We may compare the rates of growth between 1921 and 1937 of the population of those towns and of the 15 Regions into which Great Britain is divided by the Registrars General.

We find that in only 21 of the towns did the population increase more rapidly than in the regions in which they are situated, and in perhaps 4 of these * this was entirely due to an extension of boundaries. If we take the 1921 boundaries, we thus find that nearly two-thirds of the towns expanded less rapidly than the surrounding regions, while, even if we make no allowance for the boundary changes, we still find that well over one-half of the towns expanded less quickly than the surrounding regions.

The population has in many cases, however, increased relatively rapidly on the outskirts of large towns, and it may be that the areas in which this has happened have been outside the boundaries (even where these have been extended) of the large towns we have considered. We must therefore enquire whether these rapid increases have been sufficient to make the large towns, in a wider sense,

* Owing to the nature of the statistics furnished to the Commission by the Registrars General it is not possible to be more precise than this.

expand more rapidly than the surrounding regions. Some relevant figures are shown in Table IV for the seven largest conurbations, explanatory details being given below the table. Some of the figures are for the period 1921–37, others for the shorter period 1921–31 only. Where figures are readily available for a conurbation according to more than one definition, that area has been chosen that shows the greatest proportionate increase. No general tendency is apparent for the population of the conurbations to increase more quickly than that of the surrounding country.

TABLE IV

Increase (+) or decrease (—) in mid-year resident population (per cent.)

Conurbation				Whole region
(1) London, 1921–37	+17·7	Rest of South-East Region	+18·5	+17·9
(2) Birmingham, 1921–37	+12·2	Rest of Midland I Region	+ 7·1	+ 9·3
(3) Tyneside, 1921–37	— 0·4	Rest of North I Region	— 3·0	— 1·4
(4) Glasgow, 1921–31	+ 1·6	Rest of West Central Scotland Region	— 2·2	— 0·2
(5) Merseyside, 1921–31	+ 4·2 }	Rest of North IV Region *	+ 2·5	+ 2·1
(6) Manchester, 1921–31	+ 0·5 }			
(7) West Yorkshire, 1921–31	+ 1·0	Rest of North III Region	+ 6·6	+ 4·2

(1) Area falling approximately within a circle of 25 miles radius from Charing Cross.

(5), (6) and (7) As defined by Professor Fawcett.

(2) and (3) As defined by Professor Fawcett plus all other contiguous urban areas.

(4) As defined by the Registrar General for Scotland.

(1), (2), (3) and (4) Calculated from statistics furnished by the Registrars General.

(5), (6) and (7) Calculated from Table 9B of General Tables, *Census of England and Wales*, 1931.

* A very small part of the Manchester conurbation is outside the North IV Region, but it seems fitting to compare it with that region. If figures for surrounding regions were also used, the relatively slow growth of the Manchester conurbation would appear to be even more marked.

The London conurbation expanded less quickly between 1921 and 1937 than the rest of the South-east Region. Between 1921 and 1931 the Manchester conurbation expanded less quickly than the remainder of the North IV Region (Lancashire and Cheshire), excluding the Liverpool conurbation, and the West Yorkshire conurbation expanded less quickly than the rest of the North III Region (West Riding and York County Borough). In the other four cases the growth of the conurbations in relation to the surrounding regions is not very striking. During the periods considered, the population of no conurbation rose by as much as 5 per cent. in relation to the

surrounding region. The relative growth naturally becomes even less marked if we compare the conurbations with the whole of the regions in which they are situated.

B. *The growth on the fringes of towns*

The growth on the fringes of towns in relation to the central areas seems to have been both marked and general. This fact has already been mentioned, and it may now be examined in more detail. Table V, for the London conurbation, shows the changes that have taken place between 1921 and 1937 in the resident populations of the central area and of successive rings, each with a lower density of population than those it encircles. In the small innermost area, covering only 10,000 acres, the resident population fell by 19½ per cent. In the London administrative county, excluding this central area, the population fell by 6½ per cent. In the next ring it rose by 39 per cent., and in the next ring, lying between two circles of approximately 10 and 15 miles radius from Charing Cross, the resident population rose by as much as 154 per cent. This was the area in which the resident population increased most rapidly. Farther out the rise was less rapid, 56½ per cent. in the next ring, and 28 per cent. in the outermost one.

TABLE V

London conurbation

	Acreage (thousands)	Estimated resident population (thousands). (Density in persons per acre in italics)		Increase (+) or decrease (−) in resident population, 1921–37	
		1921	1937	Thousands	Per cent.
A. Nucleus consisting of 8 inner Metropolitan Boroughs and City of London.	10	1,059 *105·9*	853 *84·5*	− 206	− 19½
B. London Administrative County, except A.	65	3,465 *53·4*	3,241 *50·0*	− 224	− 6½
C. Area recommended by Royal Commission on Local Government, 1923 (equivalent to circle of 10 miles radius)—except A and B.	180	2,681 *14·9*	3,721 *20·7*	+1,040	+ 39
D. Greater London (equivalent to circle of 15 miles radius) except A, B and C.	189	331 *1·8*	840 *4·4*	+ 509	+ 154
E. Circle of 20 miles radius from Charing Cross except A, B, C and D.	360	453 *1·3*	709 *2·0*	+ 256	+ 56½
F. Circle of 25 miles radius from Charing Cross, except A, B, C, D and E.	454	349 *0·8*	447 *1·0*	+ 98	+ 28
Total	1,258	8,337 *6·6*	9,811 *7·8*	+1,474	+ 17½

Calculated from statistics furnished to the Commission by the Registrars General.

In the other conurbations it is less easy to draw rings. Instead, we may compare population changes in administrative areas with relatively great, and in administrative areas with relatively small, population densities. The less densely populated areas correspond more or less to the fringes of the conurbations. Table VI shows the changes that have taken place between 1921 and 1937 in the Birmingham and Tyneside areas. In both the area includes the conurbation as defined by Professor Fawcett, together with all contiguous urban administrative areas.

TABLE VI

Areas as constituted in 1937	Resident population (thousands)		Increase (+) or decrease (−), 1921–37		Number of areas in which population	
					In-creased	De-creased
	1921	1937	Thousands	Per cent.	relatively to population of whole conurbation	
Birmingham conurbation						
11 administrative areas with more than 10 persons per acre in 1921	1,536	1,669	+133	+8½	1	10
16 administrative areas with less than 10 persons per acre in 1921	341	437	+96	+28	12	4
All areas	1,877	2,106	+229	+12	13	14
Tyneside conurbation						
8 administrative areas with more than 10 persons per acre in 1921	884	867	−18	−2	3	5
22 administrative areas with less than 10 persons per acre in 1921	531	543	+12	+2	13	9
All areas	1,416	1,409	−6	−½	16	14

Calculated from statistics furnished to the Commission by the Registrars General.

In the Birmingham conurbation, in 10 out of the 11 areas in which the density of population was greater than 10 persons per acre, the population either fell or rose less quickly than that of the whole conurbation. In all those areas together the resident population rose by only 8½ per cent., while in all the other less densely populated areas it rose by as much as 28 per cent. On Tyneside, the population of the 8 areas with high densities fell by 2 per cent., while the population of all the other areas rose by 2 per cent.

The figures for the Glasgow conurbation are available only for the period 1921–31, but they show the same tendency. The population of the two most densely populated areas, the parishes of Glasgow (including Barony) and Govan (including Gorbals), fell by 3½ per cent., while that of all the other areas in the conurbation (as defined

by the Registrar General for Scotland) rose by as much as 26½ per cent. The figures for the Manchester and Merseyside conurbations show a similar tendency. The West Yorkshire conurbation, although showing no such tendency when an analysis is made by administrative areas (*i.e.*, local government areas), does show it quite clearly when the figures for Wards and Civil Parishes are studied. An analysis of this type is, indeed, necessary when dealing with the smaller conurbations that do not contain a sufficient number of administrative areas for any conclusions to be drawn. An examination of the population changes between 1921 and 1931 in the Wards and Civil Parishes of the larger urban administrative areas * shows an unmistakable tendency towards the relative growth of the less densely populated areas at the expense of those most densely populated. (The very densely populated Wards and Civil Parishes, with densities of, say, 50 persons per acre or more, very often show a decline in population.) This is true of large urban administrative areas in general, whether they form part of the seven largest conurbations or not.

It seems safe to say that a relatively large growth on the fringes of towns has been general.† The inner areas, on the other hand, seem in many cases to have expanded less rapidly than the surrounding areas entirely outside the conurbations, and often, indeed, to have shown an absolute decline in population. On the question whether or not the population of the whole conurbations increased on balance more rapidly than that of the surrounding country, we have seen that this happened only in some cases and then not to any striking extent.

3. *Distribution of the working population according to place of work*

So far we have considered the whole population according to where they live. It may be that our conclusions will be different if we take only the working population according to where they work. First, it may be that the relative growth of large conurbations will appear to be greater if, for example, migration into them has been mainly of persons of working age, while migration out of them has been mainly of old persons, or if many of the additional workers in a town live right outside it and travel in to their work. Secondly, for similar reasons, the relative growth of the resident population on the fringes of towns may not mean a similar relative growth in the number of people working there.

* As shown in the County Volumes of the 1931 *Census of England and Wales*, and in the City Volumes of the 1931 *Census of Scotland*.

† The *P.E.P. Report on the Location of Industry*, analysing the 1921–31 period, states (p. 173) that "a certain amount of rapid development probably occurred on the edges of most centres."

The most important source of information consists of statistics of persons insured against unemployment submitted in evidence to the Commission by the Ministry of Labour.* The figures refer to a considerable number of large towns and to a small number of the largest conurbations. Although covering only a part of the occupied population, they give us a rough idea of where people work rather than of where they live. They must, however, be interpreted in the light of some very important considerations.†

Those in employment will normally appear in the statistics under the area in which they work, since their unemployment books will generally be exchanged by their employer each year at the employment exchange nearest to the firm's premises. Those out of work will, on the other hand, usually appear in the statistics under the area in which they live, since they will themselves normally register at the employment exchange nearest to their home, which may not also be the one nearest to their work. Therefore if there is, for example, an increase in the proportion unemployed between two dates, a relative growth in the number of insured persons returned in the more residential areas (inside or outside the conurbations), as compared with the more industrial areas, may not represent a true shift of industry and trade in the direction apparently indicated.

The figures relating to those in employment may also appear to show shifts from one area to another which have no real basis. In the first place, " changes have occurred from time to time in the number of Employment Exchanges and, consequently, in the areas served by particular Exchanges." ‡ Secondly, an employer will sometimes " change the local office at which his books are exchanged, and this happens most frequently in large industrial areas where a firm's premises may be situated at a more or less equal distance from two local offices." || Thirdly, " it is also not unusual for firms who have works, branches or depots in different towns, governed by a central organisation, to keep the Unemployment Books of their work-people at their Headquarters and exchange them at the Employment Exchange nearest thereto." || As a result of the growing integration of industry, the figures may therefore show a relative growth, both in large towns as compared with the surrounding country and in the centres of towns as compared with the outer areas, that has no real foundation.

Finally, the exclusion of agricultural workers from the figures

* Some useful statistics for the Manchester conurbation were submitted in evidence by Manchester Corporation.

† See the evidence submitted to the Commission by the Ministry of Labour, Appendix I, paragraphs 1–6 (especially 5 and 6), for a full discussion.

‡ *Loc. cit.*, paragraph 5.

|| *Loc. cit.*, paragraph 6.

will tend to raise artificially the rate of growth in the less urbanized areas. If agricultural workers, whose numbers have fallen notably, were included, the rate of growth would be reduced more in the rural than in the urban areas.*

The statistics must therefore be interpreted with great caution. Certain factors may tend to raise artificially the rate of growth in the central as compared with the outer areas of towns, or in conurbations as compared with the rest of the country, while other factors may have an opposite tendency. Since it is difficult to evaluate the relative importance of the different factors, it is obvious that no very definite conclusions can be drawn.

A. *The growth of towns*

Let us consider the first possibility. Has the number of persons working in large towns increased more quickly than the number working in the rest of the country? The figures for large towns given by the Ministry of Labour refer to the number of insured persons returned by Employment Exchanges within large administrative areas, although it must be remembered that the area covered by the Exchange or Exchanges in a town almost invariably extends beyond the administrative boundaries.† These figures certainly suggest at first sight that there has been a relative increase in the number of persons working in large towns. The total number of insured persons in towns with populations of over 150,000 ‡ in 1931 (including Greater London) increased between 1923 and 1937 by 28 per cent., while in the rest of the country the increase was only 18 per cent. The rapid growth of Greater London played, of course, a very important part in raising the rate of growth of the towns as a whole (although it does not entirely account for it), and in as many as one half of the towns the rate of increase was lower than in the whole of Great Britain. Nevertheless, if we take the figures on a regional basis, we find that in most cases the towns expanded considerably more quickly than the industrial areas in which they are situated. Some details are given in Table VII.

What is the significance of the figures? Has the number working in large towns really shown so large a relative increase, both in the country as a whole and within regions? We have seen that the

* It may also be mentioned that the figures for 1923 include, while those for 1937 exclude, insured persons in the banking industry. This might perhaps be expected to reduce the rate of increase in the number of insured persons more heavily in towns, and especially in the centres of towns, than in the rest of the country, but this possibility is of only slight importance, and does not materially affect the general conclusions reached in the text.

† Appendix I, paragraph 5, of Ministry of Labour's evidence.

‡ This refers to the administrative areas of towns, not necessarily to the areas covered by the Employment Exchanges.

TABLE VII

Increase (+) or decrease (—) in numbers insured against unemployment. 1923–37. Per cent.

London and Home Counties *	+42·7
Greater London	+36·1
Midland Counties †	+28·2
Birmingham	+40·4
Stoke-on-Trent	+21·2
Leicester	+23·0
Coventry	+79·0
West Riding, Notts, and Derby	+15·0
Leeds	+28·0
Sheffield	+24·3
Nottingham	+24·5
Bradford	+ 4·1
Mid-Scotland ‡	+ 9·5
Glasgow	+14·2
Edinburgh, Leith and Portobello	+19·6
Lancashire	+ 7·6
Manchester and Salford	+19·1
Liverpool	+18·2
Bolton	+ 8·3
Northumberland and Durham	+ 4·7
Newcastle-on-Tyne	+23·4
Sunderland	+14·7
Glamorgan and Monmouth	— 4·3
Cardiff	+17·8
Swansea	+19·2
Rest of Great Britain §	+27·8
Great Britain	+22·3

Compiled from statistics given in the evidence (including supplementary evidence) of the Ministry of Labour.

* London, Middlesex, Bucks., Surrey, Kent, Essex, Herts., Beds.

† Staffs., Warwickshire, Worcestershire, Leicestershire, Northants.

‡ Counties of Lanark, Renfrew, Dumbarton, Midlothian, West Lothian.

§ The percentage rates of increase in the large towns in the "Rest of Great Britain" were as follows: Bristol, 19·2; Hull, 20·6; Portsmouth, 35·0; Dundee, 1·1; Southampton, 28·4; Aberdeen, 23·6; Plymouth and Devonport, 23·7.

statistical difficulties involved are considerable, but that some of the errors introduced are biased in one direction, some in the other. There is, however, another important consideration. It will be shown in the next section that, except in London, the number of insured persons has increased much less rapidly in the outer rings of conurbations than in the central areas, and it may be that if we had comprehensive figures for a large number of conurbations, the relative growth in the large towns as a whole would be greatly diminished, or perhaps disappear. Thus, for example, while Table VII shows that the number of insured persons returned by Exchanges within Birmingham County Borough increased by 40·4 per cent. between 1923 and 1937, the figures for the whole Birmingham conurbation show an increase of only 31·6 per cent., or little more than in the whole of the Midland Counties. Newcastle and Sunderland show increases of 23·4 per cent. and 14·7 per cent. respectively, while

in the whole of the Tyneside conurbation the rate of increase was only 3·2 per cent., or less than that for the whole of Northumberland and Durham. Glasgow shows an increase of 14·2 per cent., the Glasgow conurbation one of only 11·8 per cent. Manchester " Inner City " * shows an increase, between 1927 and 1937, of 9·9 per cent., the whole conurbation one of 0·7 per cent. only. In some of the smaller conurbations, where the main town forms a larger proportion of the whole, the difference might of course be less marked.

Table VII shows, also, that in Greater London the rate of increase was lower than in the rest of the Home Counties.† Finally, it will be seen that in the " Rest of Great Britain " (*i.e.*, the area outside the seven main industrial areas distinguished by the Ministry of Labour), which contains none of the seven largest conurbations excepting parts in Cheshire, the number of insured persons increased more quickly than in Great Britain as a whole.

In view of these considerations, and of the statistical difficulties involved, it does not seem at all safe to say that there has been any relative growth in the number of people working in large urban centres, either in the country as a whole or within regions. There is, moreover, some evidence that many smaller towns have expanded as quickly as, or more quickly than, the large ones. Dr. Brinley Thomas has remarked on the surprising extent to which certain " country towns " in the South-east, such as Bedford, Luton, Welwyn, Aylesbury and Letchworth, have received immigrant labour, especially during the period of recovery from the great depression.‡ In the Midlands, also, he has found that " considerable absorbing power has been shown by a number of independent centres scattered over the Division, *e.g.*, Rugby, Corby and Newark, which is a reminder of the result obtained for the ' country towns ' in the analysis of the South-east. There seems to be a decentralization of industry going on : sometimes old towns acquire a new lease of life, and in other cases, such as Corby, a completely new community springs into being. The population, under the influence of new forces, is redistributing itself in a manner which promises less congestion in a few areas than in the past." §

B. *Inner areas and outer rings of towns*

We may now consider the second question. Has the relative growth of the resident population on the fringes also meant a similar

* For definition, see footnote to Table VIII on p. 50.

† If a larger area than Greater London were taken to represent the London conurbation, the rate of increase would probably be higher than 36·1 per cent., but it is unlikely that it would be higher than that for London and the Home Counties.

‡ *Economica*, August 1937. § *Ibid.*, November 1938.

growth in industry and trade in those areas, or has it merely meant that people are now living farther out from their work? In the case of London, industry and trade certainly seem to have grown more quickly in the outer regions than in the centre. The number of insured persons increased, between 1923 and 1937, by only 24 per cent. in the County of London, while in the outer ring that comprises the rest of Greater London the increase was 78 per cent. In the rest of the Home Counties, excluding Greater London, the increase was 70 per cent. The figures are shown below.

Persons insured against unemployment (*aged* 16–64)

	1923 (thousands)	1937 (thousands)	Increase 1923–37 (thousands)	Increase 1923–37 (per cent.)
County of London	1,507	1,863	356	24
Greater London, excluding County of London	443	790	347	78
Home Counties,* excluding Greater London	470	799	329	70

Compiled from statistics given in the evidence, including supplementary evidence, of the Ministry of Labour.

* See Table VII on p. 47 for definition.

This happened despite the probable tendency for unemployment books to be exchanged more and more by the headquarters of firms in the centre of London. Some evidence is also provided by figures given in the Board of Trade's *Survey of Industrial Development*. Only 193 factories were opened in London Administrative County in 1936 and 1937, while 250 were closed. Of the latter, 81 were transferred to the fringe or to a nearby town.

In the case of the other four conurbations for which information is available, the figures given below in Table VIII suggest that industry and trade have grown not only as quickly in the central areas as in the outer regions, but actually considerably more quickly. This result may, of course, be to some extent unreal, if some unemployment books which were previously exchanged near outlying places of work are now exchanged instead near central headquarters. On the other hand, in those towns in which the proportion unemployed has increased, we should expect the outer areas to have shown a "false" expansion, for the reasons given in a previous paragraph. Moreover, the relative growth of the inner areas is so marked in each case that it must surely have some significance. We may therefore fairly safely say that the relative growth in the number of people living in the outer areas has at least not been accompanied by a similar relative growth in the number working there, while

there may easily have been a relative growth in the number working in the inner areas.*

TABLE VIII

Persons insured against unemployment (aged 16–64)

	1923 (thousands)	1937 (thousands)	Increase (+) or decrease (−), 1923–37	
			(thousands)	(per cent.)
Birmingham conurbation *				
Birmingham †	321	451	+130	+40
Rest of conurbation	270	327	+ 57	+21
Glasgow conurbation *				
Glasgow †	372	425	+ 53	+14
Rest of conurbation	86	87	+ 1	+ 1
Tyneside conurbation *				
Newcastle-on-Tyne †	103	127	+ 24	+23
Sunderland †	55	63	+ 8	+15
Rest of conurbation	178	157	− 21	−12
Manchester conurbation *				
Inner City ‡	429 §	472	+ 42 §	+10 §
Outer Ring	500 §	465	− 36 §	− 7 §

Calculated from statistics given in the evidence (including supplementary evidence) of the Ministry of Labour, and in the evidence of the Corporation of Manchester.

* The areas covered are as far as possible those defined by Prof. Fawcett.

† *I.e.*, areas covered by employment exchanges within Birmingham, Newcastle-on-Tyne and Sunderland County Boroughs, and Glasgow Large Burgh.

‡ Area covered by following Exchanges : Manchester, Eccles, Levenshulme, Newton Heath, Openshaw, Pendlebury, Prestwich, Salford, Stretford, Withington.

§ The figures for the Manchester conurbation refer to the years 1927 (not 1923) and 1937.

4. *Conclusion*

We have shown how difficult it is to decide exactly what is meant by a growth of urbanization, and we have discussed some of the statistical problems involved in measuring it. It is not easy to draw general conclusions, but some attempt may perhaps be made briefly to summarize our results.

We seem to have become more urbanized since the war of 1914–18, in the sense that the proportion of the population living in large urban centres has increased. This can be explained, however, largely by the growth of towns through the processes of natural increase and spreading; smaller towns have simply become larger towns. There has not in addition been any considerable migration into the larger towns from the rest of the country. There has been some relative growth in the population of the largest conurbations as a

* The *P.E.P. Report on the Location of Industry* suggests, however (p. 47), that an outward movement of industry has been fairly general, but gives no supporting statistics.

whole, even within unchanged boundaries, but this has not been at all large, and can be wholly explained by the relative growth of London. Outside London, probably the only "million city" that has grown relatively has been Birmingham, while only one or two of the other conurbations with more than 100,000 inhabitants, such as those of Hull and Teesmouth, have shown a relative growth. Outside London, it is in fact unlikely that the largest conurbations as a whole have grown as quickly as the rest of the country, even if we include as part of their growth those people who have become town-dwellers as a result of the spreading of towns. Thus, while we now find a considerably larger proportion of the population living in London, by far the greatest conurbation in the country, the proportion living in the other large conurbations as a whole may be actually smaller.

We have also seen that, within regions, there has been no clear tendency for the large conurbations to gain immigrants more quickly, or to lose emigrants more slowly, than the surrounding country.

It is not unlikely that the above general conclusions apply to places of work as well as to places of residence. There seems to be no convincing evidence to show that there has been any marked relative growth in the population working in the large towns, either in the country as a whole or within regions.

There has been a relative growth in the number of people living in the outer, less densely populated, areas of large towns, the number living in the most densely populated parts having in many cases declined absolutely. In London there has been a similar outward movement, at least in a relative sense, of industry and trade, but in the other largest conurbations the movement seems to have been, if anything, in the opposite direction.

Discussion on Mr. MacDougall's Paper

[Proofs of the Paper were sent to Fellows known to be interested and the comments received are given below, with Mr. MacDougall's reply.]

Sir Sylvanus Vivian: I agree, generally, with Mr. MacDougall's interpretation of the figures, which itself agrees with the official views expressed in my Department's evidence to the Royal Commission.

The paper makes no reference, however, to the by no means unimportant fact that the trend between 1921 and 1937 was specially affected by two extraneous influences of an unusual kind. In the first place, schemes under Section 46 of the Local Government Act, 1929, completely revolutionized the lay-out of county districts,

the general effect being to reduce the number and enlarge the size of such districts. My colleague, Mr. Derrick, reminds me that this is well illustrated by the Department's figures embodied in Table I, in which, it will be seen, the small towns are shown as increasing in numbers between 1921 and 1931 and as sharply declining in numbers from 1931 to 1937. The other extraneous factor is the tremendous Government housing development during the period. Much of this took the form, literally, of the forcible expulsion of population from the dense inner portions of towns towards peripheral localities.

It is not to be assumed, therefore, that the trends of population, however rightly inferred from the figures, are due to natural forces which may be expected to be usually operative.

However, the only point in Mr. MacDougall's paper on which I emphatically disagree with him is his remark that a conurbation "can be fairly clearly identified at any particular time." In our difficult task of planning our evidence for the Royal Commission Mr. Derrick and I formed an entirely different opinion. We became convinced that while the broad conception of a conurbation was perfectly intelligible, the limits and extent of a particular conurbation would depend entirely upon the judgment of the person who interpreted that conception in its application to the particular local facts and conditions.

Dr. E. C. Rhodes: The general question of the distribution of the population in this country is one which has exercised the minds of contributors to the *Journal* on many occasions, especially in the nineteenth century. The present problem of changes in urbanization since 1918 is of interest, especially on account of the known policy of clearing the centres of our large cities of small property and rehousing on the outskirts or in blocks of dwellings on the sites cleared. It will be within the observation of many that there is to some extent a tendency for the town-dweller of means to live now in a rural area. The increase in the number of motor-cars, bus services and further electrification of railway lines has no doubt contributed. On the other hand, the attraction of the town probably still encourages some of our rural community to migrate to these centres of work and leisure.

Here is an occasion for the use of *one* statistic for measuring the urbanization of the country. Mr. MacDougall has examined various statistics relating to geographical distribution of population. It would be useful if he were able to quote one figure extracted somehow from his statistics, this single figure being an index which could be compared with a similar figure for another period. It occurs to me that such a figure might be obtained by fitting a Pareto curve to the cumulative data which may be obtained from Table II, for the year 1931. A similar procedure for data relating to 1921 would afford the desired comparison. Another possibility is to make use of Gibrat's technique described in *Les Inégalites Economiques*. His method is to use a frequency curve to describe certain data, this curve being derived from the normal curve in a

simple manner. The point of interest about his method here is that one of the constants which enter into the transformation of the normal curve can be used as a measure of skewness of the frequency curve, or, as he prefers to describe it, a measure of inequality. He actually uses his method on figures relating to size of towns in pp. 250–2 of his book. His illustration relates to France, but I think that his method could be applied successfully to England and Wales.

It may be pointed out that Gibrat concludes that there is, over a period of 60 years from 1866 to 1926, in France a decline in the measure of inequality. The smaller towns have apparently grown at a greater rate than the larger towns.

Another possibility which might be appropriate in this place is to use the population figures of the administrative counties of England and Wales. If we plot the logarithms of the urban populations of the countries against the total populations, we get a graph which suggests that approximately the urban population and the total population might be connected by an equation of the form

$$U = CP^a$$

where U = urban population of a county, in thousands.
P = total population of the county, in thousands.
C and a being constants.

The following results were obtained :—

England and Wales : Counties

As constituted in

Year	1901		1911		1931	
	C	a	C	a	C	a
1891	0·149	1·230	—	—	—	—
1901	0·174	1·197	0·172	1·199	—	—
1911	—	—	0·196	1·185	0·202	1·182
1921	—	—	—	—	0·193	1·188
1931	—	—	—	—	0·192	1·184

In the formula $U = CP^a$ we have *two* statistics, C and a. If a is equal to unity, then the urban part of a county is a constant proportion of the population of the county. We find that a is greater than 1 for all the years for which these data were examined. The fact that a has declined somewhat from 1891 suggests that relatively urbanization is less concentrated in the country now than 40 or 50 years ago.

Perhaps the more significant statistic in relation to the subject of the present paper is C. We note from the table that for 1911, 1921 and 1931 the value of a is practically the same, but has declined from 0·202 to 0·193 to 0·192. From 1891 to 1901 C increased from 0·149 to 0·174 and from 1901, to 1911 C increased from 0·172 to 0·196. These figures indicate possibly the general growth of urbanization throughout the country as a whole in those periods.

Since 1911 this indicator C has declined. The suggestion given by C is that the decline occurred mostly in the decade 1911–21, and that only a slight change, if any reliance is to be placed on the difference between 0·193 and 0·192, occurred between 1921 and 1931.

These figures certainly suggest that urbanization has ceased to grow and, in fact, declined between 1911 and 1921. It would be interesting to find if the value of C for 1941 shows a further decline.

It should be emphasized that these suggestions are put forward with the object of elucidating the present problem. At the same time it is realized that the weakness, inherent in the use of any single figure, such as an average or an index number, for a group of data, is present here. Such single measures as I have suggested might perhaps show the general tendency, but at the same time they may hide details which are best exposed. Mr. MacDougall, as I see it, has been more concerned with the details; my instincts are, in a problem of this nature, to try to find an index.

Dr. E. C. Snow: Mr. MacDougall's paper brings out the difficulty of getting a clear statistical picture of movements of which most of us have, no doubt, got general impressions. I cannot suggest any alternative method of handling the data he has used—namely, that provided by the Registrar General and the Minister of Labour—which would be likely to make the picture clearer. It may be, however, that there are data in the possession of the various transport organizations which would, if properly collated, provide us with much more light. The London Passenger Transport Board and the four railways serving London from the home counties obtain the raw material which could provide—if the task were not too great—statistical data for analysis. The information furnished by the issuing of workmen's tickets, cheap day-tickets, season tickets, bus and tram tickets, should be capable of indicating the changes year after year in the distances which are travelled in coming to our daily work or to the great shopping centres. What was the average distance travelled, say, 20 years ago by those using (*a*) workmen's tickets, (*b*) first or third class season tickets, (*c*) cheap day-tickets, and what are the corresponding figures now? Curves showing the changes annually in these average distances for the classes of the population using the types of tickets referred to might be of even more interest to distant posterity, as an indication of conditions of living in the first half of the twentieth century, than the pictorial representation of life in London in the eighteenth century is to us to-day.

Mr. Calvert Spensley: Table I is frankly a local government table; but in Table II Mr. MacDougall eliminates some of the disturbing factors for the purpose of his thesis. Nevertheless, it has a fundamental fault, namely, that the areas grouped according to populations are merely the readily available local government districts, many of which form part only of large urban areas.

The areas so grouped differ only in their characteristics: the first group, "over 1,000,000 population," consists of two areas,

namely, (1) the County of London, administered by the London County Council, which forms only the inner portion of a far larger London, and (2) the City of Birmingham. It would have been better to have put London in a group by itself and to have grouped Birmingham (1,002,413) with other great cities like Liverpool and Manchester. Other groups comprise boroughs which are poles apart in dissimilarity—for example, the group of 25 boroughs with populations between 100,000 and 150,000 brings together fourteen industrial towns and seaports of which ten increased in population by 52,000 and four decreased by 17,000, one colliery district with a decrease of 21,000, four seaside health resorts with an increase of 37,000, and six boroughs in Greater London with a net increase of 134,000. Grouping of this kind cannot be expected to give any useful results.

The most striking comment made by Mr. MacDougall on this table is that "in the areas that remained rural in an administrative sense in 1931 the population had grown since 1921 actually more quickly than in the areas that were urban in 1931." Unless this comment is intended as an indication that the rural population of this country had grown more quickly than the urban it does not mean much. But the rural areas in an administrative sense contain large urban populations—how large it is not possible to determine without detailed investigation. Assuming, however, that where the population of a rural district adjacent to an urban district increased rapidly the probability is that the increase was mainly due to an extension of the urban area—physically though not yet administratively—it would only be necessary to take the populations of 28 of such rural districts (out of 638) as being in reality urban, to reverse the relative positions shown in Table II; the urban percentage would be raised from 4·9 to 5·6, while the rural percentage would be reduced from 7·8 to 4·8. This would show (for what it is worth!) that the urban population had grown more rapidly than the rural. This experiment illustrates the danger of relying on the distinction between urban and rural administrative districts in order to establish the actual distribution of urban and rural populations.

The figures used throughout the table are the preliminary enumerated populations, although better figures are available in the carefully adjusted mid-year resident populations. When pains have been taken to improve statistical data it is surely unwise not to make use of them in a paper appearing in the *Journal of the Royal Statistical Society*.

In dealing with the interesting subject of conurbations Mr. MacDougall points to the fact that between 1921 and 1931 Greater London alone amongst the 15 largest conurbations showed a net immigration (3·2 per cent). This has special significance in view of the fact that in each of the two previous decennial periods there was a net emigration from Greater London (3·5 per cent. and 4·1 per cent.).

By the way, Mr. MacDougall speaks of "areas in a conurbation"; has not the word "conurb" been coined yet as meaning a constituent part of a conurbation?

At the other end of the scale from conurbations it might have been

instructive to have ascertained what has happened to the populations of areas which are undoubtedly rural in character.

The fact that Mr. MacDougall has not been able to obtain satisfactory statistics of work-place from the information obtained in connection with unemployment insurance or elsewhere is a clear indication of the wisdom of the Society in urging the inclusion of particulars of work-place in the 1931 Census—unfortunately without success. It seems a pity that the Royal Commission (who used the statistics of work-place given in the 1921 Census) have not taken the opportunity of mentioning in their report the desirability of including work-place as a permanent feature of the decennial censuses.

THE PRESIDENT: Neither Mr. MacDougall nor those who have contributed to the discussion have applied any test of density of population for distinguishing between rural and urban districts. In a paper on Rural Population that I read to the Society in 1914, I regarded as rural all parishes where the density was less than 0·3 per acre, as urban those with density 1·0 or more, and dealt separately with densities between 0·3 and 1·0. This method is supplementary to the classification by administrative areas, as used by Mr. MacDougall.

The whole subject lends itself to cartographical treatment. If the smallest areas for which the population is enumerated are taken as units, and on a map, the scale being sufficiently large, a dot is marked for every 100 (or perhaps 1,000) persons in each area, the result, when the map is reduced by photography, would be shaded in systematic relation to density. I have found maps of this kind very useful both for visualizing the distribution of a population (whether of persons or animals) at one date, and for studying the changes over a period. Though this method cannot for many purposes replace statistical treatment, it is an aid in determining the best classification, and might help to resolve some of the difficulties apparent in this discussion.

MR. MACDOUGALL, in reply: I do not propose to discuss the most interesting contributions of the President, Dr. Rhodes and Dr. Snow, but shall confine myself to the comments made by Sir Sylvanus Vivian and Mr. Calvert Spensley.

I am grateful for Sir Sylvanus Vivian's expression of general agreement with my interpretation of the figures.

As regards his detailed comments, I agree that schemes under the Local Government Act, 1929, completely revolutionized the lay-out of country districts, and I ought, no doubt, to have mentioned this fact specifically in amplification of my general remarks on the difficulty of interpreting Table I.

The Government housing development during the period was, of course, an important factor, but my paper was concerned, in general, not with causes, but with results, and I did not attempt the difficult task of distinguishing "natural forces which may be expected to be usually operative."

I entirely agree that the extent of a conurbation is indefinite,

and that different people would define different limits. I did, in fact, mention in my paper that the Glasgow conurbation defined by the Registrar General for Scotland differed from that defined by Professor Fawcett, the difference in the population included being about 5 per cent. The Registrar General for Scotland gave his reasons, which were based on particular local facts and conditions, for departing from Professor Fawcett's definition. I am grateful to Sir Sylvanus for pointing out the inadequate impression given by the particular phrase that I used (that a conurbation could be *fairly* clearly identified). It is also most valuable to have on record the emphatic opinion that he formed on this matter while planning his evidence for the Royal Commission.

Mr. Calvert Spensley's criticism is confined to Table II and my discussion of it. I propose to dwell in some detail on this criticism, because I do not believe that it affects my argument, although it may appear at first sight to do so. Mr. Spensley's criticisms seem to fall into four groups.

(1) Mr. Spensley claims that the table " has a fundamental fault, namely, that the areas grouped according to populations are merely the readily available local government districts, many of which form part only of larger urban areas." Mr. Spensley can hardly claim that I have not recognized this limitation. In the Introduction I emphasized the fact that conurbations may include many administrative areas; in column (6) of Table II itself I made a partial allowance by excluding the largest conurbation of all, Greater London; and almost immediately after the discussion of Table II I returned to the difficulty. It was in recognition of precisely this " fundamental fault " that I referred to the conclusions suggested by Tables I and II as " provisional " and went on to discuss conurbations. Nevertheless I do regard Tables I and II, when taken together, as at least suggestive and as supplementary to the analysis of conurbations, which gives similar results. My method was to discuss the various types of evidence readily available and to assess the value of each.

(2) Mr. Spensley's second criticism of Table II is that the areas grouped differ widely in their characteristics. In the first place I am unable to understand the purpose of arguing that the County of London and the City of Birmingham should not be brought together in the first group, when I have excluded the County of London in Column (6). Mr. Spensley also objects to the other groups and selects as an illustration the group of towns with 100,000–150,000 inhabitants. This group contains six boroughs in Greater London, but these are excluded in column (6). If we leave these out of account, Mr. Spensley's objections appear to be (*a*) that four seaside health resorts are included in one group with one colliery town and fourteen industrial towns and seaports, (*b*) that the group contains fourteen boroughs in which the population increased and five in which it decreased. He concludes that " grouping of this kind cannot be expected to give any useful results."

I cannot understand these objections. As regards objection (*a*), it would certainly be interesting to group towns according to the type of economic activity on which they depend, and if we were

concerned with the relative growth of such groups of towns it would clearly be absurd to group them according to the size of their population, unless this were done for each economic class separately. I was concerned, however, with the relative rates of growth of towns of various sizes, so that it would have been absurd to group them according to the nature of their economic activity. Mr. Spensley may mean that I should have grouped the towns in both ways, but since I was concerned to show merely that there has been no general tendency for urban administrative areas with large populations to expand more or less quickly than areas with small populations, this was not necessary.

Objection (*b*) was that the dispersion in the rates of change within the groups is large. This would be a valid objection if I had been trying to obtain a positive result—*i.e.* to show that areas with large populations had expanded either more or less quickly than areas with small populations—but since my conclusion was a negative one, that no such tendency was apparent, the large dispersion seems only to strengthen it.

It is interesting to note that the towns that declined (outside Greater London) in the group selected for criticism by Mr. Spensley are all in regions the population of which either fell or increased much less rapidly than that of the whole country. There is, indeed, a moderately clear relation between the rate of growth of towns and the rate of growth of the regions in which they are situated, and this accounts for a considerable part of the dispersion noticed by Mr. Spensley in the rates of growth within the groups of Table II. Later in the paper I compared the rates of growth of the larger urban administrative areas and of the seven largest conurbations with the growth of the regions in which they are situated.

(3) Mr. Spensley's third criticism concerns " the danger of relying on the distinction between urban and rural administrative districts in order to establish the actual distribution of urban and rural populations," since "rural areas in an administrative sense contain large urban populations." I emphasized this point in my Introduction, in the very paragraph criticized by Mr. Spensley, and elsewhere in the paper. In view of this difficulty, Mr. Spensley is unwilling to draw any conclusions from the fact that " in the areas that remained ' rural ' in an administrative sense in 1931 the population had grown since 1921 actually more quickly than in the areas that were urban in 1931." I also was reluctant to draw any conclusions, since there might have been migration from rural areas far from towns to rural administrative areas, on the fringes of towns, that are in reality urban in nature. I did, however, cite the opinion of the Registrar General that this had not been the case. Therefore, although it would, I agree, be dangerous to assume that the rural population had grown more quickly than the urban, there are no grounds for deducing, from the evidence readily available, that there was any strong movement in the opposite direction.

Mr. Spensley, however, gives the results of an experiment which shows that only 28 rural administrative areas, adjacent to urban areas, need be considered as in reality urban to reverse the relative

positions shown in Table II—*i.e.* to make the urban growth appear greater than the rural. This experiment seems to suggest that the urban population, in a real sense, almost certainly expanded considerably more quickly than the rural, but it is, I believe, misleading.

In the first place, although Mr. Spensley's 28 areas represent only 4½ per cent. of all rural areas in number, it seems likely that they represent a larger proportion of the population of all rural areas, since Mr. Spensley has presumably tended to choose areas with larger rather than smaller populations in order to make the result of his experiment as striking as possible. The 28 rural areas of all types with the largest populations contained 15 per cent. of the total population in rural areas in 1931. Of these 28 areas, 9 are among the 28 areas that expanded most rapidly, and all except 7 expanded more rapidly than the total of urban areas. Mr. Spensley's 28 areas will not, of course, contain as large a proportion of the total population in rural areas as 15 per cent., but it is likely that the proportion is considerably higher than 4½ per cent.

Secondly, if, as Mr. Spensley urges should be done, resident populations are taken rather than enumerated, the ratio of the percentage rates of increase in rural and urban areas is raised from 7·8 : 4·9 to 8·2 : 4·9, while if Greater London be excluded it is further raised to 7·5 : 3·7. Mr. Spensley would have to take a still greater proportion of the population in rural administrative areas as in reality urban to make his experiment work. The proportion that would have to be taken may not be large, but it is not so ridiculously small as Mr. Spensley's experiment at first sight suggests.

Thirdly, it should be noted that Mr. Spensley has carefully selected certain rural areas that expanded rapidly. There must have been, however, many rural administrative areas, in reality urban, that did not expand rapidly, and some that declined.

For these reasons Mr. Spensley's experiment does, I believe, give a misleading impression. Mr. Spensley certainly does not prove that if urban and rural areas were carefully separated—for example, in accordance with the density criterion suggested by the President—the urban population would show any important relative increase.

In the third paragraph after that criticized by Mr. Spensley I returned again to the difficulty that he mentions, when I wrote that the "large conurbations may well have contained those administrative areas, both urban and rural and of all sizes, the populations of which increased most rapidly." I decided that the problem of conurbations must be treated separately, and found, among other things, that the population living outside conurbations with more than 50,000 inhabitants increased only slightly less quickly than the population within those conurbations. (If Greater London is excluded, the population outside the conurbations shows a considerable relative increase.) To say that this represents a growth in the rural population nearly as great as that in the urban population would be to stretch rather far the usual conception of "rural," but it does suggest a possible method of distinguishing urban and rural (and greater and less urbanization), which is supple-

mentary to that suggested by the President, and to the admittedly unsatisfactory distinction between urban and rural administrative areas. It is important for many purposes to know not only whether a man is living in a more or less densely populated area, but also how many people are living with him in a fairly continuous urban area. In my paper I was more concerned to show that there has been a relatively strong growth of population in the small urban centres and "the country" taken together, than to attempt any clear-cut distinction between urban and rural populations in the narrower sense. It is for this reason, as well as for the detailed reasons I have discussed, that Mr. Spensley's third criticism does not, I believe, affect my argument.

(4) Mr. Spensley's fourth criticism is that I should have used figures of mid-year resident populations in Table II rather than preliminary figures of enumerated population. The difference between the preliminary and final figures of enumerated population is in general slight, so that I felt justified in using the convenient tables given in the Preliminary Report, although I did mention in the note below Table II that the two sets of figures differed slightly. I agree that for many purposes the figures of resident population are more satisfactory than those of enumerated population, but for my particular purpose it was preferable to use the latter. I emphasized in the note below Table II that the figures referred to enumerated populations and were therefore subject to the difficulty connected with the taking of the 1921 and 1931 Censuses at different times of the year. It was for this very reason that I used figures of enumerated population, since I wished to show the effect of eliminating certain factors that have a profound effect on Table I. By grouping, in Table II, areas as constituted in 1931 according to their populations in 1931, I showed that the relative growth of the larger urban administrative areas apparent in Table I was no longer evident. If I had taken resident populations, the growth of such areas would have been, in general, even smaller, for reasons described in a footnote to page 39, but it would not have been clear how much of the difference between Tables I and II resulted from the different method of grouping and how much from the substitution of resident for enumerated populations.

MISCELLANEA

CONTENTS

Population Mathematics—I

By E. C. Rhodes, D.Sc.

Chapter I

1. We can trace the growth of population of a simple community which starts off with two married couples. If each couple has four children, two male and two female, and if these all survive, and if the males and females pair off and have four children to each couple, two male and two female, and if these survive, and if the process of marrying and having children goes on from one generation to another, we can easily write down the number of each generation. Starting with 4, the next generation consists of 8 persons, the next of 16, the next of 32, and so on.

2. At each stage of the development of this hypothetical population we have to assume survival of offspring, the possibility of pairing male and female, and the possibility of children being born to these unions.

3. Let us consider the life-history of a female child, from the point of view of the future of the population. First of all, it is exposed to the risk of dying before reaching the reproductive age. If it survives, it is exposed to the risk of marriage. If it achieves the marital state, it is exposed to the risk of having female children until the end of the reproductive period. Also, even without being married there is the chance of having female children.

4. For purposes of argument, we can combine all these various chances into a single one, the chance, at birth, of a female child giving birth, at an age between x and $x + dx$ years, to a female child. Let us call this chance $\phi(x)dx$. If l years is the beginning of the reproductive period and L years is the end of this period, then $\phi(x)$ is zero for all values of x except for $l < x < L$. $\phi(x)$ takes different values as x changes from l to L. $\phi(x)$ is dependent on mortality, nuptiality, marital fertility, and fertility of the unmarried. From the point of view of the future population, we need only be concerned with mortality and fertility generally.

5. We need only concern ourselves with the changes in the

number of females, assuming that the number of males will run parallel with that of females.

Let us call $\phi(x)$ the *net fertility function*. [An illustration of $\phi(x)$ is given in the Appendix.]

6. Now consider 100,000 females born at the same time. A number will die before reaching reproductive age. The total number of female children born to this group of females, when they are between x and $x + dx$ years old, is expected to be $100{,}000\phi(x)dx$. The total number of female children born during the course of the lifetime of this 100,000 is $100{,}000 \int_l^L \phi(x)dx$.

If we call $\int_l^L \phi(x)dx = R_0$, then the total number of female children produced by the 100,000 original female babies is $100{,}000\ R_0$. R_0 is called the *net-reproduction rate*. If $R_0 = 1$, then the original 100,000 female children give birth during their lifetime to another 100,000 female children, who in their turn might be expected eventually to be mothers. If $R_0 > 1$, then, in the future, we may expect more potential mothers than at the present, at the time when the original 100,000 are born. If $R_0 < 1$, we may expect fewer potential mothers than at the present time.

The *net-reproduction rate*, R_0, gives a simple indication of present tendencies of population growth.

7. Let us trace the number of female births, year by year, which follow from N female children born at time O, the net fertility function being $\phi(x)$, $\phi(x)dx$ representing the chance, at birth, of a female birth when the mother's age is between x and $x + dx$.

8. If $B(t)$ stands for the number of births at time t, then at the start $B(O) = N$, and $B(t) = O$ for $O < t < l$, this representing the time interval during which the original group of children are growing to maturity.

9. At a time t between l and $2l$, female children may be expected from the original group. The number born at time t is $N\phi(t)$.

10. From the year $2l$ onwards to $3l$ we may expect also female children of the second generation, because some children born in the years after year l will have reached maturity. We may obtain the number of such female children as follows. If $B(t)$ represents the number of female children born at time t, then the number of female children born to them when they are aged x to $x + dx$—*i.e.*, in the time interval $t + x$ to $t + x + dx$—is $B(t)\phi(x)dx$. Similarly if $B(t - x)$ stands for the number of female children born at time $t - x$, then the number of female children born to them when they

are aged x to $x + dx$ is $B(t - x)\phi(x)dx$, at time t, $t + dx$. Here necessarily $t > x > l$.

11. In our present case, when $2l < t < 3l$, the potential mothers born since time l are between the ages l and $t - l$. (Thus suppose the lowest reproductive age l is 15 years, and $t = 40$, then at 40 years from the start of our population, the female children born between the years 15 and 25 are now potential mothers, being aged between 25 and 15 years.) The total number of female births, then, at time t, of the second generation—*i.e.*, to mothers who were born between the years l and $t - l$—is

$$\int_l^{t-l} B(t - x)\phi(x)dx.$$

This expression may be re-written, putting $t - x = u$, as

$$\int_l^{t-l} B(u)\phi(t - u)du.$$

But the expression $B(u)$ under the integral sign, where u ranges from l to $t - l$, and where $t - l < 2l$, since $t < 3l$, is given by $N\phi(u)$, as we have already seen (para. 9).

Therefore we have

$$\int_l^{t-l} N\phi(u)\phi(t - u)du$$

as the contribution to the births at time t from the female children born between time l and $t - l$.

In addition, at time t between $2l$ and $3l$, assuming that $3l$ is less than L, as is the case if l is 15 and L is 50 (and these values of l and L are quite generally used to represent the limits of the reproductive period), the original group N may be expected to produce $N\phi(t)$ female children.

Thus, for $2l < t < 3l$, $B(t) = N\phi(t) + \int_l^{t-l} N\phi(u)\phi(t - u)du.$

Let us write :

$$\phi_1(t) = \int_l^{t-l} \phi(u)\phi(t - u)du.$$

Then, for $2l < t < 3l$, $B(t) = N(\phi(t) + \phi_1(t))$.

12. Now consider the period $3l < t < L$, assuming $L < 4l$. (This is true if $L = 50$ and $l = 15$.) We have, again, some births at this time, t, due to the original group of N, some due to children born between l and $2l$, and now, some to mothers who were born since $2l$. We can write, as before :

$$B(t) = N\phi(t) + \int_l^{t-l} B(t-x)\phi(x)dx,$$
$$= N\phi(t) + \int_l^{t-l} B(u)\phi(t-u)du,$$

where $t - x = u$.

This may be written as :

$$B(t) = N\phi(t) + \int_l^{2l} B(u)\phi(t-u)du + \int_{2l}^{t-l} B(u)\phi(t-u)du,$$

where $B(u)$ under the first integral has the value $N\phi(u)$, since here $l < u < 2l$, and $B(u)$ under the second integral has the value $N(\phi(u) + \phi_1(u))$, since now $2l < u < t - l < 3l$.

Therefore :

$$B(t) = N\phi(t) + \int_l^{2l} N\phi(u)\phi(t-u)du + \int_{2l}^{t-l} N(\phi(u) + \phi_1(u))\phi(t-u)du$$
$$= N\phi(t) + \int_l^{t-l} N\phi(u)\phi(t-u)du + \int_{2l}^{t-l} N\phi_1(u)\phi(t-u)du.$$
$$= N(\phi(t) + \phi_1(t) + \phi_2(t)),$$

where
$$\phi_2(t) = \int_{2l}^{t-l} \phi_1(u)\phi(t-u)du.$$

13. In the period $L < t < 4l$ there are no births due to the original N children, since the survivors of this group have passed reproductive age. We may write then :

$$B(t) = \int_l^{t-l} B(t-x)\phi(x)du = \int_l^{t-l} B(u)\phi(t-u)du,$$

as before, because the children born at time t are all born to mothers themselves born from time l to time $t - l$, who are now aged $t - l$ to l.

We may write, then :

$$B(t) = \int_l^{2l} B(u)\phi(t-u)du + \int_{2l}^{t-l} B(u)\phi(t-u)du$$

where
$$t - l < 3l, \text{ since } t < 4l.$$

Inserting under the integral signs the values of $B(u)$ appropriate to the periods l to $2l$ and $2l$ to $3l$ respectively, we have :

$$B(t) = N(\phi_1(t) + \phi_2(t)).$$

14. In the period $4l < t < L + l$, we have again :

$$B(t) = \int_l^{t-l} B(t-x)\phi(x)du = \int_l^{t-l} B(u)\phi(t-u)du,$$

the ages of the mothers of the children born at time t being from l to $t - l < L$. This may be written as:

$$B(t) = \int_{l}^{2l} B(u)\phi(t-u)du + \int_{2l}^{3l} B(u)\phi(t-u)du + \int_{3l}^{t-l} B(u)\phi(t-u)du$$

since $t - l < L$.

Taking the appropriate value for $B(u)$ under each integral, we have:

$$B(t) = \int_{l}^{2l} N\phi(u)\phi(t-u)du + \int_{2l}^{3l} N(\phi(u) + \phi_1(u))\phi(t-u)du + \int_{3l}^{t-l} N(\phi(u) + \phi_1(u) + \phi_2(u))\phi(t-u)du,$$

$$= N\int_{l}^{t-l} \phi(u)\phi(t-u)du + N\int_{2l}^{t-l} \phi_1(u)\phi(t-u)du + N\int_{3l}^{t-l} \phi_2(u)\phi(t-u)du,$$

$$= N(\phi_1(t) + \phi_2(t) + \phi_3(t)),$$

where $$\phi_3(t) = \int_{3l}^{t-l} \phi_2(u)\phi(t-u)du.$$

15. In the period $L + l < t < 5l < L + 2l$, we have now:

$$B(t) = \int_{l}^{L} B(t-x)\phi(x)du,$$

where the integral is taken throughout the whole reproductive period. When $t > L + l$, there are survivors of children of the original N children who are older than L, the upper limit of the reproductive period. Suppose $l = 15$, $L = 50$ and $t = 70$. Then in year 70 there are females of all ages from 0 to 55, these older ones having been born in year 15. In this group, then, there are females of all reproductive ages from 15 to 50.

$B(t)$ is now written as:

$$\int_{t-L}^{t-l} B(u)\phi(t-u)du,$$

where $t - x = u$.

This is written as:

$$\int_{t-L}^{2l} + \int_{2l}^{3l} + \int_{3l}^{L} + \int_{L}^{t-L},$$

since $t - L > l$ and $t - L < 2l$, and $t - l > L$, and $t - l < 4l$.

We have, then, inserting the appropriate values of $B(u)$ under the various integrals:

$$B(t) = \int_{t-L}^{2l} N\phi(u)\phi(t-u)du + \int_{2l}^{3l} N(\phi(u) + \phi_1(u))\phi(t-u)du + \int_{3l}^{L} N(\phi(u) + \phi_1(u) + \phi_2(u))\phi(t-u)du + \int_{L}^{t-l} N(\phi_1(u) + \phi_2(u))\phi(t-u)du$$

$$= N\int_{t-L}^{L} \phi(u)\phi(t-u)du + N\int_{2l}^{t-l} \phi_1(u)\phi(t-u)du + N\int_{3l}^{t-l} \phi_2(u)\phi(t-u)du$$

$$= N(\psi_1(t) + \phi_2(t) + \phi_3(t)),$$

where $$\psi_1(t) = \int_{t-L}^{L} \phi(u)\phi(t-u)du.$$

16. When we consider

$$\psi_1(t) = \int_{t-L}^{L} \phi(u)\phi(t-u)du$$

we observe that it is the same as $\phi_1(t)$, which is

$$\int_{l}^{t-l} \phi(u)\phi(t-u)du,$$

for values of $t > L + l$. For now

$$\phi_1(t) = \int_{l}^{t-L} \phi(u)\phi(t-u)du + \int_{t-L}^{L} \phi(u)\phi(t-u)du + \int_{L}^{t-l} \phi(u)\phi(t-u)du$$

since $t - L > l$ and $t - l > L$.

But, when u ranges from L to $t - l$, $\phi(u) = 0$, and when u ranges from l to $t - L$, $\phi(t-u) = 0$ since $t - u > L$. Thus $\phi_1(t)$ for $t > L + l$ is :

$$\int_{t-L}^{L} \phi(u)\phi(t-u)du = \psi_1(t).$$

Therefore, $B(t)$ may be written as $N(\phi_1(t) + \phi_2(t) + \phi_3(t))$, for $L + l < t < 5l$.

17. In the period $L + l < 5l < t < L + 2l$, we have similarly :

$$B(t) = N(\phi_1(t) + \phi_2(t) + \phi_3(t) + \phi_4(t)),$$

where $$\phi_4(t) = \int_{4l}^{t-l} \phi_3(u)\phi(t-u)du.$$

We can go on in this way, writing down the expression for $B(t)$ for different values of t. The result generally may be written :

$$B(t) = N(\phi(t) + \phi_1(t) + \phi_2(t) + \phi_3(t) + \ . \ . \ .),$$

where
$$\phi_n(t) = \int_{nl}^{t-l} \phi_{n-1}(u)\phi(t-u)du,$$

for $n \geq 2$,

$$\phi_1(t) = \int_{l}^{t-l} \phi(u)\phi(t-u)du.$$

In this expression for $B(t)$ it is to be understood that the ϕ's which enter on the right hand side are zero for certain ranges of values of t. Thus $\phi(t)$ is zero for $t > L$, $\phi_1(t)$ is zero for $t < 2l$ and $t > 2L$, $\phi_3(t)$ is zero for $t < 3l$ and $t > 3L$, and so on.

18. If $\phi(t)$ is such a function that $\phi(l) = 0$ and $\phi(L) = 0$, $B(t)$ is continuous, since $\phi_n(t) = 0$ for $t = (n+1)l$, etc.

On the other hand, there are discontinuities in the derivatives of $B(t)$. For instance, $B'(t)$ is discontinuous for $t = L$ unless $\phi'(L) = 0$.

Again,

$$\psi_1'''(L + l) = -\phi'(l)\phi'(L) + \int_{l}^{L} \phi(u)\phi'''(L + l - u)du$$

and

$$\phi_1'''(L + l) = \phi'(l)\phi'(L) + \int_{l}^{L} \phi(u)\phi'''(L + l - u)du.$$

These are not equal unless either $\phi'(l)$ or $\phi'(L)$ is zero. Thus the third derivative of $B(t)$ is not continuous at $t = L + l$. The function $B(t)$ takes different forms appropriate to particular ranges of values of t.

19. We may consider the functions derived from $\phi(t)$. $\phi_2(t)$, which is defined as $\int_{2l}^{t-l} \phi_1(u)\phi(t-u)du$, can be written as $\int_{l}^{t-2l} \phi_1(t-x)\phi(x)du$, when $t - u$ is replaced by x.

Generally,

$$\phi_n(t) = \int_{nl}^{t-l} \phi_{n-1}(u)\phi(t-u)du = \int_{l}^{t-nl} \phi_{n-1}(t-x)\phi(x)dx.$$

Also

$$\phi_3(t) = \int_{3l}^{t-l} \phi_2(u)\phi(t-u)du = \int_{3l}^{t-l} \phi(t-u)du \int_{2l}^{u-l} \phi_1(\theta)\phi(u-\theta)d\theta.$$

This is a double integral which corresponds to the volume of a solid figure standing on a triangular base in the plane of θu. The sides of the triangle are the lines whose equations are $\theta = 2l$, $u = t - l$, $u - l = \theta$. This volume, and therefore the integral, may be represented by

$$\int_{2l}^{t-2l}\phi_1(\theta)d\theta\int_{\theta+l}^{t-l}\phi(t-u)\phi(u-\theta)du.$$

If we put $u = \theta + v$, we may write this as :

$$\int_{2l}^{t-2l}\phi_1(\theta)d\theta\int_{l}^{t-\theta-l}\phi(v)\phi(t-\theta-v)dv.$$

But this is

$$\int_{2l}^{t-2l}\phi_1(\theta)\phi_1(t-\theta)d\theta.$$

Thus $\phi_3(t)$ may be written as :

$$\int_{2l}^{t-2l}\phi_1(u)\phi_1(t-u)du.$$

20. Generally

$$\phi_n(t) = \int_{nl}^{t-l}\phi_{n-1}(u)\phi(t-u)du = \int_{nl}^{t-l}\phi(t-u)du\int_{\overline{n-1}\,l}^{u-l}\phi_{n-2}(\theta)\phi(u-\theta)d\theta,$$

where this integral corresponds to the volume of a solid figure standing on a triangular base in the θu plane, which is defined by the lines $\theta = \overline{n-1}\,l$, $u = t - l$, $u - l = \theta$. The integral may therefore be written as :

$$\int_{\overline{n-1}\,l}^{t-2l}\phi_{n-2}(\theta)d\theta\int_{\theta+l}^{t-l}\phi(t-u)\phi(u-\theta)du.$$

Or, putting $u = \theta + v$, we have :

$$\int_{\overline{n-1}\,l}^{t-2l}\phi_{n-2}(\theta)\phi_1(t-\theta)d\theta,$$

which may also be written :

$$\int_{2l}^{t-\overline{n-1}\,l}\phi_{n-2}(t-u)\phi_1(u)du.$$

Again,

$$\int_{\overline{n-1}\,l}^{t-2l}\phi_{n-2}(u)\phi_1(t-u)du$$

may be written as :

$$\int_{\overline{n-1}l}^{t-2l} \phi_1(t-u)du \int_{\overline{n-2}l}^{u-} \phi_{n-3}(\theta)\phi(u-\theta)d\theta,$$

which corresponds to the volume of a solid figure standing on a triangular base in the θu plane, given by the lines $\theta = \overline{n-2}l$, $u = t - l$, $u - l = \theta$. This volume may be written as :

$$\int_{\overline{n-2}l}^{t-3l} \phi_{n-3}(\theta)d\theta \int_{\theta+l}^{t-2l} \phi_1(t-u)\phi(u-\theta)du.$$

Replacing u by $\theta + v$, this becomes

$$\int_{\overline{n-2}l}^{t-3l} \phi_{n-3}(\theta)d\theta \int_{l}^{t-2l-\theta} \phi(v)\phi_1(t-\theta-v)dv,$$

which is

$$\int_{\overline{n-2}l}^{t-3l} \phi_{n-3}(\theta)\phi_2(t-\theta)d\theta.$$

This may be written as

$$\int_{3l}^{t-\overline{n-2}l} \phi_{n-3}(t-u)\phi_2(u)du.$$

We have, generally

$$\phi_n(t) = \int_{nl}^{t-l} \phi_{n-1}(u)\phi(t-u)du = \int_{\overline{n-s}l}^{t-\overline{s+1}l} \phi_{n-\overline{s+1}}(u)\phi_s(t-u)du$$

$$= \int_{\overline{s+1}l}^{t-\overline{n-s}l} \phi_{n-\overline{s+1}}(t-u)\phi_s(u)du.$$

21. There are simple connections between the semi-invariants of the various ϕ functions. The semi-invariants are defined by the relation

$$s_0 e^{\lambda_1 x + \lambda_2 \frac{x^2}{2!} + \lambda_3 \frac{x^3}{3!} + \cdots} = s_0 + s_1 x + s_2 \frac{x^2}{2!} + s_3 \frac{x^3}{3!} + \cdots$$

where s_n stands for the sum of the nth powers of the variables.

Let us call

$$\int_{\overline{n+1}l}^{\overline{n+1}L} \phi_n(t)t^p dt = s_{np}, \quad \int_{l}^{L} \phi(t)t^p dt = s_{0p},$$

then $s_{00} = R_0$.

Let us consider

$$s_{n0} e^{\lambda_{n1} x + \lambda_{n2} \frac{x^2}{2!} + \lambda_{n3} \frac{x^3}{3!} + \cdots}$$

$$= s_{n0} + s_{n1}x + s_{n2}\frac{x^2}{2!} + s_{n3}\frac{x^3}{3!} + \ldots$$

$$= \int_{\overline{n+1}l}^{\overline{n+1}L} \phi_n(t)\left(1 + tx + \frac{t^2x^2}{2!} + \frac{t^3x^3}{3!} + \ldots\right)dt$$

$$= \int_{\overline{n+1}l}^{\overline{n+1}L} \phi_n(t)e^{tx}dt = \int_{\overline{n+1}l}^{\overline{n+1}L} e^{tx}dt \int_{nl}^{t-l} \phi_{n-1}(u)\phi(t-u)du.$$

This double integral corresponds to the volume of a solid standing on a base in the (tu) plane bounded by the lines $u = nl$, $u = nL$, $u = t - l$, $u = t - L$, because $\phi_{n-1}(u)$ has values for u ranging from nl to nL, and $\phi(t-u)$ has values for $t - u$ ranging from l to L, and t ranges from $\overline{n+1}l$ to $\overline{n+1}L$.

So the integral may be written

$$\int_{nl}^{nL} \phi_{n-1}(u)du \int_{u+l}^{u+L} e^{tx}\phi(t-u)dt.$$

Write $t - u = v$, then the integral becomes:

$$\int_{nl}^{nL} \phi_{n-1}(u)du \int_{l}^{L} e^{(u+v)x}\phi(v)dv$$

$$= \int_{nl}^{nL} \phi_{n-1}(u)e^{ux}du \int_{l}^{L} e^{vx}\phi(v)dv.$$

Each of these integrals may be expressed in a form involving semi-invariants as above, where we had:

$$s_{n0}e^{\lambda_{n1}x + \lambda_{n2}\frac{x^2}{2!} + \lambda_{n3}\frac{x^3}{3!} + \ldots} = \int_{\overline{n+1}l}^{\overline{n+1}L} \phi_n(t)e^{tx}dt.$$

Thus we get

$$s_{n0}e^{\lambda_{n1}x + \lambda_{n2}\frac{x^2}{2!} + \lambda_{n3}\frac{x^3}{3!} + \ldots} = s_{n-1\cdot0}e^{\lambda_{n-1\cdot1}x + \lambda_{n-1\cdot2}\frac{x^2}{2!} + \lambda_{n-1\cdot3}\frac{x^3}{3!} + \ldots} \times s_{00}e^{\lambda_{01}x + \lambda_{02}\frac{x^2}{2!} + \lambda_{03}\frac{x^3}{3!} + \ldots}$$

$$= s_{n-1\cdot0}s_{00}e^{(\lambda_{n-1\cdot1} + \lambda_{01})x + (\lambda_{n-1\cdot2} + \lambda_{02})\frac{x^2}{2!} + (\lambda_{n-1\cdot3} + \lambda_{03})\frac{x^3}{3!} + \ldots}.$$

Equating terms in x we have $s_{n0} = s_{n-1\cdot0}s_{00}$ and $\lambda_{np} = \lambda_{n-1p} + \lambda_{0p}$ for $p = 1, 2, 3, \ldots$

Since $s_{00} = R_0$, we have immediately $s_{n0} = R_0^{n+1}$, for values of $n = 1, 2, 3, \ldots$ and $\lambda_{np} = \overline{n+1}\lambda_{0p}$, for values of $n = 1, 2, 3, \ldots$ and values of $p = 1, 2, 3, \ldots$

If we refer to the semi-invariants of ϕ as $\lambda_1, \lambda_2, \lambda_3, \ldots$ instead of $\lambda_{01}, \lambda_{02}, \lambda_{03}, \ldots$ we may write $\lambda_{np} = \overline{n+1}\lambda_p$. Thus the average of $\phi_n(t)$ is $\lambda_{n1} = (n+1)\lambda_1 = (n+1) \times$ average of $\phi(t)$.

The standard deviation of $\phi_n(t)$ is :

$$\sqrt{\lambda_{n2}} = \sqrt{n+1} \times \text{standard deviation of } \phi(t).$$

The β_1 (in Pearson's notation) of $\phi_n(t)$ is :

$$\frac{\lambda_{n3}^2}{\lambda_{n2}^3} = \frac{1}{n+1} \times \beta_1 \text{ of } \phi(t).$$

The $\beta_2 - 3$ (in Pearson's notation) of $\phi_n(t)$ is :

$$\frac{\lambda_{n4}}{\lambda_{n2}^2} = \frac{1}{n+1} \times (\beta_2 - 3) \text{ of } \phi(t).$$

Thus, with increasing n, the $\phi_n(t)$ distribution becomes more and more nearly normal.

If we call the average and standard deviation of $\phi(t)$, m and σ respectively, we can say, for large values of n :

$$\phi_n(t) = \frac{R_0^{n+1}}{\sqrt{2\pi}\sqrt{n+1}\,.\,\sigma} e^{-\frac{1}{2}\frac{(t-\overline{n+1}m)^2}{\overline{n+1}\sigma^2}}, \text{ approximately.}$$

22. For large values of t, $B(t) = S\phi_n(t)$, includes a limited number of the ϕ functions. The earlier functions do not contribute. Suppose $(n+1)L > (n+r+2)l > t > nL > (n+r+1)l$, then $B(t) = N(\phi_n(t) + \phi_{n+1}(t) + \ldots \phi_{n+r}(t))$.

Further, the terms of this series derived from some of the ϕ functions are very small, owing to the fact that the ordinates of a normal curve quickly become very small when the abscissa is removed from the average by a few multiples of the standard deviation. Suppose we take account only of those values of $\phi_{n+s}(t)$ in the expression for $B(t)$ for which

$$-\lambda < \frac{t - \overline{n+s+1}m}{\sqrt{n+s+1}\,.\,\sigma} < \lambda,$$

where λ is to be given a value such as 3 or 4.

This would mean that values of s would be included for which

$$(t - \overline{n+s+1}m)^2 < \lambda^2(n+s+1)\sigma^2.$$

This gives :

$$\frac{t}{m} - \lambda\frac{\sigma}{m}\sqrt{\frac{t}{m}} < n+s+1 < \frac{t}{m} + \frac{\lambda\sigma}{m}\sqrt{\frac{t}{m}},$$

where we are assuming that $\frac{\lambda^2\sigma^2}{m}$ is small compared with t [if $\lambda = 3$, $\sigma = 6$, $m = 30$, $t = 2{,}000$ then $\frac{\lambda^2\sigma^2}{m} = 10{\cdot}8$].

Thus we are reasonably entitled to write, when t is large,

$$B(t) = N \mathop{\mathrm{S}}_{s=0}^{r} \frac{R_0^{\,n+s+1}}{\sqrt{2\pi}\sqrt{n+s+1}\,.\,\sigma} e^{-\frac{1}{2}\frac{(t-\overline{n+s+1}\,m)^2}{(n+s+1)\sigma^2}}$$

$$= N \mathop{\mathrm{S}}_{s=s_1}^{s_2} \frac{R_0^{\,n+s+1}}{\sqrt{2\pi}\sqrt{n+s+1}\,.\,\sigma} e^{-\frac{1}{2}\frac{(t-\overline{n+s+1}\,m)^2}{(n+s+1)\sigma^2}}$$

where

$$n + s_1 + 1 = \frac{t}{m} - \frac{\lambda\sigma}{m}\sqrt{\frac{t}{m}},\quad n + s_2 + 1 = \frac{t}{m} + \frac{\lambda\sigma}{m}\sqrt{\frac{t}{m}}.$$

If we put $n + s + 1 = \frac{t}{m} + q$.

we may write :

$$B(t) = N \mathop{\mathrm{S}}_{q=-\frac{\lambda\sigma}{m}\sqrt{\frac{t}{m}}}^{+\frac{\lambda\sigma}{m}\sqrt{\frac{t}{m}}} \left(\frac{R_0^{\frac{t}{m}+q}}{\sqrt{2\pi}\sqrt{\frac{t}{m}+q}\,.\,\sigma} e^{-\frac{1}{2}\frac{q^2m^2}{\left(\frac{t}{m}+q\right)\sigma^2}} \right)$$

Since q numerically is less than $\frac{\lambda\sigma}{m}\sqrt{\frac{t}{m}}$, it is small compared with $\frac{t}{m}$. (We have already neglected $\frac{\lambda^2\sigma^2}{m}$ compared with t.) If we neglect q compared with $\frac{t}{m}$ in the expression for $B(t)$, we have :

$$B(t) = N \mathop{\mathrm{S}}_{q=-\frac{\lambda\sigma}{m}\sqrt{\frac{t}{m}}}^{+\frac{\lambda\sigma}{m}\sqrt{\frac{t}{m}}} \left(\frac{R_0^{\frac{t}{m}}}{\sqrt{2\pi}\sqrt{\frac{t}{m}}\,.\,\sigma} e^{-\frac{1}{2}\frac{q^2m^2}{\frac{t}{m}\sigma^2}} \right)$$

If we call

$$\frac{\sigma}{m}\sqrt{\frac{t}{m}} = \alpha,$$

we have :

$$B(t) = N \mathop{\mathrm{S}}_{q=-\lambda\alpha}^{\lambda\alpha} \left(\frac{R_0^{\frac{t}{m}}}{m\sqrt{2\pi}\,.\,\alpha} e^{-\frac{1}{2}\frac{q^2}{\alpha^2}} \right).$$

We have then :

$$B(t) = N \frac{R_0^{\frac{t}{m}}}{m} \mathop{\mathrm{S}}_{q=-\lambda\alpha}^{\lambda\alpha} \left(\frac{1}{\sqrt{2\pi}\,.\,\alpha} e^{-\frac{1}{2}\frac{q^2}{\alpha^2}} \right).$$

The term under the summation sign represents the addition of a number of ordinates of a normal curve spaced at equal intervals, multiplied by $\frac{1}{\alpha}$, where α is the standard deviation (q moves from

— $\lambda\alpha$ to $\lambda\alpha$ by unit changes). This total is equivalent to the total area of this normal curve, which is unity.

Thus we get, for large values of t,

$$B(t) = N\frac{R_0^{\frac{t}{m}}}{m} \text{ approximately.}$$

The birth function becomes, when t is large, the same as the exponential function :

$$\frac{N}{m}e^{\frac{t}{m}\log R_0}.$$

If this is written $\frac{N}{m}e^{r_0 t}$, then $mr_0 = \log R_0$. (If $R_0 = 1$, $B(t) \to \frac{N}{m}$, when t is large.)

23. It is useful at this stage to give an illustration of the development of the function $B(t)$, taking a simple expression for $\phi(t)$. The expression chosen is

$$A \sin \frac{\pi\,(t-l)}{L-l}.$$

Here $\phi(l) = \phi(L) = 0$, and $\phi(t)$ is symmetrical about the value $t = \frac{L+l}{2}$, so that $\phi(t) = \phi(L + l - t)$. For the sake of simplicity we will call $\frac{\pi}{L-l} = \mu$, so that

$$\phi(t) = A \sin \mu(t-l).$$

We have :

$$\begin{aligned}\phi_1(t) &= \int_l^{t-l} A^2 \sin \mu(u-l) \,.\, \sin \mu(t-u-l)du \\ &= \frac{A^2}{2\mu}(\sin \mu(t-2l) - \mu(t-2l)\cos \mu(t-2l)).\end{aligned}$$

This may be written :

$$\phi_1(t) = \frac{A}{2\mu}(\phi(t-l) - (t-2l)\phi'(t-l)).$$

And $$\phi_1'(t) = -\frac{A(t-2l)}{2\mu}\phi''(t-l).$$

But $$\phi''(t-l) = -\mu^2\phi(t-l),$$

therefore $$\phi_1'(t) = \frac{A\mu}{2}(t-2l)\phi(t-l).$$

24. We can obtain a similar expression for $\phi_2'(t)$ and, by induction, for $\phi_n'(t)$.

We have, eventually :

$$\phi_n'(t) = \frac{A\mu}{2n}(t - \overline{n+1}l)\phi_{n-1}(t-l).$$

We can also show that

$$\phi_n(t) = \frac{A}{\mu}\cdot\frac{2n-1}{2n}\cdot\phi_{n-1}(t-l) - \frac{A^2}{2n(2n-2)}\cdot(t-\overline{n+1}l)^2\phi_{n-2}(t-2l).$$

$\phi_2(t)$, $\phi_3(t)$, etc., can be obtained easily from $\phi(t)$, $\phi_1(t)$, etc.

25. Let us consider the special case when $A = \frac{\mu}{2}$.

With this value of A, we have :

$$\int_l^L \phi(x)dx = \int_l^L \frac{\mu}{2}\sin\mu(x-l)dx = 1.$$

In this case, then, R_0, the net-reproduction ratio, is unity. If we take $N = 10{,}000$ and compute the values of $B(t)$ for each year from $t = 15$, we get the results below, assuming $l = 15$, $L = 51$.

TABLE I

26. *Number of Births each year starting with* 10,000 *female children born in year* 0. *Net reproduction ratio is unity*

$$\phi(x) = \frac{\pi}{72}\sin\frac{\pi}{36}(x-15)$$

Year (t)	$B(t)$	Year (t)	$B(t)$	Year (t)	$B(t)$	Year (t)	$B(t)$
15	0	39	394	63	339	87	272
16	38	40	380	64	347	88	273
17	76	41	364	65	353	89	274
18	113	42	346	66	357	90	278
19	149	43	327	67	359	91	281
20	184	44	307	68	360	92	285
21	218	45	287	69	358	93	288
22	250	46	265	70	355	94	292
23	280	47	244	71	351	95	297
24	308	48	222	72	345	96	301
25	334	49	200	73	339	97	306
26	357	50	178	74	333	98	309
27	378	51	157	75	325	99	313
28	395	52	174	76	318	100	317
29	410	53	191	77	311	101	319
30	421	54	209	78	304	102	321
31	430	55	226	79	297	103	322
32	435	56	243	80	291	104	321
33	437	57	260	81	286	105	321
34	436	58	276	82	281	106	321
35	433	59	291	83	278	107	320
36	426	60	304	84	275	108	318
37	418	61	318	85	274	109	315
38	407	62	329	86	273	110	313
						111	310

27. We note that the number of births rises to a maximum of 437 in year 33, declines to 157 in year 51, rises again to a new maximum, 360, in year 68, declines to 272 in year 87, rises again to 322 in year 103. The intervals between the maxima are both 35 years. The interval between the minima is 36 years.

There is some evidence of a periodic movement in the annual number of births, and there is some evidence that the annual number of births is tending to a constant value, about 300. We note that since $\phi(t)$ is symmetrical, and $l = 15$, $L = 51$, the average of (ϕt) is 33 years. In para. 22 we saw that when $R_0 = 1$, $B(t) \to \frac{N}{m}$. In our case this is

$$\frac{10{,}000}{33} = 303.$$

Chapter II

28. Some stress was laid on the fact that the birth function, $B(t)$, was not to be represented by the same single function of t at all stages of its development. But, on the other hand, we saw that if $\phi(l)$ and $\phi(L)$ are both zero, the function is certainly continuous. With increasing complexity of the successive expressions for $B(t)$, we get greater smoothness than at the beginning. We saw in para. 18 that at $t = L + l$, there is discontinuity of the third derivative of $B(t)$.

29. It is therefore worth while to examine the possibility of the *approximate* representation of $B(t)$ throughout its range by means of a single function. We can obtain, in a simple manner, such a function.

We saw in para. 15 that for values of $t > L + l$, the equation

$$B(t) = \int_l^L B(t-x)\phi(x)$$

held good.

Let us now assume that, in this equation, the same function $B(t)$ is substituted in both places where $B(t)$ enters. We know that this is an incorrect assumption, but we hope to derive a solution of the equation which will be approximately correct.

The exponential function is suggested by the conclusion of para. 22.

30. We can see at once that a function of the form $B(t) = SQ_n e^{r_n t}$, where the summation refers to n, will satisfy the equation under

certain conditions. Substituting this value of $B(t)$ in the equation we get

$$SQ_n e^{r_n t} = \int_l^L SQ_n e^{r_n(t-x)}\phi(x)dx = SQ_n e^{r_n t}\int_l^L e^{-r_n x}\phi(x)dx.$$

This equation is therefore satisfied so long as

$$\int_l^L e^{-r_n x}\phi(x)dx = 1,$$

for each value of r_n.

We may say, then, for all values of t, $B(t) = SQ_n e^{r_n t}$, where the values of r_n are obtained from the equation

$$\int_l^L e^{-rx}\phi(x)dx = 1.$$

Theoretically, there is an infinite number of values of r_n.

31. There is only one *real* value of r which satisfies this equation. For, supposing r to be *real*, if we call

$$f(r) = \int_l^L e^{-rx}\phi(x)dx,$$

then

$$\frac{df}{dr} = -\int_l^L e^{-rx}x\phi(x)dx.$$

Now, $\phi(x)$ is a positive function of x, zero at l and L, e^{-rx} is positive, and x is positive, therefore $\frac{df}{dr}$ is negative throughout the whole range of possible real values of r. Therefore the equation $f(r) = 1$ can only have one real solution. Call this r_0.

32. Further, this real solution has its sign determined by the size of R_0. We defined

$$R_0 = \int_l^L \phi(x)dx.$$

When r_0 is positive, $e^{-r_0 x}$ is less than unity for all values of x from l to L, therefore

$$\int_l^L e^{-r_0 x}\phi(x)dx,$$

which is equal to unity gives

$$\int_l^L \phi(x)dx > 1\text{—}i.e., R_0 > 1.$$

Similarly, when r_0 is negative, e^{-r_0x} is greater than unity for all values of x in the range of integration, and

$$\int_l^L \phi(x)dx < 1\text{—}i.e., R_0 < 1.$$

When $r_0 = 0$, we have, of course,

$$\int_l^L e^{-r_0x}\phi(x)dx = 1$$

becomes

$$\int_l^L \phi(x)dx = 1,$$

and $R_0 = 1$.

Thus, the real value of r_0 is such that $r_0 \gtreqless 0$ according as $R_0 \gtreqless 1$.

33. We can also deduce something respecting the complex roots of the equation

$$\int_l^L e^{-rx}\phi(x)dx = 1.$$

Let us suppose a complex root is $u + iv$. Then we have

$$\int_l^L e^{-ux}\cos vx \,.\, \phi(x)dx = 1, \quad \int_l^L e^{-ux}\sin vx \,.\, \phi(x)dx = 0.$$

It follows that $u - iv$ is also a complex root. Since $\cos vx$ is less than $+ 1$, and since

$$\int_l^L e^{-r_0x}\phi(x)dx = 1,$$

we must have $e^{-ux} > e^{-r_0x}$ for each value of x in the range of integration. Thence $u < r_0$.

Thus the real part of every complex root is less than r_0. In particular, if $R_0 \leqq 1$ and $r_0 \leqq 0$, then all the u's are necessarily negative.

34. It follows that $B(t) = SQ_ne^{r_nt}$

$$= Q_0e^{r_0t} + SQ_ne^{u_nt}(\cos v_nt + i \sin v_nt)$$
$$= Q_0e^{r_0t} + Se^{u_nt}(C_n \cos v_nt + D_n \sin v_nt),$$

tends to $Q_0e^{r_0t}$ when $t \rightarrow \infty$.

Relatively when $R_0 > 1$ and $r_0 > 0$, and absolutely when $R_0 \leqq 1$ the periodic terms in $B(t)$ are damped down with increasing t. Thus, the birth function $B(t)$ tends to become merely an exponential growth function with a rate of increase r_0, since if $B(t) = Q_0e^{r_0t}$,

$$\frac{B'(t)}{B(t)} = r_0.$$

35. In the foregoing, we have pointed out the approximate nature of the function $B(t)$ when it is expressed as a series of exponentials. This was a consequence of the earlier development of the birth function from a simple hypothesis of a number of female children, all born at the same time, as the foundation of the subsequent population. But, even though we modified the original assumption and based our population development on a population with females of different ages, the exponential solution must still be regarded as approximate, though it may be a reasonable approximation. For, the more complex basis may be regarded as an aggregate of a number of female children born at different times, and the essential character of the development of each set of births from the simpler hypothesis still holds. And this gives a birth function which is not represented by the same expression at all stages of its development. On the other hand, it is likely that the degree of approximation will be close, especially for larger values of t. The procedure in paras. 30 to 34 still holds in the more general case.

36. We may derive a formula for finding Q_n in the following somewhat indirect manner.

We assume $B(t) = SQ_n e^{r_n t}$, where the r's satisfy

$$\int_l^L e^{-rx}\phi(x)dx = 1,$$

throughout the whole range of values of t.

Let us consider the integral

$$\int_l^{L+l} B(t)e^{-r_m t}dt,$$

where r_m is a special value of r.

This is :

$$\int_l^{L+l} (S'Q_n e^{\overline{r_n - r_m}t} + Q_m)dt = S'Q_n\left(\frac{e^{\overline{r_n - r_m}\,\overline{L+l}} - e^{\overline{r_n - r_m}l}}{r_n - r_m}\right) + Q_m L,$$

where S' relates to the summation of n, omitting m.

Now consider the integral

$$\int_{2l}^{L+l}\left(\int_l^{t-l} B(t-x)\phi(x)dx\right)e^{-r_m t}dt =$$

$$\int_{2l}^{L+l}\left(\int_l^{t-l} SQ_n e^{r_n(t-x)}\phi(x)dx\right)e^{-r_m t}dt.$$

This integral corresponds to a volume standing on a triangular base in the tx plane bounded by the lines $x = l, x = t - l, t = L + l$. This volume may be written as :

$$\int_l^L \left(SQ_n e^{-r_n x}\phi(x) \int_{x+l}^{L+l} e^{\overline{r_n - r_m}t} dt \right) dx$$

$$= S'Q_n \int_l^L e^{-r_n x}\phi(x) \cdot \frac{e^{\overline{r_n - r_m}\,\overline{L+l}} - e^{\overline{r_n - r_m}\,\overline{x+l}}}{r_n - r_m} \cdot dx +$$

$$Q_m \int_l^L e^{-r_m x}\phi(x) \cdot (L - x)dx$$

$$= S' \frac{Q_n}{r_n - r_m} \left(e^{\overline{r_n - r_m}\,\overline{L+l}} \int_l^L e^{-r_n x}\phi(x)dx - e^{\overline{r_n - r_m}l} \int_l^L e^{-r_m x}\phi(x)dx \right) +$$

$$Q_m L \int_l^L e^{-r_m x}\phi(x)dx - Q_m \int_l^L xe^{-r_m x}\phi(x)dx$$

$$= S' \frac{Q_n}{r_n - r_m} (e^{\overline{r_n - r_m}\,\overline{L+l}} - e^{\overline{r_n - r_m}l}) + Q_m L - Q_m \int_l^L xe^{-r_m x}\phi(x)dx,$$

since

$$\int_l^L e^{-r_n x}\phi(x)dx = 1 \text{ and } \int_l^L e^{-r_m x}\phi(x)dx = 1.$$

Thus

$$\int_{2l}^{L+l} \left(\int_l^{t-l} B(t-x)\phi(x)dx \right) e^{-r_m t} dt =$$

$$\int_l^{L+l} B(t)e^{-r_m t}dt - Q_m \int_l^L xe^{-r_m x}\phi(x)dx.$$

Hence

$$Q_m = \frac{\int_l^{L+l} B(t)e^{-r_m t}dt - \int_{2l}^{L+l} \left(\int_l^{t-l} B(t-x)\phi(x)dx \right) e^{-r_m t}dt.}{\int_l^L xe^{-r_m x}\phi(x)dx}.$$

37. This is substantially the formula given by Lotka in "The Progeny of a Population Element," *American Journal of Hygiene*, VIII, No. 6, 1928, p. 891, and quoted from Hertz, *Math. Ann.*, Vol. 65, p. 85.

Since

$$B(t) = \int_l^L B(t-x)\phi(x)dx,$$

this formula may be written :

$$Q_m = \frac{\int_l^{2l} B(t)e^{-r_m t}dt + \int_{2l}^{L+l} \left(\int_{t-l}^L B(t-x)\phi(x)dx \right) e^{-r_m t}dt}{\int_l^L xe^{-r_m x}\phi(x)dx}.$$

38. The value of Q_m may therefore be obtained from a knowledge of the values of $B(t)$ in the range $t = l$ to $t = L + l$.

For example, let us take the case of a population started with N female children, all born at time 0, the case which was considered in Chapter I.

We must assume that, in this special range of t from l to $L + l$, although $B(t)$ is not exactly represented by $SQ_n e^{r_n t}$, the agreement between the various expressions for $B(t)$ in this range and the exponential series is sufficiently close for us to get correct values of Q_m, or at any rate, that the integrals will be the same, whether we use for $B(t)$ the exponential series or the functions previously developed in Chapter I.

The numerator in the formula for Q_m may be written :

$$\int_l^{2l} B(t)e^{-r_m t}dt + \int_{2l}^{L+l}\left(\int_{t-L}^{l} B(u)\phi(t-u)du\right)e^{-r_m t}dt,$$

replacing $t - x$ by u in the second integral.

Now, in the second integral we have :

$$\int_{t-L}^{l} B(u)\phi(t-u)du,$$

where t may have a value from $2l$ to $L + l$. But u ranges from $t - L$ to l, and if $t < L$, this means that u ranges from negative values through 0 to l. But in such a range $B(u)$ is zero, except when $u = 0$, where $B(0) = N$. Thus for any value of t in the range $2l$ to L,

$$\int_{t-L}^{l} B(u)\phi(t-u)du = N\phi(t).$$

And, for values of t from L to $L + l$,

$$\int_{t-L}^{l} B(u)\phi(t-u)du$$

is an integral with u ranging from a positive value of t less than l to l, and in this range $B(u) = 0$. Consequently, this part of the integral is zero. Thus,

$$\int_{t-L}^{l} B(u)\phi(t-u)du = N\phi(t), \text{ for } 2l < t < L$$

$$= 0, \qquad \text{for } L < t < L + l.$$

Also, in the range $l < t < 2l$, $B(t) = N\phi(t)$, as we saw earlier, (para. 9).

Hence, the numerator of the expression for Q_m is

$$\int_l^{2l} N\phi(t)e^{-r_m t}dt + \int_{2l}^{L} N\phi(t)e^{-r_m t}dt$$

This is

$$\int_l^L N\phi(t)e^{-r_m t}dt = N.$$

Thus, in our special case :

$$Q_m = \frac{N}{\int_l^L xe^{-r_m x}\phi(x)dx}.$$

39. Let us now consider the special illustration given in Chapter I, where

$$\phi(x) = \frac{\mu}{2}\sin\mu(x - l),\ \mu = \frac{\pi}{L - l}.$$

The equation for finding the value of r is

$$\int_l^L e^{-rx}\frac{\mu}{2}\sin\mu(x - l)dx = 1.$$

This gives

$$e^{-rl} + e^{-rL} = 2\left(1 + \frac{r^2}{\mu^2}\right).$$

The real value of r is zero, since $R_0 = 1$ (see para. 25).

Corresponding to this value of r, the denominator in Q_0,

$$\int_l^L xe^{-r_0 x}\phi(x)dx \text{ becomes } \int_l^L x\frac{\mu}{2}\sin\mu(x - l)dx = \frac{L + l}{2}.$$

We took $l = 15$, $L = 51$, $N = 10{,}000$, so we get :

$$Q_0 = \frac{10{,}000}{33} = 303{\cdot}03.$$

Thus, corresponding to the real root, we have, in the exponential expression for $B(t)$, a constant term 303·03 to which $B(t)$ tends as t increases in value.

In para. 27 we suggested that $B(t)$ was tending to a constant value about 300.

40. Now let us consider the complex roots of

$$e^{-rl} + e^{-rL} = 2\left(1 + \frac{r^2}{\mu^2}\right).$$

Write $r = -u + iv$. (We know that the real part of a complex root will be negative, since $R_0 = 1$.)

We get :

$$e^{ul}\cos vl + e^{uL}\cos vL = 2\left(1 + \frac{u^2 - v^2}{\mu^2}\right)$$

$$e^{ul}\sin vl + e^{uL}\sin vL = \frac{4uv}{\mu^2}.$$

41. The problem of finding values of u and v which will satisfy these equations presents difficulties. In practice the solutions are

obtained by successive approximations. We observe that while the various sines and cosines in the equations are numerically less than unity, v might assume large values, and, to correspond with large values on the right hand side of the first equation due to the term $-\frac{v^2}{\mu^2}$, we should have large values for e^{ul} and e^{uL}. Thus we might anticipate large values of v being associated with not so large values of u. If we assume, for the moment, that e^{uL} would be more important than e^{ul} (we are thinking of $L = 51$, $l = 15$), we can write, approximately,

$$e^{uL} \cos vL = 2\left(1 + \frac{u^2 - v^2}{\mu^2}\right)$$

$$e^{uL} \sin vL = 4\frac{uv}{\mu^2}$$

from which we obtain, (approximately), $\tan vL = -\frac{2u}{v}$, assuming for the moment that v^2 is much greater than u^2, and that $\frac{v^2}{\mu^2}$ is much greater, than 1. $\left(\mu = \frac{\pi}{36}\text{ in our illustration}\right)$. Thus vL will lie between $2n\pi + \frac{\pi}{2}$ and $2n\pi + \pi$, since $\sin vL$ is positive and $\cos vL$ is negative, from our approximate formulae. We then have, assuming that $\frac{u}{v}$ is small, $vL = 2n\pi + \pi - \theta$, where θ is approximately $\frac{2u}{v}$.

Hence $e^{uL} = \frac{2v^2}{\mu^2}$, approximately,

and this gives

$$uL = \log_e 2 + 2 \log_e (2n + 1)\left(1 - \frac{l}{L}\right), \text{ approximately.}$$

With hints derived from such considerations as these we are enabled to get approximations to the values of u and v, and then, fairly quickly, obtain the necessary solutions.

42. Theoretically there is an infinite number of solutions of these equations, but we have confined ourselves to the computation of three solutions, which suffice for our illustration.

We get :

$$u = 0{\cdot}031407,\ v = 0{\cdot}17607$$
$$u = 0{\cdot}061442,\ v = 0{\cdot}29711$$
$$u = 0{\cdot}076644,\ v = 0{\cdot}42445.$$

43. In order to complete the solution, we need to find the values of the coefficients Q_m. These involve

$$\int_l^L xe^{-rx}\phi(x)dx.$$

This may be obtained from

$$f(r) = \int_l^L e^{-rx}\phi(x)dx,$$

where
$$-\frac{df}{dr} = \int_l^L xe^{-rx}\phi(x)dx.$$

But
$$f(r) = \frac{e^{-rl} + e^{-rL}}{2\left(1 + \frac{r^2}{\mu^2}\right)}$$

in our present case. Thus

$$2f(r)\left(1 + \frac{r^2}{\mu^2}\right) = e^{-rl} + e^{-rL}.$$

From this we obtain :

$$2\frac{df}{dr}\left(1 + \frac{r^2}{\mu^2}\right) + 4f(r)\,.\,\frac{r}{\mu^2} = -(le^{-rl} + Le^{-rL}).$$

But, when we substitute in $f(r)$ the values of v appropriate to our problem $f(r) = 1$. We thus get :

$$\int_l^L xe^{-rx}\phi(x) = \frac{le^{-rl} + Le^{-rL} + \frac{4r}{\mu^2}}{2\left(1 + \frac{r^2}{\mu^2}\right)},$$

in our present case.

When $r = 0$, this becomes $\frac{L+l}{2}$, a result which we anticipated in para. 39.

44. For the case of a complex root, $r = -u + iv$, we have :

$$Q_m = \frac{N}{a + ib},$$

where
$$a + ib = \int_l^L xe^{-rx}\phi(x)dx.$$

Thus

$$2(a + ib)\left(1 + \frac{u^2 - v^2 - 2iuv}{\mu^2}\right) =$$

$$le^{ul}\cos vl + Le^{uL}\cos vL - i(le^{ul}\sin vl + Le^{uL}\sin vL) - 4\left(\frac{u - iv}{\mu^2}\right).$$

We obtain these equations for a and b :

$$2a\left(1+\frac{u^2-v^2}{\mu^2}\right)+4b\frac{uv}{\mu^2}=le^{ul}\cos vl+Le^{uL}\cos vL-\frac{4u}{\mu^2}$$

$$4a\frac{uv}{\mu^2}-2b\left(1+\frac{u^2-v^2}{\mu^2}\right)=le^{ul}\sin vl+Le^{uL}\sin vL-\frac{4v}{\mu^2}.$$

For purposes of computation it is simpler to leave these equations in the present form, in which it is an easy matter to insert the values of the various functions of u and v, some of which have already been obtained during the process of solving the equations for u and v.

45. We mentioned in para. 33 that the complex roots of the equation for r occurred in pairs. The value of Q_m appropriate to the value $r=-u-iv$ is $\frac{N}{a-ib}$. We shall have from the two roots the following terms in the expression $B(t)=SQ_me^{r_mt}$,

$$\frac{N}{a+ib}e^{-ut}(\cos vt+i\sin vt)+\frac{N}{a-ib}e^{-ut}(\cos vt-i\sin vt).$$

This is the same as

$$\frac{2N}{a^2+b^2}e^{-ut}(a\cos vt+b\sin vt)$$

$$=\frac{2Ne^{-ut}}{\sqrt{a^2+b^2}}\cos(vt+\alpha),$$

where $\tan\alpha=-\frac{b}{a}$.

46. We observed in para. 41 that with increasing size of u and v, $vL\to(2n+1)\pi$ and uL increases with $\log(2n+1)$.

In the equations for a and b, we have, when u and v are large,

$$-2a\frac{v^2}{\mu^2}=-Le^{uL},$$

since $\cos(vL)\to-1$

$$-2b\frac{v^2}{\mu^2}=Le^{uL}\sin vL\to 0$$

i.e., $a\to L$, and $b\to 0$, so $\sqrt{a^2+b^2}\to L$.

Thus the term corresponding to conjugate complex roots in the expression for $B(t)$ tends with large values of u and v to become

$$\frac{2N}{L}e^{-ut}\cos(vt+\alpha).$$

The point of interest is that this part of the expression for $B(t)$ only tends to disappear in the course of time on account of the exponential term. The constant in the expression does not ultimately become small.

47. We are now in a position to find the contributions to $B(t)$

given by the complex roots for which the values of u and v have been given in para. 42. In the table below there are shown the results of the calculations. Here $N = 10{,}000$.

Values of u	Values of v	Values of a	Values of b	Values of $\sqrt{a^2+b^2}$	Values of α	Values of $\frac{2N}{\sqrt{a^2+b^2}}$
0·031407	0·17607	38·157	− 14·113	40·683	20°17′·88	491·61
0·061442	0·29711	50·072	− 2·973	50·160	3°23′·85	398·73
0·076644	0·42445	52·245	− 5·796	52·565	6°19′·8	380·48

The periodicities indicated by the various values of v are, for $v = 0{\cdot}17607$, 35·68 years, for $v = 0{\cdot}29711$, 21·15 years, and for $v = 0{\cdot}42445$, 14·80 years. The dominant period, owing to the smaller value of u is 35·68 years. We observed previously, para. 27, that in the development of $B(t)$ there appeared to be a periodicity of 35 to 36 years.

48. The values of the terms in the expression for $B(t)$ contributed by the three pairs of complex roots of the equation for r were computed for the range $t = 66$ to $t = 111$ for comparison with the values obtained previously and given in para. 26 (Table II).

49. It is seen that the exponential expression for $B(t)$ agrees with the straightforward development of the birth function in this range, the differences being due to the exponential terms which are not included, but which would contribute very little, except possibly to the first part of our range, and to errors arising on account of the figures being taken to the nearest whole number. This illustration demonstrates the power of the expression $B(t) = SQ_n e^{r_n t}$ to represent the birth function, even though this function theoretically cannot properly be represented by the same expression at all stages of time. We may, however, regard the exponential expansion as something in the nature of a smoothing function, just as a logarithmic function may be regarded for practical purposes as a smoothing function which rounds off the corners of a particular polygon, " practical purposes " being determined by the degree of accuracy required.

50. Finally, if we assume that the function $B(t)$ may be expressed in the form

$$\frac{N}{\frac{L+l}{2}} + S\frac{2N}{\sqrt{a^2+b^2}}e^{-ut}\cos(vt+\alpha),$$

we see that ultimately it tends to a constant value $\frac{2N}{L+l}$.

It is a simple matter to compute, in our particular case, at what

TABLE II

Values of $\frac{2N}{\sqrt{a^2+b^2}}e^{-ut}\cos(vt+\alpha)$ *and approximations to*

$$B(t)=\frac{N}{33}+S\frac{2N}{\sqrt{a^2+b^2}}e^{-ut}\cos(vt+\alpha)$$

Values of t	$u = 0{\cdot}031407$ $v = 0{\cdot}17607$	$u = 0{\cdot}061442$ $v = 0{\cdot}29711$	$u = 0{\cdot}076644$ $v = 0{\cdot}42445$	$B(t)$ [adding 303·0]	Values of $B(t)$ (para. 26)	Difference
66	+ 51·4	+ 4·7	− 2·4	357	357	0
67	+ 54·9	+ 2·9	− 2·2	359	359	0
68	+ 56·4	+ 1·0	− 1·6	359	360	− 1
69	+ 56·2	− 0·8	− 0·8	358	358	0
70	+ 54·1	− 2·3	− 0·0	355	355	0
71	+ 50·6	− 3·4	+ 0·6	351	351	0
72	+ 45·8	− 4·1	+ 1·1	346	345	+ 1
73	+ 39·8	− 4·4	+ 1·3	340	339	+ 1
74	+ 32·9	− 4·2	+ 1·3	333	333	0
75	+ 25·5	− 3·7	+ 1·0	326	325	+ 1
76	+ 17·7	− 3·0	+ 0·7	318	318	0
77	+ 9·8	− 2·1	+ 0·2	311	311	0
78	+ 2·1	− 1·1	− 0·2	304	304	0
79	− 5·2	− 0·1	− 0·5	297	297	0
80	− 11·9	+ 0·8	− 0·7	291	291	0
81	− 17·8	+ 1·5	− 0·8	286	286	0
82	− 22·8	+ 2·0	− 0·7	281	281	0
83	− 26·8	+ 2·2	− 0·5	278	278	0
84	− 29·7	+ 2·3	− 0·2	275	275	0
85	− 31·5	+ 2·1	+ 0·0	274	274	0
86	− 32·3	+ 1·8	+ 0·2	273	273	0
87	− 32·0	+ 1·4	+ 0·3	273	272	+ 1
88	− 30·7	+ 0·9	+ 0·4	274	273	+ 1
89	− 28·5	+ 0·3	+ 0·4	275	274	+ 1
90	− 25·6	− 0·1	+ 0·3	278	278	0
91	− 25·1	− 0·6	+ 0·2	280	281	− 1
92	− 18·1	− 0·9	+ 0·0	284	285	− 1
93	− 13·9	− 1·1	− 0·1	288	288	0
94	− 9·4	− 1·2	− 0·2	292	292	0
95	− 4·9	− 1·2	− 0·2	297	297	0
96	− 0·5	− 1·0	− 0·2	301	301	0
97	+ 3·6	− 0·8	− 0·2	306	306	0
98	+ 7·4	− 0·6	− 0·1	310	309	+ 1
99	+ 10·6	− 0·3	− 0·0	313	313	0
100	+ 13·4	− 0·1	+ 0·0	316	317	− 1
101	+ 15·6	+ 0·2	+ 0·1	319	319	0
102	+ 17·2	+ 0·4	+ 0·1	321	321	0
103	+ 18·1	+ 0·5	+ 0·1	322	322	0
104	+ 18·4	+ 0·6	+ 0·1	322	321	+ 1
105	+ 18·2	+ 0·6	+ 0·1	322	321	+ 1
106	+ 17·4	+ 0·6	+ 0·0	321	321	0
107	+ 16·1	+ 0·5	+ 0·0	320	320	0
108	+ 14·4	+ 0·4	− 0·0	318	318	0
109	+ 12·3	+ 0·2	− 0·1	315	315	0
110	+ 10·0	+ 0·1	− 0·1	313	313	0
111	+ 7·5	− 0·0	− 0·1	310	310	0

stage the function is within given narrow limits of this value. The most important of the periodic components is

$$491{\cdot}61e^{-0{\cdot}031407t}\cos(0{\cdot}17607t + 20°17'{\cdot}88)$$

This is less than unity for values of t greater than t_0, given by

$$0 = \log_{10} 491{\cdot}61 - 0{\cdot}013638t_0.$$
$$t_0 = 197.$$

Thus, after the lapse of about 200 years we may feel confident that the birth function would oscillate between 303 ± 1, the size of the oscillations still decreasing with time.

51. In order to indicate the usefulness of the exponential series for $B(t)$, even in ranges for which the fundamental formula

$$B(t) = \int_l^L B(t-x)\phi(x)dx$$

does not apply, two more pairs of values of u and v were computed and the exponential terms in the range $t = 45$ to $t = 65$ were obtained.

The additional values of u and v and the derived constants are given below.

Values of u	Values of v	Values of a	Values of b	Values of $\sqrt{a^2+b^2}$	Values of α	Values of $\frac{2N}{\sqrt{a^2+b^2}}$
0·085143	0·54892	49·362	− 4·892	49·604	5°39′·59	403·19
0·093279	0·67155	49·839	− 1·983	49·878	2°16′·71	400·98

In Table III the contributions of the first five oscillatory components of $B(t)$ are shown separately.

52. The range $t = 45$ to 65 includes the year 51, when a break occurs in the smoothness of the original series of values of $B(t)$ given in para. 26, due to the cessation of contributions to the births from the original group of 10,000 children. The values of $B(t)$ computed from the exponential series are practically identical with those computed directly in the latter part of the range 45 to 65. There is considerable divergence at year 51, as might be expected.

53. We may reasonably conclude that for purposes of computing the birth function year by year, it is preferable to use the direct method in the initial stages, especially since the functions involved are few and fairly simple. In the later stages, it is better to use the exponential expansion for $B(t)$, especially as at this stage, only a few of the oscillatory components will have any practical importance, and at the same time direct computation would involve many functions of increasing complexity.

54. Finally we may say that if a female population is developed from N female children all born at time zero, ultimately the birth

TABLE III

Values of $\frac{2N}{\sqrt{a^2+b^2}} e^{-ut} \cos (vt + \alpha)$ *and approximations to* $B(t)$

Year	$u =$ 0·031407 $v =$ 0·17607	$u =$ 0·061442 $v =$ 0·29711	$u =$ 0·076644 $v =$ 0·42445	$u =$ 0·085143 $v =$ 0·54892	$u =$ 0·093279 $v =$ 0·67155	$B(t)$ [adding 303·0]	Values of $B(t)$ (para. 26)	Difference
45	− 49·2	+ 16·3	+ 11·3	+ 8·3	+ 2·4	292	287	+ 5
46	− 65·4	+ 9·4	+ 7·9	+ 7·8	+ 4·9	268	265	+ 3
47	− 78·7	+ 2·5	+ 3·7	+ 5·3	+ 4·9	241	244	− 3
48	− 88·7	− 3·7	− 0·6	+ 1·7	+ 3·0	215	222	− 7
49	− 95·3	− 9·1	− 4·2	− 1·8	+ 0·2	193	200	− 7
50	− 98·6	− 13·0	− 6·5	− 4·3	− 2·2	178	178	0
51	− 98·7	− 15·3	− 7·4	− 5·0	− 3·3	173	157	+16
52	− 98·5	− 16·0	− 6·9	− 4·7	− 2·9	174	174	0
53	− 89·9	− 15·3	− 5·3	− 3·1	− 1·4	188	191	− 3
54	− 81·9	− 13·3	− 3·1	− 1·0	+ 0·5	204	209	− 5
55	− 71·5	− 10·4	− 0·6	+ 1·1	+ 1·8	223	226	− 3
56	− 59·6	− 7·0	+ 1·6	+ 2·6	+ 2·2	243	243	0
57	− 49·7	− 3·4	+ 3·3	+ 3·1	+ 1·6	261	260	+ 1
58	− 33·1	+ 0·2	+ 4·1	+ 2·7	+ 0·5	277	276	+ 1
59	− 19·3	+ 3·2	+ 4·1	+ 1·6	− 0·6	292	291	+ 1
60	− 5·7	+ 5·7	+ 3·5	+ 0·2	− 1·3	305	304	+ 1
61	+ 7·2	+ 7·4	+ 2·3	− 1·0	− 1·3	318	318	0
62	+ 19·1	+ 8·2	+ 0·9	− 1·7	− 0·8	329	329	0
63	+ 29·6	+ 8·3	− 0·4	− 1·9	− 0·1	339	339	0
64	+ 38·7	+ 7·6	− 1·5	− 1·5	+ 0·6	347	347	0
65	+ 45·9	+ 6·4	− 2·2	− 0·7	+ 0·9	353	353	0

function is given by $B(t) = Q_0 e^{r_0 t}$, where r_0 is the real value of r satisfying the equation

$$\int_l^L e^{-rx} \phi(x) dx = 1,$$

and where

$$Q_0 = \frac{N}{\int_l^L x e^{-r_0 x} \phi(x) dx}.$$

The sign of r_0 depends on R_0, the net-reproduction ratio. If $R_0 \gtrless 1$, then r_0 is $\gtreqless 0$.

In particular if $R_0 = 1$ and $r_0 = 0$, the birth function becomes constant ultimately and equal to

$$\frac{N}{\int_l^L x \phi(x) dx}.$$

Grateful acknowledgement is made to the pioneering work of Dr. A. J. Lotka.

Appendix

An example of a Net-Fertility Schedule

The following table gives the net-fertility schedule for the Netherlands for 1935. The table is taken from Communication No. 2, Jan. 1937, Population Statistics of the Central Bureau of Statistics of the Netherlands.

The table gives the values of $1000\phi(x)$ for different values of x.

Age	15	16	17	18	19	20	21	22	23	24	25	26
1000 $\phi(x)$	0·27	0·84	3·03	6·95	13·47	20·42	26·62	36·41	43·29	49·07	56·03	63·74
Age	27	28	29	30	31	32	33	34	35	36	37	38
1000 $\phi(x)$	64·00	65·75	65·27	63·53	60·70	59·63	55·00	50·10	48·63	44·66	40·23	37·77
Age	39	40	41	42	43	44	45	46	47	48	49	Total
1000 $\phi(x)$	32·30	27·32	22·32	16·20	12·06	7·91	4·68	2·11	0·88	0·33	0·28	1101·80

Thus, according to this experience, the chance at birth of a female child having a female child at age 25 is 0·05603, and so on.

R_0, the net reproduction ratio is

$$\int_l^L \phi(x)dx = 1{\cdot}1018.$$

We note that the distribution $\phi(x)$ is not very unsymmetrical.

References.

"The Progeny of a Population Element," by A. J. Lotka, *American Journal of Hygiene*, Vol. VIII, No. 6, Nov. 1928.

"The Spread of Generations," by A. J. Lotka, *Human Biology*, Vol. I, No. 3, Sept. 1929.

The Mathematical Expectation of the Mean Square Contingency when the Attributes are Mutually Independent

By R. C. Geary

From the first two terms of the formal expansion in $\frac{1}{N}$ (N being the number in the sample), A. A. Tschuprow * surmised that, when the two attributes, or variates, X and Y, are independent, the mathematical expectation of the mean square contingency is given by

$$E\phi'^2 = \frac{(k-1)(l-1)}{N-1} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where k and l are the numbers of rows and columns, respectively, but he added that he had not succeeded in proving it. This was done by M. S. Bartlett † some time ago; as the property is rather interesting, however, it may be useful to place a proof from first principles on record here.

Let the number in the ith row and jth column of the contingency table be n_{ij} and the marginal totals for rows be denoted by $n_{i.}$ and for columns $n_{.j}$ so that

$$\sum_{j=1}^{l} n_{ij} = n_{i.}\ (i = 1, 2, \ldots, k)\,;\ \sum_{i=1}^{k} n_{ij} = n_{.j}\ (j = 1, 2, \ldots, l)$$

$$\sum_i n_{i.} = \sum_j n_{.j} = N$$

The mean square contingency is given by

$$\phi'^2 = \sum_i \sum_j \left(n_{ij} - \frac{n_{i.}\,n_{.j}}{N}\right)^2 \frac{1}{n_{i.}\,n_{.j}}$$

$$= \sum_i \sum_j \frac{n_{ij}^2}{n_{i.}\,n_{.j}} - 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If the probabilities of X and Y are, respectively, $\rho_{i.}$ and $\rho_{.j}$, so that

$$\sum_i \rho_{i.} = \sum_j \rho_{.j} = 1,$$

the mathematical expectation of n_{ij} will be $N\rho_{i.}\,\rho_{.j}$ when the attributes are independent, and the expectation of ϕ'^2 is given by

$$N!\,S \prod_i \prod_j \frac{\rho_{i.}^{\,n_{i.}}\,\rho_{.j}^{\,n_{.j}}}{n_{ij}!} \left\{ \sum_i \sum_j \frac{n_{ij}^2}{n_{i.}\,n_{.j}} - 1 \right\} \quad . \quad . \quad (3)$$

where S denotes summation to all possible samples of $N = \sum_i \sum_j n_{ij}$. This summation is effected first by keeping the marginal frequencies $n_{i.}$ and $n_{.j}$ constant, and then extending summation to all possible marginal frequencies. It is known that

* *Principles of the Mathematical Theory of Correlation* (English translation by M. Kantorowitsch), p. 112.

† *Proceedings of the Royal Society*, Series A, Vol. CLX (1937), p. 280.

$$S' \prod_i \prod_j \frac{1}{n_{ij}!} = \frac{N!}{\prod_i n_{i.}! \prod_j n_{.j}!} \quad . \quad . \quad . \quad . \quad . \qquad (4)$$

where S' indicates summation to all n_{ij} consistent with the marginal frequencies $n_{i.}$ and $n_{.j}$. In (3) set

$$n_{ij}^2 = n_{ij}(n_{ij} - 1) + n_{ij}$$

and, using (4), we find

$$E\phi'^2 = N!\, S'' \Big\{ \overline{N-2}! \Big[\prod_i \prod_j \frac{\rho_{i.}^{\,n_{i.}} \cdot \rho_{.j}^{\,n_{.j}}}{n_{i.}!\, n_{.j}!} \Big] \sum_i \sum_j (n_{i.} - 1)(n_{.j} - 1)$$

$$+ \overline{N-1}!\, kl \prod_i \prod_j \frac{\rho_{i.}^{\,n_{i.}} \cdot \rho_{.j}^{\,n_{.j}}}{n_{i.}!\, n_{.j}!} \Big\} - 1 \quad . \quad . \qquad (5)$$

where S'' denotes summation to all $n_{i.}$ $(i = 1, 2, \ldots, k)$ and $n_{.j}$ $(j = 1, 2, \ldots, l)$. Applying the multinominal theorem for the expansion of

$$(\sum_i \rho_{i.})^N = (\sum_i \rho_{i.})^{N-1} = (\sum_j \rho_{.j})^N = (\sum \rho_{.j})^{N-1} = 1$$

to (5),

$$E\phi'^2 = N!\, \overline{N-2}! \left\{ \frac{\sum_i \rho_{i.} \cdot \sum_j \rho_{.j}}{(\overline{N-1}!)^2} - \frac{\sum_i \sum_j (\rho_{i.} + \rho_{.j})}{N!\, \overline{N-1}!} + \frac{kl}{(N!)^2} \right\}$$

$$+ kl \frac{N!\, \overline{N-1}!}{(N!)^2} - 1,$$

which reduces to

$$\frac{(k-1)(l-1)}{N-1}.$$

It is rather remarkable that this expression is independent of the marginal probabilities $\rho_{i.}$ and $\rho_{.j}$. It is known, moreover, that when the marginal frequencies are constant and the expected cell frequencies not small, the frequency distribution of $z = \phi'^2$, when the attributes are independent, is given approximately by

$$\text{constant} \times z^{\frac{kl-k-l-1}{2}} e^{-\frac{zN}{2}} \delta z, \quad \cdot$$

and the mean value is $\dfrac{(k-1)(l-1)}{N}$. This suggests that, in the general case, when the marginal frequencies are not too small and vary from sample to sample, the frequency distribution of $z = \phi'^2$, when the attributes are independent, may be given approximately by

$$\text{constant} \times z^{\frac{kl-k-l-1}{2}} e^{-\frac{z(N-1)}{2}} \delta z,$$

the coefficient in the exponent being so selected that the first moment of z is $\dfrac{(k-1)(l-1)}{N-1}$.

Note on the Agricultural Position in England and Wales, as compared with the Beginning of the War in 1914

Although much has been done in the way of assisting and encouraging agriculture in recent years, the beginning of the present war finds us with a definitely smaller acreage under cultivation than in 1914. In that year the arable area in England and Wales was 11 million acres, whereas in 1939 it was under 9 million acres. The reduction has mainly occurred among crops used for feeding livestock, such as barley, oats, turnips, swedes and mangolds, though part of the land formerly used for the root crop is now devoted to sugar-beet, a crop which has so developed that about one-fourth of our sugar requirements are now provided from home-grown sources. The area under wheat in 1939 was about 7 per cent. less than it was in 1914, but this was to some extent seasonal, as the wheat area has been kept up in recent years under the stimulus of the standard price fixed by the Wheat Act, and in 1938 was actually somewhat larger than in 1914. The area under barley and oats, on the other hand, fell from 3,434,000 acres in 1914 to 2,267,000 acres in 1939.

This arable area of 9 million acres is not all tilled annually, as it included in 1939 over 2 million acres of rotation grasses—that is, grass which is ploughed up in rotation every four, five or more years. An endeavour is being made this winter to induce farmers to plough up a proportion of the poorer grassland so as to add 1½ million acres to the arable area on which crops of more value than grass can be grown. A similar effort was made in 1916–18, when the Food Production Campaign resulted in an increase in the arable area of over 1,800,000 acres.

Changes have also taken place in the last 25 years in the area of permanent pasture, of which 15,700,000 acres were returned in 1939, against 16,100,000 acres in 1914. The comparison here is not very precise. A good deal of land formerly classed as permanent pasture has deteriorated in quality, so that it is now returned as "rough grazings," and changes are also believed to have occurred as a result of greater accuracy of definition, there being no sharp distinction between the poorer qualities of cultivated grassland and "rough grazings." The exact extent of these alterations is uncertain, but the position seems to be that while there has been a decline of over 2 million acres in the arable area, and of 400,000 acres in permanent pasture, the missing area of nearly 2½ million acres has not been entirely lost to agriculture, but is partly accounted for by an increase in rough grazings. Allowing for transfers from one category to another, the net decline in the agricultural area is probably in the neighbourhood of 1 to 1½ million acres. A reduction in the total agricultural area is, of course, inevitable as a consequence of the

extension of building both for residential and industrial purposes which has been so noticeable since the last war.

Although there has been this decline in the arable area and a change which has certainly not been for the better in the area under grass, the live-stock in the country—cattle, sheep, pigs and poultry—have all increased. The number of cattle has risen from 5,878,000 in 1914 to 6,762,000 in 1939 or by 15 per cent., the principal additions occurring among cows and heifers, which are more numerous by 25 per cent. Sheep show a rise of only 4 per cent., but pigs, which have been kept in larger numbers in recent years, show an increase over 1914 of 40 per cent. Horses have definitely decreased, and are now only 60 per cent. as numerous as formerly.

Production, of course, varies from year to year according to weather and other conditions. The output of wheat in 1914 was 7,307,000 qrs., as against 8,323,000 qrs. in 1938, the latter being an exceptionally favourable year, with a higher yield per acre from a somewhat larger area. The output of most other crops, owing to the reduced acreage, was less. On the other hand, the production of meat—that is, beef and veal, mutton and lamb, and pigmeat—on the average of 1909–13 was only 17,593,000 cwts., as compared with 20,326,000 cwts. in 1936–38. Milk production in 1937–38 was put at 1,300 million gallons, and was certainly larger than it was 25 years ago, and this applies to poultry and eggs, fruit and vegetables, except potatoes, the acreage and yield of which have not varied substantially.

The live-stock are not, however, entirely fed from our own soil, and the maintenance of their numbers and output is dependent on the supply of feeding-stuffs imported from abroad. No exact figures are available, but it is evident, from the decline in the arable area on the one hand and the increase in numbers on the other, that our live-stock have been fed on imported food to a much larger extent in recent years than was the case in 1914. So long as this imported supply can be kept up or any shortage made good by the production of animal feeding-stuffs from the new arable land now being brought under cultivation, it is safe to say that the home production of human food—that is, meat, milk, eggs, and poultry—is *potentially* greater now than it was in 1914. This is, however, entirely dependent on the maintenance from one source or another of the supply of food for live-stock. In the same way, though not to the same extent, the growth of crops of all kinds is partly dependent on the importation from abroad of raw fertilisers, and the future supply of these will influence the production of wheat, vegetables and sugar-beet, as well as the production of animal food both from arable and grass-land.

R. J. T.

REVIEWS OF STATISTICAL AND ECONOMIC BOOKS

CONTENTS

1.—*British Association for the Advancement of Science Mathematical Tables.* Volume VII. *The Probability Integral.* By W. F. Sheppard. Completed and edited by the Committee for the Calculation of Mathematical Tables.

The "Tables of the Probability Integral" are based on the work of the late Dr. W. F. Sheppard, who set out to compute this integral to as many places of decimals as would ever be required. Shortly before his death his manuscripts were placed by his family at the disposal of the British Association Mathematical Tables Committee. They have completed and published his work in the belief "that his tables of the probability integral constitute just that memorial to Sheppard's unsurpassed labours in the field of mathematical statistics, which he would himself most greatly have appreciated."

The integral under consideration is the well-known "normal integral"

$$\tfrac{1}{2}(1 \quad \alpha_x) = (2\pi)^{-\frac{1}{2}} \int_x^\infty e^{-\frac{1}{2}t^2} dt \qquad . \quad . \quad (1)$$

To compute it Sheppard made use of Laplace's continued fraction for the ratio of the tail area (1) to its bounding ordinate, viz.

$$x^2 \int_x^\alpha e^{-\frac{1}{2}t^2} dt \qquad (2)$$

which is particularly suited to computing a large number of decimals. His fundamental table (Table II) gives the above ratio to 24 decimals, but at the wide interval of one-tenths of the standard deviation (or, more precisely, for $x = 0{\cdot}0\ (0{\cdot}1)\ 10{\cdot}0$). Sheppard was very fond of using reduced derivatives for purposes of interpolation, and these are tabulated in Table II, where they were easily obtained from the convergents of Laplace's continued fraction. From this fundamental table Sheppard has also produced values of the natural and common logarithms of the tail area (1) at the same tabular interval and over the same range but to 16 and 12 decimals respectively (Tables IV and V).

The Mathematical Tables Committee realized that whilst the number of decimals given by Sheppard was ample, many occasions would arise when the tabulated functions would be required to a finer interval. They have therefore sub-tabulated two of Sheppard's tables and produced 12-decimal values of the ratio (2) over the range $x = 0{\cdot}00\ (0{\cdot}01)\ 10{\cdot}00$ and 8-decimal values of the common logarithm of the tail area (1) (Tables I and VI). For interpolation reduced derivatives are given in the first table, whilst second differences are provided for the logarithms of the tail area. The conversion of logarithms into a large number of significant figures of the tail area should afford no difficulty if appropriate auxiliary tables are used. It will be noted that at least 7 significant figures can be obtained from Table VI over the whole range $x = 0{\cdot}00\ (0{\cdot}01)\ 10{\cdot}00$, which is the advantage of the logarithmic form used as compared with the direct form adopted by the best of the existing tables giving the tail area in terms of the standard deviation. In order to obtain from the ratio (2) values of the tail area (1) large-decimal values of the ordinates $z(x) = (2\pi)^{-\frac{1}{2}} \exp\{-\frac{1}{2}x^2\}$ are required which have to be computed by interpolation from tables of the exponential function or antilogarithms. The last page of the book of tables under review (which is left blank) could have been profitably filled with a table of the distribution function $z(x)$.

The question may rightly be asked whether the large number of decimals published will ever be required. Three or possibly four decimals are, of course, quite sufficient for the practical purposes of the statistician, unless, indeed, he should find the higher decimals of the quantities tabulated in Table II (say) useful as a table of random numbers. But it was not the direct application to practical problems that Sheppard had in mind when he set out to calculate this integral, which is of fundamental importance in practically every branch of natural science. There is a variety of problems (mainly physical) for which the validity of the normal law has to be established to a very high degree of accuracy in order that rigorous conclusions may be drawn from the theory, and for such cases high decimal accuracy of the probability integral is essential. To give an example, a generalization of Airy's theory of absorption spectra leads to the integral

$$\int_0^{x_1} \cos\,(ax - b\alpha_x)dx$$

which has to be evaluated for large values of the parameters a and b, and the number of decimals required for Sheppard's α_x is therefore appreciable.

More examples of this kind could be added to show that Sheppard was wise when calculating this integral to an accuracy which at first sight may appear extravagant. In thanking him and the Committee for having produced what may be termed the fundamental tables of the science of probability we concur, we are convinced, with research workers in practically every branch of natural science.

H. O. H.

2.—*Tracts for Computers*. No. XXIV. *Tables of Random Sampling Numbers*. By M. G. Kendall and B. Babington Smith. Cambridge University Press, 1939.

The first table of random sampling numbers was prepared by L. H. C. Tippett and published in 1927 (*Tract for Computers*, No. XV). It originated in early work when certain theoretical distribution functions had to be compared with results obtained by experimental sampling. More recently, with artificial randomization becoming an essential feature of modern experimental technique, the table has become part of the necessary equipment of the statistician. Its use does away with less satisfactory processes of randomization, such as the tossing of a coin or the shuffling of cards. With modern experiments extending in scale it was found in many cases that the scope of Tippett's tables was not sufficient, and the present table has been prepared mainly to satisfy the need for a longer series of random sampling numbers.

The occasion has also been taken as an opportunity for revising the technique of producing such numbers. It will be remembered that the old table was constructed from census material providing, as it were, random figures in the sense that they were "unlikely to follow any law of choice." Such a source of randomness is, however, viewed with suspicion by the authors of the new table, and they prefer a technique by which randomness is ensured through mechanical means particularly designed for the purpose and described in detail elsewhere.* With the help of this technique the authors produce 100,000 random digits arranged in 100 groups of 1,000 each. The mechanical procedure is then tested by submitting the resulting random numbers to four tests designed to ensure certain necessary properties of random arrangements. These are: the expected frequency of each of the 10 digits (frequency test), the frequency of pairs of digits (serial test), the frequency of the occurrence of sets of four digits (poker test) and finally the frequency distribution of the gaps between consecutive zeros (gap test). Although the complete entity of 100,000 random digits is seen to satisfy these tests, five of the 100 groups (of 1,000 digits each) are found to deviate from expectation when tested as individual groups. This behaviour, called poor local randomness, is of course not surprising. The authors advise the user of the table to avoid these

* "Randomness and Random Sampling Numbers," *R.S.S. Journal*, 1938, p. 101.

groups of poor randomness when selecting short series of random numbers, but the appropriate procedure would appear to be to use the table as it stands and to select the beginning of each random series strictly at random.

The presentation of the tables is perfect. Each group of 1,000 random digits covering half a page is arranged in 25 rows and 40 columns. Appropriate spacing greatly facilitates the selection of one-, two-, and four-figure numbers. H. O. H.

3.—*Probability and Frequency*. By H. C. Plummer, M.A., F.R.S. London: Macmillan. 1940. vii + 277 pp. 15*s*. net.

This book is deceptively entitled. One opens it (apprehensively) expecting to find that another Fellow of the Royal Society has joined in the scrimmage round the question whether probability and relative frequency are the same thing. Actually Professor Plummer is not concerned with this question at all, what little he says about the foundations of the theory of probability being severely orthodox. His main object is to give " an approach to probability and statistics from a mathematical point of view," which amounts, in his modest treatment, to an account of the mathematics required in probability, the theory of errors and the elementary statistical theory of frequency distributions and correlation.

Chapters 1 and 2 deal with the probabilities of discontinuous and continuous populations, and as Professor Plummer himself indicates, are such as might be found in any serious text-books of algebra or integral calculus. They are nearly all occupied with direct probabilities, and we meet the usual problems whose solutions depend on counting the number of ways in which events can happen—the probability that two knights placed at random on a chess-board will attack each other; the chance that if a stick is broken in two places the resulting pieces will form a triangle; the St. Petersburg paradox; Buffon's problem; and so on. Professor Plummer obtains Bernoulli's theorem and Bayes's theorem in the orthodox way; but he does not venture on a complete discussion of their use in statistical inference.

In his Chapter 3 the author deals with the normal law of error and the concomitant topics of precision, rejection of observations and least squares. The fourth and fifth chapters treat of the normal and Poisson laws, the Pearson family of curves, and correlation.

There is nothing much in this book which has not appeared elsewhere, but students of statistics may find it useful to have some of the basic mathematics assembled under one cover.

M. G. K.

4.—*Modern Machine Calculation with the Facit Calculating Machine Model Lx*. By H. Sabielny, translated and revised by L. J. Comrie, M.A., Ph.D., and H. O. Hartley, Ph.D. London: Scientific Computing Service, Ltd. 1939. 74 pp. 5*s*. net.

This manual is a sort of guide-book to the most efficient and economical use of the Facit Model Lx, which has a capacity $10 \times 10 \times 19$. After a few pages on the operation and care of the

machine, it describes the methods of carrying out addition, multiplication and division, with various short cuts, and then proceeds to some of the commoner types of calculation required in business and commerce. Tables for use in work with British weights and measures are appended. The names of Dr. Comrie and Dr. Hartley, who have added a good deal to the Swedish text, are sufficient to ensure that within its prescribed limits the manual is complete.

M. G. K.

5.—*The Economics of a Declining Population.* By W. B. Reddaway. London: Allen and Unwin. 1939. 268 pp. 8*s.* 6*d.*

This book, as its title says, is a study of the economic consequences which may be expected from the decline in the population of Great Britain, now generally assumed to be inevitable in a few years' time. It does not deal with the political and military consequences, which would be the chief considerations if this was one of the great Continental Powers. Naturally, Mr. Reddaway begins with Malthus, and reminds us that in the nineteenth century an increase in population was taken for granted. At the beginning of that century people were anxious to restrict this growth within manageable proportions.

Mr. Reddaway himself explains the scope of his book in the Preface. First he describes how estimates of future population are made, and what the position is in this country. In the second part he examines in turn the probable developments in each of the more important parts of our economic life. Where it seems appropriate, he suggests measures which would help to maximize the advantages and minimize the disadvantages.

In general Mr. Reddaway accepts the conclusions reached by Dr. Enid Charles and Mr. Colin Clark, though he is optimistic enough to think that means will be found to check the catastrophic fall which Dr. Charles predicts a generation or two ahead.

One is apt to regard the death-rate and the birth-rate as the two governing factors, but this view is mistaken. A town's death-rate is no criterion of its healthiness, as the rate depends on the age of the population; thus Bath and Bournemouth have comparatively high death-rates because they are places to which old people "retire." From the mortality-rate, which is based on age-groups, we can calculate how many persons of a given age will be alive at future dates. It is certain, from the composition of these age-groups, that the crude death-rate will rise for some years to come, because the proportion of old people is steadily rising. Mr. Reddaway regards the fertility-rate as more important, since we want to know how many children are likely to be born, and we find a continuous fall in fertility-rates. In England and Wales the birth-rate has declined from 33·5 in 1881–85 to 14·7 in 1933–36. Yet the fall in fertility has been decidedly greater, the birth-rate being kept up by the increased proportion of potential mothers in a population with relatively fewer children.

Some things in the near future are practically certain. Thus we can estimate the number of births for the next ten years or so

with reasonable confidence, and, failing a deliberate and determined social policy to raise fertility-rates, the future population is certain both to be smaller and to have an older age-composition. The net reproduction rate in England and Wales is now between 0·7 and 0·8, whereas a rate of unity is required to maintain the population. Mr. Reddaway believes the birth-rate will fall still further, even though fertility-rates are maintained. In Scotland, however, he sees no reason to expect a decline for many years. He examines Dr. Charles's classic "study," and finds that one of her assumptions has been disproved by later facts; the births each year have been in excess of her estimates. Of course there must be a certain amount of guesswork, because you cannot know what motives will be most powerful ten or twenty years hence. But you do know what numbers there will be then in the various age-groups, and especially the numbers of women of child-bearing age.

Our author does not accept Dr. Charles's alarmist conclusions, one of which would bring down our population to five millions in a hundred years, but he does admit "a steady decline in the proportion of dependent children (0–14) and a steady rise in the proportion of pensioners." A decline in the population is inevitable unless "fertile marriages produce an average of nearly three children. This is almost impossible if a family of two continues to be typical." A decline in the total population will not begin immediately, and will be small for many years, but the change in age composition *has* begun.

Many questions may be asked about the consequences of the present trend in our population, *e.g.* what do we want? will a smaller population mean more unemployment and heavier burdens, or more jobs available? Here each person may give his own answer. Mr. Reddaway is not anxious about the immediate future, but thinks that we must make up our minds what we mean to do now, if we do not want the decline to become beyond the country's power to arrest. As an academic economist, he is more concerned with diagnosis than with prescription, but he does enumerate various remedies. A higher birth-rate might be encouraged in various ways, such as family allowances, larger income-tax allowances and improved maternity benefit. These might be considered desirable in themselves, apart from their influence on the population. There are also more coercive measures, such as the heavy taxation of bachelors (why not of spinsters too?); then posts in public services might be open only to married men, and finally there is the drastic proposal to exclude women from certain employments.

Our author devotes much of his book to general economic questions, particularly unemployment, without bringing them into very close touch with the subject of a declining population. Evidently he holds that the economics of a declining population cannot be understood without a fair acquaintance with ordinary economic laws, which is doubtless true. J. E. A.

6.—*The Theory of Prices. An Examination of the Central Problems of Monetary Theory.* Volume I. By Arthur W. Marget, Ph.D.

New York: Prentice Hall; London: P. S. King. 1938. $9\frac{1}{4}'' \times 6''$. xxv + 624 pp. 18*s*.

A large part of the abundant literature which has appeared in recent years on the subject of monetary theory has been concerned with the alleged failure of the existing body of theory to deal adequately with the problems of the post-war world and, particularly, of the years since 1929. Much has been written of the primary necessity of finding new analytical concepts and a new approach to the essential problems of monetary and trade-cycle theory. The traditional theory, however, has not lacked its champions, and in this, the first of two volumes, Professor Marget seeks to provide a reasoned defence of the substance of received doctrine upon the theory of money and prices against its critics and detractors.

It is natural that Mr. Keynes, who has led the more recent criticism of traditional theory, should be the principal target for Professor Marget's strictures. Indeed, as the author admits, much of this book is nothing more than a polemical tract directed against Mr. Keynes' *Treatise*. Unfortunately, to achieve its object such a tract should be brief, incisive and up-to-date, and this book possesses none of these virtues. The style is leisurely and verbose, the pages overcrowded with footnotes and, worst defect of all, the criticism is directed against a position which Mr. Keynes had already vacated to a large extent when this book was published.

The first part of the book is devoted to a detailed defence of the "quantity equations" against their critics, and the author stresses particularly the distinction between the "quantity equations" and the so-called "quantity theory of money." He argues that, when correctly presented, the "quantity equations" are neither purely mechanistic nor static. They represent, he says "a summary of the slow growth, over a period of centuries, of our knowledge with respect to the forces determining prices," and he holds that their usefulness can be tested only by the usefulness of the body of analysis which lies behind each of their terms.

In the second part of the book the author proceeds to an examination of the doctrine which lies behind the "quantity equations"—an examination which comprises a scholarly and well-documented account of the factors which determine the quantity of money M (including a most interesting discussion of the rôle of the rate of interest), the velocity of circulation V and the volume of transactions T. It is here that we see Professor Marget at his best, and the high standard of most of this section leads the reader to deplore all the more the "pamphleteering" tone of the remainder and of the early chapters. The author's original researches into the history and development of monetary theory have been most fruitful, and part of the book will almost certainly become the *locus classicus* of its subject-matter. Special reference may be made to the chapters on the "income" and "cash balance" approaches to the theory of prices, which include a masterly and exhaustive account of the forces which determine the proportion of his wealth which any individual chooses to hold in the form of money.

In the second volume of this work the author proposes to deal with

the theory of the effect of money upon output, with the theory of savings and investment and with Mr. Keynes' *General Theory.* In fairness, criticism of Professor Marget's general standpoint must await the publication of this second volume. J. E. W.

7.—*Contemporary Monetary Theory.* By R. J. Saulnier. New York: Columbia University Press; London: Humphrey Milford. 1938. $8\frac{3}{4}'' \times 5\frac{3}{4}''$. 416 pp. 20*s.*

The object of this book is to present a critical description of the contributions to monetary and trade-cycle theory of four leading contemporary economists—Messrs. R. G. Hawtrey, F. A. von Hayek, J. M. Keynes and D. H. Robertson. The author has devoted some 100 pages to the writings of each economist. He discusses in each case first the theoretical approach, terminology and analytical concepts, then the explanation of the nature and causes of the trade cycle, and concludes with the methods which are considered necessary to modify the fluctuations of industry and trade.

In general Mr. Saulnier has acquitted his most difficult task in a praiseworthy manner. It is true that the presentation of his data has caused him some difficulty. For example, it is not easy to know at times whether it is the actual theory which is presented or Mr. Saulnier's criticism of that theory. Moreover, as is to be expected in a book of this nature, the standard of treatment of the theories of the economists under review differs appreciably. A comparison of the sections on Professor Robertson and Professor Hayek would illustrate this point. Nevertheless, the author has clearly taken considerable pains to present his statements accurately; his criticisms are always well reasoned and, in many cases, justified.

Mr. Hawtrey's writings are neatly presented and ably summarized, the author being assisted by the fact that the former has always given us a complete and up-to-date account of the development of his thought. Mr. Saulnier could perhaps have given rather greater emphasis to what is possibly Mr. Hawtrey's major contribution to monetary theory—his original development and insistence upon the importance of the so-called "cash balance" approach.

On the other hand, the treatment of Professor Hayek (the other exponent of the monetary aspects of the trade cycle) is less satisfactory, and is tinged with that lack of sympathy which has been typical of the attitude of many Anglo-Saxon economists towards the Austrian school. It would appear that Mr. Saulnier has relied mainly upon *Prices and Production* and *Monetary Theory*, and has tended to neglect those important modifications of his theories which Professor Hayek has presented in various non-English periodicals since the publication of the above two books. Professor Knight's criticisms of the Austrian theory of capital are discussed at some length.

The section on Professor Robertson's writings is well done and shows a tendency on the part of the author to favour an approach to trade-cycle problems in terms of "real" and institutional factors. He lays, with justice, especial emphasis upon Professor Robertson's rôle as a creator of important "theoretical tools of analysis" which point the direction to careful studies of the behaviour of the economic system.

The fourth section, on Mr. Keynes, which is devoted mainly to a discussion of the *General Theory* is perhaps the least satisfactory part of the book, probably because any attempt to do justice to the subtleties and complexities of the *General Theory* in some seventy pages was foredoomed to failure. The author's conclusion is that " Keynes has not been altogether successful in his re-formulation of economic theory." The same might be said of Mr. Saulnier's criticism of the *General Theory*, which could have been greatly strengthened by a more adequate appreciation of the rôle played by risk and uncertainty in Mr. Keynes' analysis.

In the final chapter of the book the author gives a brief account of what he regards as the principal defects of the theories which he has examined. These are the failure to take sufficient account of institutional factors and of the dependence of monetary theory upon the general theory of value and distribution and to distinguish accurately between the operation of monetary and non-monetary forces during the course of the cycle; the inadequate treatment of the time factor and, most important of all, the widely different statements of what constitutes an appropriate policy to overcome the alternation between prosperity and depression. The justice of this latter criticism at least will not be disputed. J. E. W.

8.—*Prosperity and Depression*. New Revised and Enlarged Edition. By Gottfried von Haberler. Geneva: League of Nations. 1939. $9\frac{1}{4}'' \times 6\frac{1}{4}''$. xix + 473 pp.

The first edition of Professor Haberler's now well-known analysis of trade-cycle theories was reviewed in Part IV of the 1937 *Journal* (p. 671). As Mr. Loveday points out in the preface to the new edition, the manuscript of Part I of the book was substantially completed by December 1935, and of Part II by the middle of 1936. The subsequent period has produced not only a veritable torrent of literature on monetary and trade-cycle theory provoked largely by Mr. Keynes' *General Theory*, but also a wealth of economic experience as a result of the violent conjunctural movements in the United States and the cessation of the years of prosperity in the United Kingdom. Accordingly, Professor Haberler has not only revised the entire manuscript in the light of criticism received, but has added an entirely new chapter of some ninety pages dealing with more recent contributions to trade-cycle theory.

In the new chapter the author deals largely with Mr. Keynes' *General Theory* and the literature which emanated from that work, and provides a most useful summary of the recent controversies on such matters as savings and investment, hoarding, the rate of interest, the multiplier and wage policy. Professor Haberler is able to show with a considerable degree of success that many of the ostensible differences between the various schools of thought on these matters are, in fact, differences of terminology and emphasis and not of substance. He concludes this new section with a brief statement of the difficulties of constructing a really dynamic theory of the trade cycle which " would do some justice to the enormous complexity of the real world." J. E. W.

9.—*Monetary Equilibrium.* By Gunnar Myrdal. London: William Hodge. 1939. xi + 214 pp. 12*s*. 6*d*.

When Professor Myrdal's book was published in Sweden in 1932 the Cambridge school was still struggling with the ideas of Keynes' *Treatise on Money*, but even to-day it is of much more than historical interest. We find in it that investment and saving, expressed in the appropriate terminology of the Swedish school, are always equal; we see how increase in investment activity affects income and consumption (multiplier), and how the increased rate of consumption affects investment (relation). Professor Myrdal saw that people generally save a greater proportion out of a higher income, that the workers are more concerned with money wages than with real wages, and that a cut in money wages may reduce the expectations of entrepreneurs to such an extent that unemployment will actually be increased. There are many other points in this book in which the Swedish school anticipated the Cambridge developments.

The core of the book is the analysis of Wicksell's concept of monetary equilibrium, in which, combining great admiration with frank criticism, Professor Myrdal tries to build a new Wicksellian theory which is self-consistent and true.

Wicksell defined monetary equilibrium as a situation in which the normal rate must: (1) equal the marginal technical productivity of real capital; (2) equate the supply and demand for savings; (3) guarantee a stable price level, mainly of consumption goods. In Professor Myrdal's definition Wicksell's normal rate becomes the marginal efficiency of capital, or, in his terminology, the "yield of real capital." Wicksell's first condition then becomes a situation in which the value of the existing capital and its cost of reproduction are equal. When not in equilibrium the gap between the two provides the incentive for investment. Wicksell's third condition is incompatible with the first, and Professor Myrdal discusses the dilemma of monetary policy: the choice between a stable price level and the full consequences of a change in the anticipated yield of capital, suggesting, as of particular statistical interest, an index "weighted with regard to the stickiness of various prices and their significance for profitability and real investment."

Wicksell's second condition can be re-stated by using the *ex ante* and *ex post* technique. The author claims that the elaboration of this technique is the most original contribution of his book. Saving and investment are always equal *ex post*, by definition, while they may differ *ex ante*. The difference between *ex ante* saving and *ex post* saving is made up by movements in incomes during the period. In equilibrium: *ex ante* saving equals *ex post* saving which is equal to *ex post* investment. This condition implies no change in the level of incomes. The terminology is extremely clear, but the "*ex ante*" concept is inferior to Keynes' "propensities" in so far as it does not indicate causal relationships. Professor Myrdal recognizes the importance of movements in income, but neglects his own idea of "effective consumption," and so his treatment of unemployment is isolated from his main argument; unemployment in his view arises as a consequence of the monopolistic position of trade unions and the mono-

polistic or monopsonistic policy of the employers. Interesting pages follow about the indeterminateness of the monetary equilibrium so defined, and the conclusion is that equilibrium of the system is possible on any level of activity.

This book is remarkable for the clarity of its analysis and the great intellectual modesty of its author. The translators, Mr. R. B. Bryce and Mr. N. Stolper, also deserve commendation. G. A. B.

10.—*How to Pay for the War.* By Evan F. M. Durbin. London: Routledge, 1939. $7\frac{1}{2}'' \times 5''$. 119 pp. 3*s.* 6*d.*

Here is a book on an extremely topical if not in every sense a popular subject; it gives the guiding principles which a well-known economist thinks should be applied in financing the present war. He starts by explaining, lucidly and forcefully, the fundamental problem of industrial mobilization—that is, the necessary diversion of industrial resources from peace to war employments and the corollary that, after allowing liberally for absorption of men and machines at present unemployed, this diversion must mean a seriously reduced consumption of peace-time goods and services. Here he is on solid ground. But when he goes on to assert that talk about spreading the burden of the war over time is nonsense, and suggests that any objection the public would have to increasing annual taxation by £2000 millions would be dangerously irrational, his attitude to taxation and tax burden seems dangerously naïve.

After discussing the three possible ways, taxation, industrial control, and loans, to finance the war, Mr. Durbin argues that the first two have overwhelming advantages in the long run, but, as time is an important factor, loans must be employed, particularly in the earlier stages. He shows how large State expenditure and rising money income are not incompatible with low interest rates, provided that the State borrows directly from the commercial banks at very low or nominal interest rates and at the same time rigorously controls their ordinary commercial lending. Under these conditions a moderate expansion of money income is desirable to stimulate private enterprise and savings. Other recommendations are that the direct control of industry should be extended and taxation increased as fast as possible; there is no mention of compulsory personal savings, which came into the limelight just after the book was written.

Disclaiming any desire to produce rabbits out of a hat, Mr. Durbin produces a table based on certain assumptions, including the doubling of money income and the trebling of voluntary saving, to show how five years of war could be paid for entirely by loans at a cost of a mere 20 per cent. increase in the debt burden. But with an increase of real taxation and a post-war capital levy which he recommends, the proportionate money burden of the debt might even be reduced below the pre-war level. And all the other post-war ills are to be avoided by never allowing total expenditure (public and private combined) to fall below the wartime peak level.

Some of the detailed calculations may seem less reassuring. Commencing with the assumption that the Government will require

for war purposes the same additional share of the national income as in 1914–18, which he estimates at 40 per cent., then, as nearly 25 per cent. was being absorbed in 1938–39, he argues rather precipitately that, failing substantial economies in non-military expenditure, "the Government must absorb and emit 65 (25 + 40) per cent. of all our money payments." The reader is warned that the figures are only rough, and is referred to an Appendix for their statistical foundation. There, in the basic estimate of war expenditure in 1914–18, audacity of assumption and lightheartedness of calculation quickly yield the above-mentioned and oft-recurring figure of 40 per cent., but the precise calculation of this basic figure is a little mystifying. One wonders, indeed, if there is an error in simple division. If so, it would seem that the proportion of the national income which the Government, under Mr. Durbin's assumptions, would have to absorb would be even higher than his figure of 65 per cent.

C. O. G.

11.—*Savings in Great Britain, 1922–35.* By E. A. Radice. Oxford University Press. 1939. $8\frac{3}{4}'' \times 5\frac{1}{2}''$. 146 pp. 8*s*. 6*d*.

This book is the second in the series of Oxford Studies in Economics, and sets out to measure the influence of various factors on the growth of British savings in the period after the Great War.

In the early chapters certain familiar (in the main) general hypotheses are brought together and repeated in mathematical form to provide a foundation for the subsequent statistical work, and there is a useful comparison between the equation systems of Mr. Keynes, Dr. Hicks, and Dr. Marschak.

In Chapter IV the relevant statistical problems are discussed, with due emphasis on the limitations of the methods to be applied. Chapter V is a general review of the statistical sources, which are partly published figures and partly private estimates.

In the subsequent chapters multiple correlation analysis is applied to each type of saving in turn in order to discover the influence of changes in the levels of income, rates of interest, etc. The probable errors of many of the calculated coefficients are disturbingly large, but this is to be partly attributed to the paucity of the material. The estimated regression equations serve mainly to indicate possible lines of further research if and when more data are available. The following warning on page 35 is typical of many which are given: "In no case can any results be established with complete certainty, and all kinds of combinations must be attempted before even preliminary results can be given. Finally, if the results accord reasonably well with established economic theory and with practical experience, they are more acceptable than if they lead to strange and inconsistent conclusions."

In his concluding chapter the author suggests that, stated in terms of consumption, none of the results confirm the arguments of the over-investment theorists, that the proportion of income consumed rises at some time during the boom, although none of the years considered are years of full employment and this gap in the data might provide a loophole for such theories. The under-consumption theories are

to some extent confirmed in their implication that the gap between income and consumption becomes progressively wider (p. 120).

On p. 92 a calculated Multiplier is given for the years 1929–36, showing that its value rises in times of slump when investment falls, but as the minimum is 2·5 and the maximum 3·5, Mr. Keynes' suggestion (*General Theory*, pp. 121–2), that 2 or 3 is a rather low figure, is not confirmed.

Reference is made to the diminution in foreign lending since 1930 and to the alternative outlet in residential house-building, the view being expressed that in the absence of a rearmament programme there appeared to be a grave danger of the savings accumulated in building societies running to waste, unless investment was centrally encouraged or saving reduced by a re-distribution of incomes in favour of the lower wage groups. The outbreak of war has, however, reversed the problem completely.

The book has a useful mathematical appendix, an index to the notation used in the mathematical treatment, and a convenient bibliography. The author is to be congratulated on his industry and in getting so much into some 140 pages. The work deserves a place on the shelf of every serious student of econometrics alongside of the parallel investigations of Professor Tinbergen. H. W. G. G.

12.—*The Coal-Mining Industry*, an International Study in Planning. By J. Harry Jones, G. Cartwright, and P. H. Guénault. London: 1939. Pitman. $8\frac{1}{2}'' \times 5\frac{1}{2}''$. Pp. x + 394. 16*s*.

This book, according to the Preface, "consists of a survey of the efforts made by different coal-producing countries to deal with the problems of depression in their respective mining industries. It is concerned with the organization of capital, not the organization of industry." The volume consists of three parts: The British Coal-Mining Industry; The Industry Abroad; and Conclusion. The first part is divided into two sections, the first of which, dealing with the position up to 1928, is mainly statistical and is "virtually a reprint of a paper" by Professor Jones, read before this Society in November 1929. Two years later "the West Yorkshire Coal Owners' Association granted an endowment to the University of Leeds for research into industrial fluctuations, with special reference to the coal-mining and allied industries." The results of this praiseworthy benefaction are embodied in the second section, which was prepared by Mrs. Cartwright and Mr. Guénault, who were engaged in the enquiry, in consultation with Professor Jones. In the second part Mr. Guénault describes the coal industries of Poland, France, and Belgium, which were investigated by his colleague and himself, and they also are responsible for the chapter on Canada, which is based mainly on material submitted to the Nova Scotia Royal Commission of Economic Inquiry, 1934, of which Professor Jones was chairman. The section on Germany is based on two memoranda prepared by Professor Jones, who is also responsible for that on the United States and for the "reflective analysis" of the third part. The examination of the condition of the huge coal industry of the United States occupies only twenty-seven pages,

but Professor Jones hopes "to publish, in due course, another volume which will deal more fully with the industry of the United States and compare the relative merits of recent experiments in the British and American industries, which have more to learn from each other than either has to learn from recent continental experience."

The experience of State control of industry during the European War of 1914–18 and the losses caused by excessive competition during the following years of disorganization and the depression of 1930–32 turned the minds of men engaged in the great industries of the United Kingdom away from the Victorian ideas of *laissez-faire*, and gave a tremendous impetus to the movement towards consolidation of enterprises and their organization in the interests of an industry as a whole apart from the several interests of the persons making their living out of it. With this development went a parallel movement towards the extension of State control over industries for the protection of the community as a whole. What may be the economic effects of the present war is still on the lap of the gods, but it is useful to have a base-line, such as the present authors have drawn, from which future changes can be measured. Professor Jones is so well known as an authority on the coal industry and has received so much assistance from the Government Departments dealing with coal in Great Britain, the United States, and Canada, from the West Yorkshire Coal Owners' Association, and from the Rhenish-Westphalian Coal Syndicate that this book can be taken as authoritative. There is a sufficiency of statistics to illustrate the descriptive matter, but the text is not over-burdened with figures. The authors have aimed at giving an unbiassed account of the various schemes adopted in different countries and of their working, confining criticism to "matters of comparative detail." It would be out of place here to subject their survey to detailed examination, but prospective readers may be assured that they can rely on the fairness and independence of those three writers.

Professor Jones's "reflective analysis," which forms the third part of the book, opens with the confession of an individualist. "In the absence of special circumstances I attach great value to the force of competition as a corrective working within the economic system. In an expanding industry I believe that survival of the test of competition is the best criterion of efficiency that we possess. The burden of proof always rests upon those who desire to restrict freedom of enterprise; the presumption must always be in favour of freedom, no less in the sphere of economic activity than in other spheres of human life. . . . But, like many others, I was driven by unpleasant facts to the conclusion that the circumstances of the (coal) industry had changed so completely as to make its case exceptional rather than typical. . . . It appeared to me that there are overwhelming arguments against allowing the force of unregulated competition to eliminate the excess of producing capacity in the coal-mining industry." Consequently, control of output became necessary, but as that still left the possibility that

local surpluses would have to be sold at reduced prices, it had to be followed by price control, the most efficient form of which is the establishment of sales agencies. The part of the British Act of 1930 enabling the Coal Commission to effect consolidations proved ineffective, but the Act of 1934 gave greater powers. Professor Jones thinks that "the experience of the new sales agencies will exercise a greater and more rapid influence upon concentration of ownership than that exercised by the Commission itself," and that "control of royalties has now given the Coal Commission an entirely new status and far greater power over the organization of the industry," without, however, rendering superfluous the control of prices and output. The United States legislation set up an independent National Coal Commission to which the district associations controlling prices were responsible, whereas in the British scheme the administration was left wholly to bodies set up by the industry itself, subject to a right of appeal by coal users to an independent tribunal. "It seems to me," says Professor Jones, "that the American system is to be preferred to the British system. . . . An independent national authority, in possession of all the facts, and able to prevent abuse rather than deal with an abuse presumed to have been already committed, is in a better position than an appeal tribunal to safeguard the interests of the consumers."

H. W. M.

13.—*The Canadian Balance of International Payments*, a Study of Methods and Results. Ottawa: Dominion Bureau of Statistics. 1939. $10'' \times 6\frac{3}{4}''$. 251 pp. $1.25.

We can heartily congratulate Mr. R. H. Coats, the Dominion Statistician, and Mr. Herbert Marshall, Chief of the Internal Trade Branch of the Bureau, both of whom are Fellows of this Society, on this valuable report, which, one can well believe, "represents the results of several years' effort." This is also an appropriate occasion for congratulating Mr. Marshall on the award to him of the annual medal of the Professional Institute of the Civil Service of Canada for his work on this report, as "an outstanding contribution to science."

The annual estimate made by the British Board of Trade shows "the net balance on all transactions of a revenue character between the United Kingdom and other countries," but the Canadian scheme is much more comprehensive. To quote the report: "A complete balance of payments statement is tripartite. It should contain first a statement of international indebtedness. That is to say, it should show by categories the amount of a country's indebtedness to other countries through borrowings and investments and, on the other hand, the value of its investments in other countries. Secondly, there should be a current account showing a year's transactions in all the visible and invisible items constituting its international trade. In the third place, there should be a current capital account which will contain the record of the year's capital movements inward and outward." Only a bare outline of the procedure adopted in making the calculations can be given here. First

with regard to capital indebtedness, the Bureau of Statistics, in taking the annual Industrial Census, requires particulars to be furnished respecting the capital employed, and then it ascertains by direct enquiry what firms have some British or foreign interest. "Current changes are ascertained by means of a monthly return on the purchase and sales of securities between Canada and other countries which is made through the co-operation of the Investment Dealers' Association, the Canadian Bankers' Association . . . and other agencies." Further, though the field has not been entirely covered, "a considerable amount of information has been obtained by the Bureau from the trust companies in Canada and from other sources on the extent of the assets in Canada administered for non-residents." "Direct investments"—*i.e.*, in "branch plants"—are very important, and particulars as to the geographical distribution of ownership of shares are obtained by means of questionnaires, but there are some gaps in respect of unregistered bonds. "Portfolio investments" present peculiar difficulties and are excluded from the record. On the other side of the account there are uncertainties respecting the amount of Canadian investments in other countries, especially as regards the holding of British and foreign securities by Canadian corporations and individuals; fortunately the effect on the balance of payments statement is not serious.

Commodity trade forms the chief item in the current account, and note is made in the Report of various features of the Canadian trade accounts, affecting the valuation of particular items. Gold transactions are treated separately, and here the co-operation of the Bank of Canada has been helpful. The value of the international tourist trade is ascertained as follows: the method is "to use the detailed records of the numbers of tourists crossing the international boundaries of Canada and to calculate the total expenditures of these by means of average sample expenditures collected directly from individual tourists." The calculation of inward and outward freights by rail and by sea presents a whole swarm of troubles, the description of which and an account of the means taken to overcome them occupy 27 pages—the longest chapter in the Report. With the "excellent assistance" of the shipping industry, particulars as to freight earnings are now obtained by "reports directly from the firms concerned." Interest and dividend receipts and payments are calculated for payments from the Bureau's record of Canadian bond issues held abroad, from information furnished by British financial institutions, from official returns of dividends paid to investors overseas by Canadian companies, and from returns to the income tax department. Receipts, except from United States investors, appear to contain some measure of doubt. The minor elements in the account, such as receipts and payments for migrants' remittances, for educational, charitable and missionary contributions, for films, etc., involve estimates to a greater or less degree.

The annual capital account includes the sale of new issues of Canadian securities outside of Canada, the redemption of Canadian securities held abroad, the international trade in outstanding securities, the movements of capital connected with international

direct investments and trust companies, insurance transactions, international short-term assets and liabilities, and investments abroad of Canadian banks. "Statistics on the amounts of new issues originally sold abroad are at best rather arbitrary approximations in many cases. . . . In compiling statistics of new issues, especially in the early years of the period covered, it has been necessary to rely for the most part upon data available from various external secondary sources." Particulars of redemptions "have been obtained from the bond records showing geographical distribution of ownership." With regard to outstanding securities, "through the co-operation of the Canadian Bankers' Association, the Investment Dealers' Association and the principal stock exchanges in Canada it has been possible to secure monthly reports from these organizations consolidating the trade of their individual members, and these are supplemented by returns from the minority of independent firms. As for direct investments, a schedule was sent in 1936 to all companies and firms falling in the group of direct investments. "Excellent co-operation was received from the many hundreds of firms to which the schedule was sent, and a fairly complete record of the relevant transactions during the period has been obtained. Similar reports have been secured in subsequent years." Particulars as to insurance transactions are also obtained by schedule.

Statistical tables covering the years 1926–37 occupy 71 pages. It is interesting to observe that in 1937 the total of British and foreign investments in Canada was 6,765 million dollars, of which 2,685 million were British and 3,932 United States. Included therein were "direct investments" valued at 2,809 million dollars, of which 2,471 million were under non-Canadian control (1,886 million United States, 446 British), at the end of 1937. Canadian investments abroad at the end of 1937 were 1,758 million dollars (of which 1,098 million were in the United States), including a total of direct investments, 511 million. In the current account commodity trade (corrected) on a 12-year average accounted for 54·52 per cent. of the debits and 62·41 of the credits, while tourist expenditure was 6·42 per cent. of the debits and 16·04 of the credits, and interest and dividends averaged 20·17 of the debits. A debit or credit "balancing item" each year represents in part the capital inflow or outflow, but it also absorbs all the errors in the estimated details. The total sales and purchases of securities between Canada and other countries were, respectively, 507 and 511 million dollars in 1937 and 369 and 340 million dollars in 1938, and 77 per cent. of those transactions were with the United States.

The Bureau is to be congratulated on the powers which it possesses, and both the Bureau and Canadian manufacturers, traders and financiers are to be congratulated on the way in which they cooperate. In appreciating the results of their joint efforts we may use the modest words of the Preface: "While it cannot be claimed that all problems of measurement have been solved, and while in certain parts of the field it is still necessary to resort to estimates, a solid statistical basis has been built up of sufficient comprehensiveness to warrant considerable confidence in the results of the study as a whole,

and to reduce residual errors and omissions to relatively small proportions." H. W. M.

14.—*The National Income of Hungary.* By Matthias Matolcsy and Stephen Varga. London: P. S. King & Son. 1938. vii + 116 pp. 15*s.*

This is an English edition of a Hungarian work revised and enlarged so as to cover the financial years 1924–25 to 1936–37. Definitions of the national income of different countries vary greatly, being apparently formulated to suit the conditions prevailing in the respective countries. Remembering that the traditional Hungarian definition of Dr. Fellner and his compatriots excludes all derivative incomes from the national dividend, it is not surprising to find that the authors maintain that ". . . the machinery required for keeping order and securing safety, as well as the whole civil service . . . ensures only the maintenance of the present economic and social order and the maintenance of the present level of production." But, even so, there is another stumbling-block: "Concerning the expenses of education, it seemed to us that, as part of the population, possessing a certain standard of knowledge, dies year by year, the costs of equipping successive generations with the same amount of knowledge are similar in character to the maintenance charges of capital goods and could just as little be regarded as an addition to the national income as those." The same principle is applied to doctors, lawyers, etc., and one wonders how far the authors would logically carry it in respect of increases in population, of improvements in the standard of knowledge, or of the non-luxury goods consumed.

All figures are given both in value and in "volume." The latter is the value expressed in prices based on the three-years' average 1924–25 to 1926–27. The "volume" of the whole national income is, therefore, a better estimate of the real income than the usual procedure of inflating the money national income with a general index. Volume and value both increased from 1924–25 to 1928–29, declined during the next four years, and then recovered. There are great divergences between value and volume, the latter being greater than value by about 4 per cent. in 1924–25 and about 62 per cent. in 1936–37. The explanation offered is that roughly 40 per cent. of the total volume is represented by agricultural products. The steady industrialization of the country is shown by a continuous rise of about 0·5 per cent. per annum in the share of the nominal value of industrial products in the national income, from 25 per cent. in 1924–25 to 31 per cent. in 1936–37.

The value of domestic work is estimated at between 6 and 12 per cent. of the national income on the assumption that unoccupied married women spend half the day in domestic work, employed married women spend a quarter, and domestic servants the whole day. The value of one day's domestic work in the country is supposed to be equal to half the daily wage of female agricultural labourers, while in towns the earnings of domestic servants are taken.

The value of indirect taxation, which forms roughly 8 per cent. of the total value, was included to enable the authors to obtain

the real income. The Hungarian edition of this work came out much earlier than Colin Clark's *National Income and Outlay*, and this treatment in both is similar. Again, with regard to capital accumulation, up till 1930 capital investment plus increase in gold and foreign assets was entirely offset by the increase in foreign indebtedness. From 1931 onwards borrowing from abroad ceased, and 3–5 per cent. of the total national income went yearly into net investment.

The whole calculation of the national income is based on the Census method with occasional evaluations on "income" basis, with an estimated maximum error of about 10–12 per cent. Unusually detailed information is provided for many branches of Hungarian production, but the agricultural section is undoubtedly the most valuable part of this work. G. A. B.

15.—*The Dynamics of Automobile Demand.* Published by General Motors Corporation, New York. 1939. 11″ × 8½″. 139 pp.

With over 25,000,000 private cars registered, one for every five inhabitants, and the most complete statistical service in the world, it might be thought that the United States offered the ideal field for a statistical evaluation of demand. But the factors are too complex. There have not been wide enough swings in the prices of cars. The real price of cars has been chronically in only one direction—down. This is the general conclusion of S. M. Dubril on this collection of four interesting studies, presented to the American Statistical Association in 1938. S. L. Horner, in the first essay, states the problem as "an attempt to isolate and measure the effect of price and price changes upon the demand for automobiles." C. F. Roos and V. von Szeliski contribute a brilliant analysis of the factors governing changes in the domestic demand. The demand for automobiles is a derived demand. The primary demand is for individual transportation. If the primary demand is satisfied by greater durability, then in periods of falling national income, price adjustment is ineffective in stimulating demand. (This is confirmed by the 1939 estimate of the Automobile Manufacturers' Association of U.S.A. that replacements of old vehicles accounted for all the new cars sold in the domestic market during 1938.) A. L. Court attempts the difficult task of computing an index of automobile prices per unit of quality (comfort, performance, durability, etc.), referred to as a "Hedonic Price Index." Finally, S. M. Dubril sums up and interprets the previous studies, illustrating them by an analysis of the financial statements of various automobile manufacturers.

S. J. A.

16.—*Management and Labour.* By K. G. Fenelon. London: Methuen. 1939. 7¼″ × 5″. vi + 276 pp. 7*s*. 6*d*.

This book has developed from lectures delivered at the Manchester College of Technology, of which the author is the Director of the Department of Industrial Administration. The object is to present a survey of the problems involved in the practice of industrial management and the administration of industrial enterprises.

In order to illustrate how the consideration of these problems has gradually become of first-class importance, the author traces the history of systematic methods of management, from the first tentative suggestions and experiments early in the last century to the highly complicated science it has become to-day. The survey covers every feature of industrial organization, both as regards systems of management and administration and problems connected with industrial relations and labour management, special attention being given also to problems of working conditions and environment. The development and necessity of scientific training for management and administration are emphasized, and various systems for recruiting and training foremen, apprentices and for the selection of labour are discussed. The importance of psychological and scientific research in problems connected with the employment of labour, both from the point of view of output and of general welfare, are clearly shewn; and examples are given of many interesting results from tests and experiments. Chapters are devoted to industrial diseases and accidents and measures adopted for prevention and compensation. A comprehensive survey with a historical background is given of the organization of labour, including the Trade Union Movement, and the gradual development of methods of arbitration and conciliation in cases of industrial dispute. The two last chapters are devoted to the methods promoted for obtaining economic security among the working classes, and also methods of securing contacts between management and labour in firms operating on a large scale.

The book, which the author states has been written for students of industrial management and administration, should not only be valuable in this respect, but should appeal to a wider circle. The survey is not only most comprehensive, but is also so lucidly written that it can be appreciated by the general reader interested in the many and varied problems presented by modern industrial life.

D. P. E.

17.—*Other New Publications.**

International Institute of Agriculture.

[The International Year Book of Agricultural Statistics for 1938–39 is arranged in the same way as were preceding editions, but it is mentioned in the Introduction that an endeavour has been made to complete all the statistical tables with the most recent data, making possible the calculation of world totals of production up to 1938 in the northern hemisphere and to the first months of 1939 in the southern hemisphere. Complete figures of imports and exports in 1938 are also given so far as they are available. The volume extends to over 1000 pages. In addition to much miscellaneous information, it gives the area, production, and number of live stock in each country in the world separately, and also the total world production by bringing together particulars for each country under headings such as cotton, wheat, cattle, etc. The world trade in all the important agricultural products is given, showing imports, exports, and balance of trade for each country. Particulars for Germany are given: (1) for the area included in the 1937 frontiers and (2) for Austria; Czecho-slovakia is treated separately.]

* *See also* "Additions to Library," pp. 137 *et seq.*

Research and Statistical Methodology: Books and Reviews of 1933–1938. Edited by O. K. Buros, Rutgers University Press, New Brunswick, U.S.A. 100 + vi pp. $1.25.

[This is a catalogue of books published in English during the period 1933–8 on the methodology of research and on statistical methods, together with excerpts of reviews of those books. At a time when it is difficult to keep pace even with reviews the compilation will be of interest and value to statisticians, and will, we hope, be continued for future years.]

University of Liverpool: Social Science Department. The Economic Status of Coloured Families in the Port of Liverpool. Liverpool University Press, 1940. $8\frac{1}{2}'' \times 5\frac{1}{4}''$. 23 pp. 1*s.*

[This Report is designed to elucidate a problem which is very real in the ports of this country. It supplements the account of an investigation made in 1930 (for the Liverpool Association for the Welfare of Half-Caste Children) on the Colour Problem in Liverpool and other Ports, and is especially confined to Liverpool and to the present economic conditions. The enquiry was undertaken under the auspices of the Liverpool Association for the Welfare of Coloured People, and was organized by Mr. Caradog Jones and his staff. It covered 225 families—"all that could be readily traced and visited"—and the questions asked were those used for the Merseyside Survey. Out of 214 heads of families whose birthplace was ascertained 155 were West African, 34, West Indian, 13 were English half-castes, the rest were from India, East Africa, the United States and Canada. All came as seamen, and as they are usually unfitted or disinclined for any other calling there is much unemployment. The difficulty of finding work for half-caste adolescents aggravates the problem. Figures relating to income, rent, budgets, employment, etc., are given, and, as in the Merseyside Survey, some detailed accounts of particular persons give the statistical skeleton a humane aspect.]

STATISTICAL NOTES

(1) British Official Statistics

On page 118 we give a table summarizing the oversea trade of the United Kingdom for the years 1938 and 1939. Figures are only available for trade in *merchandise* since the outbreak of war, and the particulars given in the table for bullion and specie and for shipping movements accordingly relate to the twelve months ended August 1939, and not to the calendar year. The trade accounts published since the outbreak of war give information concerning the value of each class and group of merchandise imported, exported or re-exported, but no details are given of the quantity and value of the imports or exports of particular descriptions of goods. Owing to the suppression, doubtless for good reasons, of information relating to quantities, it is impossible to give the usual quantitative analysis of the trade of the country last year. Certain facts stand out, however, from the published value figures.

First of all the trade balance. Up to August there had been a fall of £11·2 million in the adverse balance compared with the corresponding period of last year. A further fall of £6·7 million was recorded in September as a result of much-reduced imports. Since then there has been a considerable rise each month—£4·1 million in October, £13·9 million in November and £13·7 million in December. For the year as a whole the adverse balance rose from £387 million in 1938 to £401 million last year. The year 1940 will undoubtedly see a very noticeable further rise.

The outbreak of war, in 1939 as in 1914, had an immediate drastic effect on both our import and our export trade; imports in September were 33 per cent. lower in value than a year earlier, the corresponding falls for exports and re-exports being 42 and 41 per cent. respectively. Subsequently while re-exports remained little more than half what they were a year earlier, imports and exports rose considerably, and in December were respectively 17 and 3 per cent. higher than in December 1938. Shipping was probably the main cause of the reduction in trade at the outbreak of war. Many vessels were requisitioned for the transport of the Expeditionary Force to France, the Mediterranean was closed for three weeks, and the necessity for entering all goods at the Customs before shipment caused much delay and congestion at the docks. The necessity for

the introduction of the convoy system so early in the war caused an immediate loss of efficiency in the employment of shipping and the loss in carrying capacity has been reflected in the trade figures. Other difficulties arose from the institution of import and export licensing systems, which covered in each instance a very wide range of goods, and as these restrictions could not be brought into operation gradually, as they were during 1914–18, they accordingly resulted in a great hold-up of trade, particularly as regards exports. To what extent that hold-up caused the trade returns for November and December to be more favourable than would otherwise be the case cannot be stated, but there is no doubt that our trade recovered from the initial shock much more rapidly on this occasion than it did in 1914. The following table shows the relationship to the average for the three months before the war of the value of imports and exports in each of the first four months of war in 1914 and 1939:—

War month	Imports		Exports	
	1914	1939	1914	1939
	Per cent.	Per cent.	Per cent.	Per cent.
First	72	62	57	59
Second	76	77	63	63
Third	88	104	68	96
Fourth	95	108	58	103

It was not till May 1916 that our exports exceeded this pre-war average, whereas on this occasion the pre-war figure was exceeded in the fourth month of the war. For imports the pre-war figure was exceeded in the fifth month of the last war and the third of this. One reason for the more rapid improvement this time is that prices, of imports in particular, have risen more rapidly than they did in 1914. In the absence of the usual calculation by the Board of Trade and the quantitative data normally published, the extent of the rise cannot be ascertained, but the rise of 25 per cent. in wholesale prices between August and December indicates that this is likely to be substantial. For food and industrial raw materials the rises were 31 and 22 per cent., respectively, and as the quotations on which these are based are very largely for imported goods, a large rise is clearly indicated for imports, though this may well not be wholly reflected in the trade returns. For the group described by the Board of Trade as "manufactured articles," comprising the more finished goods for which wholesale prices are available, the rise has been 12 per cent., and this again is probably considerably

greater than the increase in average values of exports, partly because of the time lag and partly because exports comprise mainly more highly finished articles than those included in the wholesale prices index number.

Prior to the outbreak of war, retained imports of food, drink and tobacco had fallen by £15·2 million (6 per cent.), the reduction of £12·5 million for grain and flour being outstanding. A further fall of £14·0 million occurred after August, the largest reductions during this period being £7·5 million for tobacco and £5·8 million for grain and flour. In December, however, there was a rise of £7·3 million, the only substantial decline being for tobacco.

For raw materials there was a fall of £5·9 million (4 per cent.) in retained imports in the eight months before the war, cotton and rubber each showing decreases of about £4½ million and wool a rise of nearly £5 million. Subsequently there has been a rise of £5·2 million, mainly as a result of larger imports of cotton—up by over £9 million. Timber fell by £6½ million and wool by £3 million.

Retained imports of manufactured goods showed the opposite movement to raw materials, an increase of £15·8 million before the war being followed by a fall of £3·4 million subsequently. Many of the groups show the effect of import restrictions, and the fall would have been doubled had it not been for a rise of £3·3 million in retained imports of iron and steel.

An increase of £1·7 million was recorded in exports of food, drink and tobacco in the eight months before the war, followed by a fall of £1·9 million, though in the latter period exports of beverages—essentially spirits—showed a rise of nearly £1 million. Exports of raw materials, which had risen by £1·2 million, fell during the first four months of the war by £3·7 million. Exports of coal were maintained in value, but there was a marked fall of £1·5 million in exports of raw wool, etc., and most other groups showed smaller declines.

Exports of manufactured articles rose by £3·1 million up to August and fell by £30·9 million subsequently. Heavy declines were recorded during the last four months of the year for vehicles (£8·7 million), machinery (£8·1 million) and iron and steel (£4·9 million); exports in most of the other groups also showed a reduction, but small increases were recorded for coke and manufactured fuel, silk and artificial silk goods, foot-wear and rubber manufactures. Cotton goods showed a negligible decline, and in December were £1·3 million more than a year earlier.

Movements and Classes	Twelve Months ended December, 1938	Twelve Months ended December, 1939	Increase (+) or Decrease (−)
Imports, c.i.f.—	£'000	£'000	£'000
Food, drink and tobacco	430,117	399,460	(−) 30,657
Raw materials and articles mainly unmanufactured ...	247,923	241,491	(−) 6,432
Articles wholly or mainly manufactured	233,811	237,889	(+) 4,078
Other articles	7,658	7,104	(−) 554
Total Imports	919,509	885,944	(−) 33,565
Exports, f.o.b.—			
United Kingdom Produce and Manufactures—			
Food, drink and tobacco	35,894	35,707	(−) 187
Raw materials and articles mainly unmanufactured	56,920	54,391	(−) 2,529
Articles wholly or mainly manufactured	365,244	337,452	(−) 27,792
Other articles	12,697	11,256	(−) 1,441
Imported Merchandise—			
Food, drink and tobacco	12,299	10,866	(−) 1,433
Raw materials and articles mainly unmanufactured ...	30,251	24,544	(−) 5,707
Articles wholly or mainly manufactured	18,277	9,930	(−) 8,347
Other articles	698	586	(−) 112
Total Exports	532,280	484,732	(−) 47,548

Bullion and Specie—	Twelve months ended December, 1938	Twelve months ended August, 1939
Imports	257,732	245,072
Exports	331,619	682,607

Movements of Shipping in the Foreign Trade—	Number of Vessels	Thousand Tons Net	Number of Vessels	Thousand Tons Net
Entered with cargoes—				
British	24,215	38,908	24,596	39,359
Foreign	26,004	29,464	25,974	29,123
Total entered	50,219	68,372	50,570	68,482
Cleared with cargoes—				
British	29,183	34,511	29,812	35,078
Foreign	21,521	24,370	22,459	25,815
Total cleared	50,704	58,881	52,271	60,893

Bullion and specie figures are not available for the year; up to August imports amounted to £165 million and exports to £461 million, showing a net loss of £295 million in gold and £1 million in silver. In the year 1938 exports had exceeded imports by £74 million.

There was very little change in the general level of *wholesale prices* during the first eight months of 1939, a fall of 1·5 per cent. in the prices of articles of food and tobacco between December 1938 and August 1939 practically balancing a slight rise (0·5 per cent.) in the prices of industrial materials and manufactures. Thus, measured by the Board of Trade index-number (1930 = 100), which stood at 98·3 for December 1938, the fall in general prices had only amounted to 0·2 per cent. by August, when the number stood at 98·1. From the commencement of the war, however, prices advanced steeply, and by December the index number had mounted to 120·9, an advance of about 23 per cent. both as compared with December 1938 and with August 1939. The increases in price over the four months September to December were greatest in cereals (60·7 per cent.) and in textiles generally—cotton 50·4 per cent., wool 27·9 per cent., and other textiles 37·9 per cent. Most other commodities advanced in price appreciably, especially meat and other articles of food, and in some cases (*e.g.* sugar, butter, and bacon) it was soon found necessary for the Government to fix maximum prices and to establish rationing. As far as industrial materials were concerned, control of many of them was taken over by the Government, prices being stabilized in some cases and supplies to consumers regulated by licence or otherwise. In other cases the Government became the purchaser of the actual and prospective supplies. As a consequence, although in most cases market prices are still quoted, it is not always clear that supplies are available in desired quantities to would-be purchasers or holders of licences, and it would seem that there is, from one aspect, some artificiality in index numbers quoted under such conditions. There is, however, no reason to suppose that transactions on any large scale take place except at the fixed prices or range of prices, and it is not likely that the variation in the quantities sold or available would alter to any extent the weights used in the Board of Trade Index Number at any rate for the present. The *Board of Trade Journal*, while continuing to publish the index numbers for the various groups of commodities, has ceased to print the actual monthly prices of the commodities themselves, but fairly reliable figures are available from other sources as to the current prices of many of them. The smallest increases in price over the

first four months of the war were in the coal and iron and steel groups, which advanced 8·6 per cent. and 10·3 per cent., respectively. A further advance in iron and steel prices of about 10 per cent. is, however, arranged for February. The increases were highest, as has been stated, in cereals and textiles, *English Gazette* averages for barley and oats showing advances of over 100 per cent. and English wheat of over 50 per cent., but the increase in the prices of imported cereals were considerably less. The rise in the price of American cotton has also been very considerable, and "middling American," which at the end of August 1939 was $5\frac{1}{2}d.$ per lb. at Liverpool, was $8\frac{3}{4}d.$ at the end of December—an advance of nearly 60 per cent.—Egyptian raw cotton advanced 46 per cent.

The Board of Trade index numbers of wholesale prices from July 1939 are given below :—

(Averages for the year 1930 = 100)

Date	Total Food	Total not Food	All Articles	Basic Materials	Inter-mediate Products	Manu-factured Articles	Building Materials
July 1939	91·2	101·7	98·1	94·3	102·0	109·1	103·3
Aug. ,,	90·4	102·2	98·1	94·5	104·0	108·7	104·1
Sept. ,,	101·2	107·7	105·6	101·6	113·3	111·3	107·3
Oct. ,,	109·4	111·4	110·9	107·8	116·2	114·2	107·7
Nov. ,,	115·3	118·0	117·2	117·5	121·7	119·8	109·5
Dec. ,,	120·1	121·1	120·9	121·9	125·0	122·0	110·3
Dec. 1938	*91·8*	*101·7*	*98·3*	*89·9*	*102·5*	*111·2*	*103·5*
,, 1937	*104·4*	*109·2*	*107·6*	*106·9*	*107·7*	*114·1*	*105·0*
Year 1939	*97·5*	*105·1*	*102·6*	*98·1*	*106·5*	*111·3*	*104·8*
,, 1938	*97·3*	*103·5*	*101·4*	*92·9*	*104·5*	*112·1*	*104·1*
,, 1937	*102·2*	*112·0*	*108·7*	*122·9*	*108·9*	*111·4*	*104·2*

The figures of certain other British index numbers and the official index number for the United States (Bureau of Labour) are given below for comparison. The French and German index numbers are not available for the period of the war.

Date	Board of Trade (1930 = 100)	*Economist* (1927 = 100)	*Statist* (1866–77 = 100)	*The Times* (1913 = 100)	U.S.A. (Bureau of Labour) (1926 = 100)*
Dec. 1938	98·3	68·6	89·1	113·8	76·9
July 1939	98·1	69·3	88·7	111·9	75·3
Aug. 1939	98·1	70·3	90·4	114·5	74·8
Dec. 1939	120·9	91·7	120·1	142·5	79·0

Average of weekly figures.

Figures compiled by the Wholesale Textile Association in collaboration with the Bank of England, and showing by index numbers the *value* of wholesale textile trading, are now published monthly in the *Board of Trade Journal.* "Practically all the important houses in the large centres of distribution are contributors," and the index numbers for the years 1935–39 for Great Britain are set out below, the average monthly sales in 1937 being taken as 100.

Year	Home Trade	Export Trade	Total Trade
1935	96	83	96
1936	99	90	99
1937	100	100	100
1938	94	82	93
1939	105	82	104

The export index number for September 1939 (74) showed a heavy decrease as compared with that for September 1938 (100), but for the following three months the index numbers showed considerable increases on those for the corresponding months of 1938, due for the most part, no doubt, to the increased prices of many classes of textile goods.

The Ministry of Agriculture have published Part I of their Agricultural Statistics for 1938, containing figures relating to the acreage and production of crops, number of live-stock and of agricultural workers, and output and prices of Agricultural Produce in England and Wales. In addition to the detailed statistics for the year 1938, summary tables are given for the principal items covering the ten years 1929–38 for Great Britain and the United Kingdom as well as England and Wales. Many of the figures have been published separately in advance of this publication, and particulars of the fluctuations in the index numbers and of the estimated value of the agricultural output have already appeared in this *Journal* (Part II, 1939, p. 326, and Part III, 1939, p. 477).

This volume of agricultural statistics, which is accompanied by a short report, covers all the main items of statistics relating to agriculture in 1938 except those relating to supplies of agricultural produce and requirements imported from overseas and their relation to home production. These questions are reserved for Part II, the latest issue of which refers to 1936.

At January 1st, 1940, the average level of *retail prices* of articles of working-class consumption had risen about 12 per cent. compared with prices at September 1st, 1939. According to the Ministry of Labour index number, retail prices of food had advanced 14 per cent., those of articles of clothing 20 per cent., fuel and light 10 per cent., and miscellaneous articles of expenditure nearly 6 per cent. Taking the general level of these prices at July 1914 as 100, the index number for January 1st, 1940, was 174 and that for September 1st, 1939, was 155. The principal increases in food prices were in respect of sugar and bacon, which advanced 49 and 31 per cent., respectively. Most of the increase in sugar was due to the increased duty of 1*d.* per lb. imposed in September. Maximum retail selling prices have been fixed for meat, sugar, butter, margarine, eggs, bread, and potatoes, the increased cost of some of them being partly met by a Government subvention. Up to now only butter, bacon, and sugar have been rationed to consumers, the rationing of meat not coming into operation until March 11th. By January 1st the prices of the cheaper portions of beef and mutton had risen 14 to 19 per cent. and of the dearer joints 8 to 10 per cent. The index numbers for the various classes of expenditure are given below at the undermentioned dates, the prices for July 1914 being taken as 100.

Date	Food	Rent and Rates	Clothing	Fuel and Light	Other Items	All Items
Jan. 1st, 1939	138	161	205–210	180–185	175	155
Sept. 1st, ,,	138	162	205–210	180–185	180	155
Jan. 1st, 1940	157	162	250	200	190	174

The net effect of all the changes* in *wages* reported to the Ministry of Labour in 1939 was a weekly increase of nearly £900,000. About 5,482,000 workpeople received increases and 68,000 received decreases. Most of the increases were arranged since the commencement of the war, and in the four months commencing September about four and three-quarter million workpeople received advances totalling to about £830,000 per week. This is the largest increase in rates of wages since 1920. It is estimated by the Ministry of Labour that in all industries (including agriculture) for which information is available the average level of weekly full-time rates of wages was at the end of 1939 from 4 to 4½ per cent. higher than at the end of 1938. In 30 of the 47 districts into which

* Excluding changes in the wages of agricultural labourers, domestic servants, Government employees, shop assistants, and clerks.

England and Wales is divided for the purposes of the Agricultural Wages Regulation Act of 1924, advances of wages to Agricultural labourers of from 1*s*. to 3*s*. 6*d*. per week have taken place since the beginning of September.

About 380,000 workpeople had their hours of labour reduced to the extent of about 3½ hours per week. These workpeople were chiefly juvenile workers under 16 years of age and their hours were reduced to a maximum of 44 per week from July 1st, 1939, under a provision of the Factories Act of 1937. The working hours were also reduced of certain classes of surface workers at coal mines in Cumberland, and at shale mines in Scotland.

The net number of workpeople directly involved in *trade disputes* causing stoppage of work in 1939 was 245,000, and 91,000 more were thrown out of work at the establishments where the disputes occurred, but were not parties to the disputes. The number of working days lost in consequence of trade disputes in 1939 was about 1,340,000, compared with 1,330,000 similarly lost in 1938. Disputes in the coal-mining industry accounted for over three-fifths of the workpeople involved in the disputes and for rather more than two-fifths of the working days lost.

Except for the first two months of the year, *employment* in 1939 was consistently better than in the corresponding month of 1938, and the mean rate of unemployment in the insured trades in Great Britain and Northern Ireland* in 1939 was 10·5 per cent. as compared with 12·9 per cent. in 1938 and 10·8 per cent. in 1937. From February to August the improvement in employment was continuous, the proportion unemployed dropping from 13·3 per cent. in the former month to 8·7 per cent. in the latter. Following on the outbreak of war, however, unemployment increased to an appreciable extent, and the proportion unemployed rose to 9·1 per cent. at September 12th and to 9·6 per cent. at October 17th. There was a slight improvement in November and a greater one in December 1939, but at the end of the year the rate of unemployment (9·1 per cent.) was still above that for August. The increase in unemployment was principally among women and girls, and is stated to have been due to large numbers of women, who had not previously been in the insured trades, offering themselves for work in connection with the war. The number of men out of work was slightly less in December than in August, and this no doubt was due to a considerable extent to the calling up of men for service with the colours. It is quite obvious that the con-

* Excluding Agriculture.

siderably increased numbers of men employed in munition work of all kinds had not at the end of the year compensated for the dislocation in industry caused by the outbreak of war. Unemployment at the end of 1939 was, however, much less than at the end of 1938 (9·1 per cent. compared with 12·9 per cent.). Except in London, unemployment in 1939 showed a decline in all the administrative areas. It was lowest in the South-Western and South-Eastern Divisions, and in the Midlands (6·5, 7·7, and 7·4 per cent., respectively), and highest, as usual, in Northern Ireland (23·0 per cent.). It was 18·5 per cent. in Wales, 15·7 per cent. in the Northern Division, 13·5 per cent. in Scotland, 13·6 per cent. in the North-Western, 10·1 per cent. in the North-Eastern, and 8·1 per cent. in the London Division. Unemployment in agriculture in 1939 showed little change as compared with 1938. The proportion out of work varied from 2·6 per cent. in the South-Western Division to 10·3 per cent. in Wales; in Northern Ireland it was 22·3 per cent.

As compared with December 1938 employment at the end of 1939 had improved in 90 out of the 105 groups into which the Ministry of Labour divides the insured trades. In the remaining groups, covering about 2,000,000 workpeople, there was a decline, which was most marked in dressmaking and millinery, in printing and bookbinding, and in laundries. In eleven groups covering nearly 2½ million workpeople the rate of unemployment was over 15 per cent.; in two groups—public works construction and entertainment trades, etc.—it was 24·5 per cent. and 20·7 per cent., respectively, and in the building trades 15·8 per cent.

The percentages unemployed in the insured trades in Great Britain and Northern Ireland from July to December 1939 are set out below :—

Date	Percentage Unemployed in Great Britain and Northern Ireland of Workpeople Insured under				
	General Scheme	Agricultural Scheme	General and Agricultural Scheme		
			Males	Females	Total
July 10th, 1939 ...	9·0	4·0	9·3	7·2	8·7
Aug. 14th, „ ...	8·7	3·9	8·9	7·4	8·5
Sept. 11th, „ ...	9·1	3·4	8·2	10·6	8·8
Oct. 16th, „ ...	9·6	4·0	8·9	10·4	9·3
Nov. 13th, „ ...	9·5	5·1	8·8	10·4	9·2
Dec. 11th, „ ...	9·1	7·4	8·8	9·8	9·1
Dec. 12th, 1938 ...	*12·9*	*8·9*	*13·5*	*10·4*	*12·7*

The total number of workpeople aged 16 to 64 (insured and uninsured) on the registers of the Employment Offices of the Ministry of Labour in Great Britain for the last six months of 1939 is given below. The total recorded for December 11th, 1939, included 10,102 boys and 13,595 girls between the ages of 14 and 16, and 18,844 boys and 24,717 girls between the ages of 16 and 18.

Date	Wholly Unemployed	Temporarily Stopped	Persons Normally in Casual Employment	Total
July 10th, 1939	1,013,636	190,364	52,424	1,256,424
Aug. 14th, ,, ...	968,108	211,978	51,606	1,231,692
Sept. 11th, ,, ...	1,052,218	227,099	51,611	1,330,927
Oct. 16th, ,, ...	1,221,655	146,451	62,532	1,430,638
Nov. 13th, ,, ...	1,213,345	135,233	54,010	1,402,588
Dec. 11th, ,, ...	1,170,798	143,065	47,662	1,361,525
Dec. 12th, 1938	*1,474,019*	*294,708*	*62,645*	*1,831,372*

As compared with December 1938, the numbers on the registers at the end of 1939 declined by 469,847. The number of men over 18 fell by 486,938 and the number of women increased by 20,588.

(2) Other Statistics

The general downward tendency in *Stock Exchange Values* during 1937 and 1938 continued during 1939 with some slight reaction in the middle of the year. With the outbreak of the war prices fell away steeply, and the index number * of the *Bankers' Magazine*, which stood at 107·4 at August 18th, 1939, fell to 103·3 at September 19th, a fall of 3·8 per cent. (values at December 1921 = 100). During the following three months prices recovered to some extent, and for November and December the index number stood at 108·7, or rather more than 1 per cent. above that for August, but lower than in any December since 1931.

The Group of Fixed Interest Stocks recovered to a somewhat greater extent than that for variable Dividend Securities, the index numbers for the two groups being respectively 112·4 and 100·8 in December and 110·9 and 100·2 in August. British Railway preference and ordinary stocks had improved appreciably by the end of the year, and there was an advance in the prices of United States railway shares. On balance British and Indian Funds showed little change during the four months of war, but at the end of the year they were considerably below (4·7 per cent.) the prices obtaining at

* Based on the prices of 365 representative securities.

the end of 1938. Since the end of the year, however, these stocks have advanced very appreciably, due no doubt to a considerable extent to the announcement of the British Government's proposals for the conversion of the 4½ per cent. loan.

The statistics of *retail sales* prepared by the Bank of England in conjunction with various retail distributors' Associations and co-operative societies show that over the eleven months February to December 1939 there was an increase of 3·1 per cent. in value as compared with the sales in the corresponding period of 1938. Sales of food and perishables increased 4·3 per cent. and of non-food merchandise 1·8 per cent. It would appear, so far as the sales of food are concerned, there was some decline in the quantities sold, as over the same period the Ministry of Labour index number recorded an increase in food prices of nearly 14 per cent. The two sets of figures are not strictly comparable, as the Ministry of Labour's index number relates to working-class prices only, but there is no reason to suppose that other food prices have not also advanced considerably.

No exact figures can be given as to the advance in *shipping freights*, as the index number of the Chamber of Shipping has not been published for any month later than August. What evidence is available shows that freights have advanced very considerably, especially for cargoes carried in neutral shipping.

The October issue of the *Monthly Crop Report of the International Institute of Agriculture* contains a review of the *world wheat position* in the cereal year, August 1939 to July 1940. The outbreak of war necessarily makes the forecasts more conjectural and less reliable than usual, as the normal flow of international trade has been interrupted and several Governments have suspended the publication of information relating to production and trade movements. For some countries there are no official estimates of the 1939 wheat crop, while for others there are only unrevised forecasts made before harvest. So far, however, as can be judged, the world wheat production is large, being only 8 per cent. less than the record wheat crop of last year and 15 per cent. above the 1933–37 average.

Taking the main areas separately, the European crop of 1939, though less than last year, is appreciably larger than the five- or ten-year averages. In North America also the crops, though not as large as in 1938, are the next best obtained since 1932. Pro-

duction in Asia is excellent, and the out-turn in Africa surpasses all previous records. In South America and Oceania the results are forecast as appreciably better than average.

As a result of these favourable harvests, there will be a considerable addition to the exportable supplies, which are made up partly of the surplus from the new crops, after satisfying internal requirements and providing for a normal carry-over, and partly of the large stocks remaining on August 1, 1939, from the exceptionally large crop of last year. The total quantities available for export in the 1939–40 season are accordingly estimated to amount to 1,313 million bushels as compared with 1,146 million bushels in 1938–39.

The *wheat requirements* of the importing countries are likely this year to be affected by a number of conflicting influences. Demand is likely to be reduced by the abundance of domestic wheat supplies in the majority of importing countries, as well as by restrictions on consumption and economy in purchase. On the other hand, import requirements may be increased by the natural tendency in the present situation to increase domestic reserves in anticipation of greater difficulties in future, and to make good smaller supplies of some other food-stuffs. Obviously estimates tend to be conjectural and liable to great modification in practice, but, subject to these reserves, the Institute estimates the import requirements as about 600 million bushels, as against 618 million bushels last year. It will be seen that, compared with the heavy exportable supplies, the import requirements are very low. This must result in a further increase in stocks in hand at the end of the season (*i.e.*, in the middle of 1940), even if these forecasts undergo in the actual event fairly considerable modification.

The November *Bulletin* gives some later figures, but these largely counterbalance one another, and the Institute does not suggest any change in the above general forecast.

The forest area within the boundaries of the U.S.S.R. is said to represent one-third of the world's forest lands, and although only about one-half of this is actually being exploited, timber is one of the most important items of Russian export. In the last year or two, however, a marked contraction has taken place in the *Russian Timber Trade*. Between 1930 and 1936 and also before 1913, exports amounted to 10 million cub. metres and upwards annually, whereas in 1938 they were little more than one-half this quantity. According to an article in the *Monthly Bulletin of Agricultural Economics*, published by the International Institute of Agriculture, this decline was due, in part, to the heavy fall in the prices of sawn

timber which in 1935 were less than half what they had been in 1929. The result of these low prices was the establishment of export quotas by the European Timber Exporters Convention, to which Russia adhered and under which her exports of sawn timber were reduced.

Apart from this, the policy recently adopted in regard to the exploitation of forests in Russia is said to have had the effect of reducing exports, although this policy will take many years to be fully carried out. It aims generally at a shifting of the areas of exploitation to the well-wooded but less accessible territories in the north and east of the U.S.S.R., partly with the object of developing and improving forest resources, and partly to prevent the denudation of the forest areas in the Central industrial regions. The progress of the policy is, however, likely to be slow, as it requires great capital investment, technically skilled labour, and an improvement of living conditions in the remoter forest districts by the construction of roads, houses, schools, social centres, etc., in order to make the lives of the workers more attractive. The execution of the policy is also impeded by the need for exporting timber in the interests of the balance of trade.

Another factor in the supply of wood for export is the expansion in the home demand. The carrying out of the programmes of the Five-Year Plans has required large quantities of timber of different types, and the consumption of sawn wood increased from 7·1 million cub. metres in 1913 to 29·3 million cub. metres in 1936. The use of wood is also increasing industrially for the manufacture of plywood, paper, artificial silk, and fibre and other purposes.

On the whole it is thought that the U.S.S.R. will hardly be able to export a greater proportion of its timber production than it does now without adverse effects on the steadily rising home demand.

About 59 per cent. of the total Russian exports in 1938 consisted of sawn timber, 17½ per cent. of pit props and 15 per cent. of pulp wood, leaving 8 per cent. for other descriptions. The United Kingdom is the largest customer, and in 1938 took over one-half of the total exports of sawn wood, while Holland took 16 per cent. and Germany 13 per cent.

In the pulp-wood trade there have been great changes as a result of the development of the Russian paper and cellulose industries. Exports have declined from 4½ million cub. metres in 1934 to a little over 1 million in 1938 : in 1934, Germany took 60 per cent. amounting to 2·7 million cub. metres, but by 1938 the quantity had fallen to 91,800 cub. metres or 8·7 per cent. Consignments to Britain fell in the same period from 389,000 cub. metres to 121,000 cub. metres. The exports of pit-props have also declined.

CURRENT NOTES

In 1921 Professor Pigou published a book on *The Political Economy of War* and has now issued a fresh and revised edition (Macmillan's, viii + 169 pp., 5*s*.). We would draw attention to this work as certain to be of use to those interested in tracing the economic effects of "war measures." In fourteen short and very clear chapters the author first discusses sundry general topics, such as the economic causes for war, the resources available for war, personal economies, etc., and then surveys what was done in the name of the State by taxing and borrowing, by controlling prices and rationing, by financing through new money and bank credits, and so on. A brief economic commentary accompanies the description and summary of what was done. All these matters, as Professor Pigou says, are "once more of dominating interest," for "economic factors may be expected to play a decisive part" in the present conflict.

Since 1921 various changes in economic organization have taken place and attention is directed to them. For example, "now this country is possessed of a vast medley of marketing boards. . . . Thus, *prima facie*, we are in a better position for introducing an effective control of prices than we were a quarter of a century ago." But it is added that it "is, to say the least, doubtful" whether bodies intended for the maintenance of "fair" prices can be used to prevent farmers and middlemen from earning too much. It will do no harm if readers of this little book are led to return to Sir W. H. Beveridge's larger treatise on *British Food Control* (published in 1928 by the Oxford University Press for the Carnegie Endowment for International Peace) in order to observe in greater detail the working of the various official agencies in 1914–18.

Professor Pigou subjoins a sad little Epilogue. The corresponding chapters in the original book were followed by chapters on "the aftermath of the war as regards currency, national debt and government control," but those have been omitted, for "now, at the beginning of our struggle—a struggle that may wreck the world—it is not in place." His concluding words are : "As an economist I have not the power, nor, as a man, the heart, to strain through a night so black to a dawn I shall not see." This is too melancholy an ending for a man who is only 63. *Sursum corda !*

There lies before us a document, the operative part of which reads as follows: "Universitas Litterarum Serdicensis Sancti Clementis Achridensis, quinquagesimum a quo conditum est annum celebrans, virum clarissimum nobilissimum Arthurum Lyon Bowley, Professorem Londinensem, ob merita ejus erga scientiam statisticam, Doctorem Juris honoris causa creavit atque declaravit."

We congratulate our President on his continued conquest of the Balkan States.

Looking through *Hansard* for March, one discovers a question by Sir Herbert Williams as to the average weekly number of persons placed in employment through the Central Register and the two subsidiary registers, to which the Minister of Labour, Mr. Ernest Brown, replied by giving the average weekly number of persons notified as placed from the Central Register, adding that "as different factors govern placing on the other two registers no combined average can be given." In a further reply to a supplementary question the Minister embroidered his first answer by stating that "an average may be a mathematical fact but a social and spiritual fiction." Since one definition of Statistics is the *Science of Averages* one feels that a wider publicity should be given to the Minister's *obiter dictum*, which is certainly a colourful expansion of our President's statement in Chapter V of the *Elements of Statistics* that "an average is a purely mathematical conception." Members of Parliament, whether on the front or back benches, seem to have only the haziest notions of either the meaning or the use of averages, beyond that they are something the statistical Civil Servants put in answers to parliamentary questions. One might take leave to remind them that, as our President has put it, "by the use of averages complex groups and large numbers are presented in a few significant words or figures," or, to use Mr. Yule's mode of exposition, averages are necessary in the classification of any long series of observations. Averages, however, must be properly used, for, again to quote our President: "The use of an average in suggesting fields for special investigation is common in statistics. Of itself an arithmetical average is more likely to conceal than to disclose important facts; it is of the nature of an abbreviation, and is often an excuse for laziness" (*The Measurement of Social Phenomena*, p. 46).

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ITALY—

Les Assurances Sociale, July–August, 1939—Les maladies chroniques et l'invalidité en ce qui concerne plus spécialement le rhumatisme et la tuberculose : *A. O. Zorini.*

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October, 1939—La scienza economica e l'autarchia : *C. E. Ferri.*

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Rivista di storia economica, September, 1939—Alexander Wedderburn e la teoria della rendita sino al 1777 : *W. R. Scott.* Teorie dei cicli e problemi di metodo : *M. Lamberti.*

JAPAN—

Kyoto University Economic Review, October, 1939—Characteristic features of Japanese small industries and policies for their development : *I. Otsuka.*

SWEDEN—

Ekonomisk Tidskrift, December, 1939—Om finskt näringsliv och dess jämförbarhet med svenskt : *C. A. J. Gadolin.* Några problem rörande definitioner och terminologi i teorien för de internationella kapitalrörelserna : *Karin Kock.*

SWITZERLAND—

Zeitschrift für schweizerische Statistik und Volkswirtschaft, 1939, *III.*—Reiseverkehr und Volkswirtschaft Graubündens : *H. Gurtner.* Inlandserzeugung, Einfuhr, Ausfuhr, Verbrauch und fiskalische Belastung alkoholischer Getränke in der Schweiz 1885–1938 : *A. Schellenberg, E. T. H. Wädenswil and R. F. Gosselke.*

INTERNATIONAL—

International Labour Review, January, 1940—Housing policy in war-time: *Carl M. Wright.* The compensation of war victims: Germany.

Monthly Bulletin of Statistics, January, 1940—Monthly exports of certain commodities. Central monetary gold reserves of the world.

Revue de l'Institut International de Statistique, Livr. 2–3, 1939—Le Comité d'experts statisticiens de la Société des Nations (1931–39): (avec résumé anglais): *M. Huber.* Una nuova rappresentazione grafica (avec résumé français et anglais): *F. Vinci.* Inquiry concerning statistics of wool stocks in the principal wool-producing and wool-consuming countries of the world (avec résumé français): *E. H. Godfrey.*

LIST OF ADDITIONS TO THE LIBRARY

Since the issue of Part IV, 1939, the Society has received the publications enumerated below :—

I.—OFFICIAL PUBLICATIONS

(a) United Kingdom

Colonial Office. Labour conditions in the West Indies : report by *Major G. St. Orde Browne.* London : H.M.S.O., 1939. Cmd. 6070. 9¾″ × 6″. 216 pp. 3*s.* 6*d.*

Commissioner for the Special Areas (England and Wales). Report of the Committee of Enquiry into Land Settlement. London : H.M.S.O., 1939. 9¾″ × 6″. vi + 158 pp. 2*s.* 6*d.*

Economic Advisory Council. Committee on Nutrition in the Colonial Empire. First report—Part I, Nutrition in the colonial empire. London : H.M.S.O., 1939. Cmd. 6050. 9¾″ × 6″. 210 pp. 3*s.*

Education, Board of. Educational pamphlet No. 94. An outline of the structure of the educational system in England and Wales. London : H.M.S.O., 1939. 7″ × 4¾″. 47 pp. 9*d.*

—— *Science Museum.* Hand-list of short titles of current periodicals in the Science Library. Part 1 : alphabetical. 5th ed. First supplement, compiled to the end of 1938. London, H.M.S.O., 1939. 10½″ × 7¼″. 151 pp. 2*s.* 6*d.*

Foreign Office. Germany No. 1 (1939). Final report by *Sir Nevile Henderson, G.C.M.G.*, on the circumstances leading to the termination of his mission to Berlin, September 20, 1939. London : H.M.S.O., 1939. Cmd. 6115. 9¾″ × 6″. 27 pp. 3*d.*

Miscellaneous No. 9 (1939). Documents concerning German-Polish relations and the outbreak of hostilities between Great Britain and Germany on September 3, 1939. London : H.M.S.O., 1939. Cmd. 6106. 9¾″ × 6″. xxvii + 195 pp. 1*s.*

Health, Ministry of. The Ministry of Health and its work. London : H.M.S.O., 1939. 8½″ × 5½″. 34 pp. 6*d.*

National health insurance and contributory pensions insurance : an outline of the schemes in the United Kingdom of Great Britain and Northern Ireland. London : H.M.S.O., 1939. 8½″ × 5½″. 34 pp. 6*d.*

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Overseas Trade, Department of. Reports : 733. Argentine Republic, June 1939. 194 pp. 3*s.* 6*d.* 734. Southern Rhodesia, Northern Rhodesia, and Nyasaland, May 1939. 75 pp. 1*s.* 3*d.* 735. Egypt, June 1939. 103 pp. 2*s.* 736. Canada, July 1939. 165 pp. 2*s.* 6*d.* London : H.M.S.O., 1939. 9″ × 6″. 4 parts.

Transport, Ministry of. Road traffic census 1938. Report including tables of statistics of traffic recorded on class I (including trunk) roads in Great Britain at the general census taken in August 1938. London : H.M.S.O., 1939. 13″ × 8¼″. 28 pp. 2 maps. 1*s.* 3*d.*

Treasury. War damage to property. I. Report of a conference presided over by the *Rt. Hon. Viscount Weir.* II. Statement of Government policy on the report. London : H.M.S.O., 1939. Cmd. 6116. 9¾″ × 6″. 19 pp. 4*d.*

(b) British Empire

Palestine—

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Union of South Africa—

Census and Statistics Office. Sixth census of the population of the Union of South Africa, enumerated 5th May 1936. Vol. X. Unemployment. Pretoria: 1939. 12″ × 9½″. x + 219 pp.

Report on road vehicle accident statistics 1938. Pretoria: 1939. 12″ × 9½″. xix + 36 pp. 2*s.* 6*d.*

(c) Foreign Countries

Argentina—

Dirección General de Estadistica. Estadistica industrial de la Republica Argentina correspondiente al año 1937. (Informe No. 69.) Buenos Aires: 1939. 9½″ × 6½″. 107 pp.

Buenos Aires, Municipalidad de la Ciudad de. Cuarto censo general 1936. Poblacion 22.x.1936. Tomo III. Buenos Aires: 1939. 12″ × 9″. xiv + 413 pp.

Austria—

Österreichisches Statistisches Landesamt. Der Viehbestand im Gebiete der Ostmark nach den Ergebnissen der im Deutschen Reiche durchgeführten Viehzählung vom 3 Dezember 1938. Vienna: 1939. 11¾″ × 8¼″. 112 pp.

Colombia—

Departamento de Contraloria. Informe financiero del Contralor General de la Republica de Colombia, correspondiente al año fiscal de 1938. Bogota: 1939. 11¾″ × 8½″. 276 pp.

Direccion General de los Censos. Primer censo nacional de edificio efectuado el 20 de abril de 1938. Bogota: 1939. 9½″ × 7¾″. xvi + 393 pp.

Direccion Nacional de Estadistica. Estadistica fiscal y administrativa 1938. Bogota: 1939. 11¾″ × 8½″. 169 pp.

Denmark—

Det Statistiske Department. Danmarks Retspleje den borgerlige Retspleje 1926–35 og Strafferetsplejen 1926–32. Copenhagen: 1939. 11½″ × 9½″. 62 + 82 pp.

Hungary—

Office Central Royal Hongrois de Statistique. Recensement de la population, de la propriété foncière et du cheptel, en 1938, dans la zone nord recouvrée. Résultats, par commune, du recensement effectué sur le territoire réincorporé en vertu de l'arbitrage de Vienne du 2 novembre 1938. (Publications Statistiques Hongroises, 108). Budapest: 1939. 10½″ × 7½″. xxvi + 51 + 277 pp. Maps.

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(c) **Foreign Countries**—*Contd.*

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Sweden—

Statistiska Centralbyrån. Befolkningsrörelsen, översikt för 1921–1930. Stockholm: 1939. $9\frac{1}{2}'' \times 6\frac{1}{2}''$. 118 + 155 pp.

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Switzerland—

Eidgenössisches Statistisches Amt. Die Bevölkerung der Schweiz. Bern: 1939. $10\frac{1}{4}'' \times 7''$. 58 pp.

Die Statistik in der Schweiz. Bern: 1939. $10\frac{1}{4}'' \times 7''$. 42 pp.

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Circular No. 463. Wholesale markets for fruits and vegetables in 40 cities. 142 pp. 15*c*. No. 471. Forests and employment in Germany. 51 pp. 10*c*.

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Technical Bulletin No. 600. The influence of climate and grazing on spring-fall sheep range in Southern Idaho. 41 pp. 10*c*. No. 626. Stumpage prices of privately owned timber in the United States. 162 pp. 20*c*. No. 685. Cotton prices in spot and future markets. 71 pp. 15*c*. Washington: Government Printing Office, 1938–9. $9'' \times 6''$.

United States Tariff Commission. Imports, exports, domestic production and prices. Petroleum, coal, lumber, copper, together with excise taxes collected thereon. Washington: 1939. $12'' \times 10''$. 48 pp.

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(d) **International**

International Institute of Agriculture—

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International Labour Office—

Studies and reports Series G. no. 5. Facilities for the use of workers' leisure during holidays. $9\frac{1}{2}'' \times 6\frac{1}{4}''$. 96 pp. 2*s*. 6*d*.

Series I, No. 4. The law and women's work: a contribution to the study of the status of women. $9\frac{1}{2}'' \times 6\frac{1}{4}''$. xii + 590 pp. 12*s*. 6*d*.

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(d) **International**—*Contd.*

League of Nations—

Economic Intelligence Service—

Raw materials and foodstuffs: production by countries, 1935 and 1938. 10½″ × 8½″. 75 pp.

Statistical testing of business-cycle theories, II. Business cycles in the United States of America 1919–1932. By *J. Tinbergen.* 9½″ × 6″. 244 pp. 5*s.*

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Jeffreys (*Harold*). Theory of probability. Oxford : Clarendon Press, 1939. $9\frac{1}{4}'' \times 6\frac{1}{4}''$. vi + 380 pp. 21*s*.

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—— Tables of random sampling numbers. (Dept. of Statistics University of London, University College, Tracts for computors No. XXIV.) Cambridge : University Press, 1939. $9\frac{1}{4}'' \times 6\frac{1}{4}''$. x + 60 pp. 3*s*. 9*d*. (From Prof. E. S. Pearson.)

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Pigou (*A. C.*). The political economy of war: a new and revised edition. London: Macmillan, 1940. 7¼″ × 4¾″. viii + 169 pp. 5*s*.

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Punjab, Board of Economic Inquiry. Publication No. 63. Farm accounts in the Punjab 1936–37. xvii + 295 pp. *Rs*. 1–8. Publication No. 64. Relationship between fertility and economic and social status in the Punjab: being the results of an inquiry conducted in 275 villages in 25 districts of the Punjab and comprising about 20,000 rural families, by *S. P. Jain*. viii + 34 pp. 8 *annas*. Lahore: C. & M. Gazette, 1939.

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REVENUE OF THE UNITED KINGDOM

Net Produce in Quarters of 1939, *and in Financial Years ended March* 31, 1938-39, 1937-38, 1936-37, 1935-36

(000's omitted.)

QUARTERS, ended	March 31, 1939	June 30, 1939	Sept. 30, 1939	Dec. 31, 1939	Total for calendar year 1939
	£	£	£	£	£
Customs	56,697	61,523	71,116	65,406	254,742
Excise	26,000	26,950	34,300	39,900	127,150
Stamps and Estate etc. Duties ...	26,300	25,310	23,830	20,660	96,100
Other Inland Revenue Duties ...	770	350	140	150	1,410
Post Office	23,800	21,350	20,950	22,350	88,450
National Defence Contribution ...	8,620	5,040	7,100	7,270	28,030
	142,187	140,523	157,436	155,736	595,882
Income Tax and Super Tax ...	286,137	16,906	45,004	60,986	409,033
	428,324	157,429	202,440	216,722	1,004,915
Motor Vehicle Duties	24,039	4,629	3,882	1,310	33,860
Crown Lands	310	320	330	320	1,280
Interest on Sundry Loans ...	907	356	2,995	907	5,165
Miscellaneous Receipts	3,559	1,159	4,571	8,151	17,440
	457,139	163,893	214,218	227,410	1,062,660

YEARS, ended March 31,	1938-39	1937-38	1938-39 (compared with 1937-38)		Corresponding years	
			Increase	Decrease	1936-37	1935-36
	£	£	£	£	£	£
Customs	226,326	221,561	4,765	—	211,282	196,642
Excise	114,200	113,700	500	—	109,500	106,700
Stamps and Estate etc. Duties	98,410	113,150	—	14,740	177,130	113,720
Land Tax and Mineral Rights Duty	—	—	—	—	730	785
Other Inland Revenues ...	1,550	1,730	—	180	—	—
Post Office	89,850	87,375	2,475	—	82,950	77,750
National Defence Contribution	21,890	1,420	20,470	—	—	—
	552,226	538,926	28,210	14,920	521,592	495,597
Income Tax and Super Tax ...	398,431	355,046	43,385	—	310,777	289,094
	950,657	893,982	71,595	14,920	832,369	784,691
Excess Profits Duties ... ... / Corporation Profits Tax	—	—	—	—	1,000	1,300
Motor Vehicle Duties	35,608	34,608	1,000	—	32,727	30,752
Crown Lands	1,330	1,330	—	—	1,350	1,360
Interest on Sundry Loans ...	5,698	5,230	468	—	4,550	4,934
Miscellaneous Receipts ...	12,942	13,510	—	568	24,600	21,738
Total	1,006,235	948,660	73,063	15,488	896,596	844,775
			NET INCR. 57,575			

TRADE OF THE UNITED KINGDOM
for the years **1937-38-39**
(From the Monthly Trade Returns, December, 1939)

Values (c.i.f.) of **Imports***

	Year ended December 31, 1937	1938	1939	Increase or Decrease, 1939—1938	Increase or Decrease, 1939—1937
I. FOOD, DRINK AND TOBACCO—	£	£	£	£	£
A. Grain and flour	91,367,180	74,418,338	55,495,075	− 18,923,263	− 35,872,105
B. Feeding-stuffs for animals	11,347,741	11,419,250	8,936,037	− 2,483,213	− 2,411,704
C. Animals, living, for food	7,122,058	9,220,418	10,993,699	+ 1,773,281	+ 3,871,641
D. Meat	87,058,849	90,679,859	93,262,310	+ 2,582,451	+ 6,203,461
E. Dairy produce	72,904,974	80,013,976	76,050,261	− 3,963,715	+ 3,145,287
F. Fresh fruit and vegetables	36,485,544	37,657,557	35,425,809	− 2,231,748	− 1,059,735
G. Beverages and cocoa preparations	48,486,037	46,512,572	40,809,420	− 5,703,152	− 7,676,617
H. Other food	58,319,928	56,910,094	64,779,401	+ 7,869,307	+ 6,459,473
I. Tobacco	18,007,377	23,284,633	13,707,711	− 9,576,922	− 4,299,666
Total, Class I	431,099,688	430,116,697	399,459,723	− 30,656,974	− 31,639,965
II. RAW MATERIALS AND ARTICLES MAINLY UNMANUFACTURED—					
A. Coal	23,790	14,739	18,760	+ 4,021	− 5,030
B. Other non-metalliferous mining and quarry products and the like	5,260,533	4,711,225	5,306,096	+ 594,871	+ 45,563
C. Iron ore and scrap	12,657,597	11,152,127	9,714,936	− 1,437,191	− 2,942,661
D. Non-ferrous metalliferous ores and scrap	19,143,144	16,335,920	17,928,480	+ 1,592,560	− 1,214,664
E. Wood and timber	61,772,285	42,852,348	37,064,204	− 5,788,144	− 24,708,081
F. Raw cotton and cotton waste	48,729,690	29,579,628	34,268,898	+ 4,689,270	− 14,460,792
G. Wool, raw and waste, and woollen rags	52,139,314	42,618,055	40,585,048	− 2,033,007	− 11,554,266
H. Silk, raw, knubs and noils	2,265,175	2,062,164	2,521,379	+ 459,215	+ 256,204
I. Other textile materials	12,526,091	11,775,441	12,819,073	+ 1,043,632	+ 292,982
J. Seeds and nuts for oil, oils, fats, resins and gums	35,214,712	30,595,876	31,070,858	+ 474,982	− 4,143,854
K. Hides and skins, undressed	25,168,989	18,233,113	15,496,115	− 2,736,998	− 9,672,874
L. Paper-making materials	15,943,241	16,042,767	15,669,641	− 373,126	− 273,600
M. Rubber	12,248,226	11,503,292	9,547,316	− 1,955,976	− 2,700,910
N. Miscellaneous raw materials and articles mainly unmanufactured	12,142,975	10,446,972	9,480,684	− 966,288	− 2,662,291
Total, Class II	315,235,762	247,923,667	241,491,488	− 6,432,179	− 73,744,274
III. ARTICLES WHOLLY OR MAINLY MANUFACTURED—					
A. Coke and manufactured fuel	82,237	8,298	6,055	− 2,243	− 76,182
B. Pottery, glass, abrasives, &c.	8,237,437	7,341,104	5,183,779	− 2,157,325	− 3,053,658
C. Iron and steel and manufactures thereof	19,805,244	14,501,669	17,597,629	+ 3,095,960	− 2,207,615
D. Non-ferrous metals and manufactures thereof	55,713,076	40,816,966	38,662,140	− 2,154,826	− 17,050,936
E. Cutlery, hardware, implements and instruments	7,754,287	7,052,969	5,683,377	− 1,369,592	− 2,070,910
F. Electrical goods and apparatus	4,082,970	3,104,940	2,801,727	− 303,213	− 1,281,243
G. Machinery	24,222,057	21,836,980	24,530,881	+ 2,693,901	+ 308,824
H. Manufactures of wood and timber	8,390,594	6,286,483	4,996,274	− 1,290,209	− 3,394,320
I. Cotton yarns and manufactures	3,428,524	3,135,264	2,277,699	− 857,565	− 1,150,825
J. Woollen and worsted yarns and manufactures	4,229,228	3,838,109	3,438,940	− 399,169	− 790,288
K. Silk yarns and manufactures	5,723,994	4,946,149	3,611,780	− 1,334,369	− 2,112,214
L. Manufactures of other textile materials	4,974,894	4,605,997	7,113,597	+ 2,507,600	+ 2,138,703
M. Apparel	8,383,278	8,027,854	5,657,249	− 2,370,605	− 2,726,029
N. Footwear	2,494,619	2,794,347	2,756,320	− 38,027	+ 261,701
O. Chemicals, drugs, dyes and colours	13,857,534	13,268,241	15,933,268	+ 2,665,027	+ 2,075,734

* The value of the Imports represents the cost, insurance and freight; or, when goods are consigned for sale, the latest sale value of such goods.

Values (c.i.f.) of **Imports**—*Contd.*

	Year ended December 31, 1937	1938	1939	Increase or Decrease, 1939—1938	Increase or Decrease, 1939—1937
III. ARTICLES WHOLLY OR MAINLY MANUFACTURED—*Contd.*	£	£	£	£	£
P. Oils, fats and resins, manufactured	46,524,452	44,070,485	46,794,428	+ 2,723,943	+ 269,976
Q. Leather and manufactures thereof	9,788,744	6,440,292	7,735,717	+ 1,295,425	− 2,053,027
R. Paper, cardboard, &c.	17,198,797	14,841,996	15,648,588	+ 806,592	− 1,550,209
S. Vehicles (including locomotives, ships and aircraft...	6,459,447	4,532,639	5,492,140	+ 959,501	− 967,307
T. Rubber manufactures	740,164	783,231	1,144,784	+ 361,553	+ 404,620
U. Miscellaneous articles wholly or mainly manufactured	22,809,920	21,576,719	20,822,405	− 754,314	− 1,987,515
Total, Class III	274,901,497	233,810,732	237,888,777	+ 4,078,045	− 37,012,720
IV. ANIMALS, NOT FOR FOOD	3,030,178	3,318,458	3,100,560	− 217,898	+ 70,382
V. PARCEL POST	3,557,303	4,339,379	4,003,219	− 336,160	+ 445,916
Total	1,027,824,428	919,508,933	885,943,767	− 33,565,166	− 141,880,661

Values (f.o.b.) of* **Exports** *of Produce and Manufactures*

	Year ended December 31, 1937	1938	1939	Increase or Decrease, 1939—1938	Increase or Decrease, 1939—1937
I. FOOD, DRINK AND TOBACCO—	£	£	£	£	£
A. Grain and flour	1,731,371	1,670,308	1,204,609	− 465,699	− 526,765
B. Feeding-stuffs for animals	636,974	698,533	603,416	− 95,117	− 33,558
C. Animals, living, for food	95,862	8,535	29,512	+ 20,977	− 66,350
D. Meat	1,150,975	1,233,898	1,191,691	42,207	+ 40,716
E. Dairy produce	1,158,596	1,112,331	1,115,093	+ 2,762	− 43,503
F. Fresh fruit and vegetables	531,697	394,031	330,838	− 63,193	− 200,859
G. Beverages and cocoa preparations	15,528,080	13,824,631	15,944,933	+ 2,120,302	+ 416,853
H. Other food	12,902,897	12,036,852	10,279,387	− 1,757,465	− 2,623,510
I. Tobacco	5,038,385	4,914,904	5,007,291	+ 92,387	− 31,094
Total, Class I	38,774,840	35,891,023	35,706,770	− 187,253	− 3,068,070
II. RAW MATERIALS AND ARTICLES MAINLY UNMANUFACTURED—					
A. Coal	37,653,529	37,406,306	38,258,793	+ 852,187	+ 605,264
B. Other non-metalliferous mining and quarry products and the like	1,351,699	1,057,636	1,231,672	+ 174,036	− 120,027
C. Iron ore and scrap	890,970	594,891	344,153	− 250,738	− 546,817
D. Non-ferrous metalliferous ores and scrap	3,219,255	2,298,115	1,561,031	− 737,084	− 1,658,224
E. Wood and timber	101,462	71,266	59,848	− 11,418	− 41,614
F. Raw cotton and cotton waste	965,624	458,879	447,361	− 11,518	− 518,263
G. Wool, raw and waste, and woollen rags	9,069,350	6,262,842	4,709,995	− 1,552,847	− 4,359,355
H. Silk, raw, knubs and noils	362,063	661,414	1,357,534	+ (93,120	+ 995,471
I. Other textile materials	228,506	268,835	153,305	− 115,530	− 75,201
J. Seeds and nuts for oil, oils, fats, resins and gums	3,754,087	2,893,839	1,966,358	− 927,481	− 1,787,729
K. Hides and skins, undressed	1,800,686	1,108,125	981,683	− 126,442	− 819,003
L. Paper-making materials	1,870,972	1,177,652	937,739	− 239,913	− 933,233
M. Rubber	387,199	227,256	220,716	− 6,540	− 166,483
N. Miscellaneous raw materials and articles mainly unmanufactured	2,973,762	2,433,375	2,161,044	− 272,331	− 812,718
Total, Class II	64,629,164	56,920,431	54,391,232	− 2,529,199	− 10,237,932

* The value of the Exports represents the cost and the charges of delivering the goods on board the ship, and is known as the "free on board" value.

Values (f.o.b.) of **Exports**—*Contd.*

	Year ended December 31, 1937	1938	1939	Increase or Decrease, 1939—1938	Increase or Decrease, 1939—1937
III. ARTICLES WHOLLY OR MAINLY MANUFACTURED—	£	£	£	£	£
A. Coke and manufactured fuel...	4,234,918	3,291,723	4,000,140	+ 708,417	− 234,778
B. Pottery, glass, abrasives, &c.	9,972,129	9,610,109	9,445,763	− 164,346	− 526,366
C. Iron and steel and manufactures thereof	48,370,349	41,555,579	32,844,383	− 8,711,196	− 15,525,966
D. Non-ferrous metals and manufactures thereof	15,699,979	12,339,096	12,755,159	+ 416,063	− 2,944,820
E. Cutlery, hardware, implements and instruments	9,709,725	9,027,687	8,718,472	− 309,215	− 991,253
F. Electrical goods and apparatus	12,538,044	13,430,405	11,251,664	− 2,178,741	− 1,286,380
G. Machinery	49,740,963	57,867,565	47,340,118	− 10,527,447	− 2,400,845
H. Manufactures of wood and timber	1,316,401	1,164,556	1,055,676	− 108,880	− 260,725
I. Cotton yarns and manufactures......................	68,508,703	49,680,714	49,088,055	− 592,659	− 19,420,648
J. Woollen and worsted yarns and manufactures	35,502,846	26,813,619	26,648,979	− 164,640	− 8,853,867
K. Silk yarns and manufactures	7,217,574	5,502,161	5,899,865	+ 397,704	− 1,317,709
L. Manufactures of other textile materials	14,592,786	10,657,365	11,098,581	+ 441,216	− 3,494,205
M. Apparel..............................	10,221,144	8,515,263	8,257,085	− 258,178	− 1,964,059
N. Footwear	2,174,625	1,962,596	1,889,924	− 72,672	− 284,701
O. Chemicals, drugs, dyes and colours	24,653,095	22,060,176	22,777,595	+ 717,419	− 1,875,500
P. Oils, fats and resins, manufactured	5,877,019	5,365,496	4,743,871	− 621,625	− 1,133,148
Q. Leather and manufactures thereof	5,344,766	3,943,101	4,098,546	+ 155,445	− 1,246,220
R. Paper, cardboard, &c.	8,095,878	6,930,313	6,688,903	− 241,410	− 1,406,975
S. Vehicles (including locomotives, ships and aircraft)	39,923,718	44,627,488	39,086,192	− 5,541,296	− 837,526
T. Rubber manufactures...........	1,657,118	1,649,695	1,450,874	− 198,821	− 206,244
U. Miscellaneous articles wholly or mainly manufactured	29,304,160	29,249,323	28,312,392	− 936,931	− 991,768
Total, Class III	404,655,940	365,244,030	337,452,237	− 27,791,793	− 67,203,703
IV. ANIMALS, NOT FOR FOOD	850,152	679,364	683,053	+ 3,689	− 167,099
V. PARCEL POST	12,481,398	12,017,472	10,572,786	− 1,444,686	− 1,908,612
Total..........................	521,391,494	470,755,320	438,806,078	− 31,949,242	− 82,585,416

Values (f.o.b.) of* **Exports** *of Imported Merchandise*

	Year ended December 31, 1937	1938	1939	Increase or Decrease, 1939—1938	Increase or Decrease, 1939—1937
FOOD, DRINK AND TOBACCO—	£	£	£	£	£
A. Grain and flour	1,869,742	1,506,469	911,999	− 594,470	− 957,743
B. Feeding-stuffs for animals......	49,507	67,945	84,691	+ 16,746	+ 35,184
C. Animals, living, for food	—	—	—	—	—
D. Meat	623,058	558,722	538,255	− 20,467	− 84,803
E. Dairy produce....................	932,461	736,371	538,714	− 197,657	− 393,750
F. Fresh fruit and vegetables......	1,341,038	1,324,581	1,152,834	− 171,747	− 188,204
G. Beverages and cocoa preparations	6,569,913	6,127,237	5,617,362	− 509,875	− 952,551
H. Other food	1,312,378	1,197,053	1,097,972	− 99,081	− 214,406
I. Tobacco	489,046	780,992	923,821	+ 142,829	+ 434,775
Total, Class I	13,187,146	12,299,370	10,865,648	− 1,433,722	− 2,321,498

* The value of the Exports represents the cost and the charges of delivering the goods on board the ship, nd is known as the "free on board" value.

Values (f.o.b.) of **Exports** *of Imported Merchandise—Contd.*

	Year ended December 31, 1937	1938	1939	Increase or Decrease, 1939—1938	Increase or Decrease, 1939—1937
II. RAW MATERIALS AND ARTICLES MAINLY UNMANUFACTURED—	£	£	£	£	£
A. Coal	3,099	—	—	—	— 3,099
B. Other non-metalliferous mining and quarry products and the like	367,507	347,676	300,467	— 47,209	— 67,040
C. Iron ore and scrap	546	301	801	+ 500	+ 255
D. Non-ferrous metalliferous ores and scrap	722,108	941,447	524,565	— 416,882	— 197,543
E. Wood and timber	315,375	289,539	244,377	— 45,162	— 70,998
F. Raw cotton and cotton waste	2,036,230	1,253,572	1,155,839	— 97,733	— 880,391
G. Wool, raw and waste, and woollen rags	14,430,970	12,559,507	8,620,272	— 3,939,235	— 5,810,698
H. Silk, raw, knubs and noils	15,192	22,069	18,183	— 3,886	+ 2,991
I. Other textile materials	628,396	818,543	828,356	+ 9,813	+ 199,960
J. Seeds and nuts for oil, oils, fats, resins and gums	680,262	545,250	521,566	— 23,684	— 158,696
K. Hides and skins, undressed	11,664,849	9,568,319	7,784,733	— 1,783,586	— 3,880,116
L. Paper-making materials	50,873	62,857	25,772	— 37,085	— 25,101
M. Rubber	4,393,340	2,695,097	3,620,528	+ 925,431	— 772,812
N. Miscellaneous raw materials and articles mainly unmanufactured	1,566,669	1,146,432	898,148	— 248,284	— 668,521
Total, Class II	36,875,416	30,250,609	24,543,607	— 5,707,002	— 12,331,809
III. ARTICLES WHOLLY OR MAINLY MANUFACTURED—					
A. Coke and manufactured fuel	787	—	11	+ 11	— 776
B. Pottery, glass, abrasives, &c.	64,921	57,829	45,363	— 12,466	— 19,558
C. Iron and steel and manufactures thereof	138,421	210,100	74,806	— 135,294	— 63,615
D. Non-ferrous metals and manufactures thereof	13,638,536	9,061,682	2,605,858	— 6,455,824	— 11,032,678
E. Cutlery, hardware, implements and instruments	815,420	728,043	606,999	— 121,044	— 208,421
F. Electrical goods and apparatus	110,835	136,357	111,297	— 25,060	+ 462
G. Machinery	948,179	943,210	904,646	— 38,564	— 43,533
H. Manufactures of wood and timber	201,628	199,653	169,112	— 30,541	— 32,516
I. Cotton yarns and manufactures	290,332	195,724	117,610	— 78,114	— 172,722
J. Woollen and worsted yarns and manufactures	647,038	445,009	375,960	— 69,049	— 271,078
K. Silk yarns and manufactures	511,766	504,643	341,018	— 163,625	— 170,748
L. Manufactures of other textile materials	228,898	130,802	171,776	+ 40,974	— 57,122
M. Apparel	461,508	432,700	395,672	— 37,028	— 65,836
N. Footwear	44,519	47,370	148,424	+ 101,054	+ 103,905
O. Chemicals, drugs, dyes and colours	465,298	462,430	469,778	+ 7,348	+ 4,480
P. Oils, fats and resins, manufactured	2,125,321	1,081,294	1,049,212	— 32,082	— 1,076,109
Q. Leather and manufactures thereof	1,197,235	902,466	566,083	— 336,383	— 631,152
R. Paper, cardboard, &c.	74,907	69,245	49,001	— 20,244	— 25,906
S. Vehicles (including locomotives, ships and aircraft)	292,829	415,174	283,824	— 131,350	— 9,005
T. Rubber manufactures	12,845	18,001	12,583	— 5,418	— 262
U. Miscellaneous articles wholly or mainly manufactured	2,171,793	2,234,783	1,431,384	— 803,399	— 740,409
Total, Class III	24,443,016	18,276,515	9,930,417	— 8,346,098	— 14,512,599
IV. ANIMALS NOT FOR FOOD	628,093	698,152	585,804	— 112,348	— 42,289
Total	75,133,671	61,524,646	45,925,476	— 15,599,170	— 29,208,195

FOREIGN EXCHANGES.—*Quotations as under,* LONDON *on Paris, Berlin, Calcutta and Hong Kong; New York on* LONDON, **1939.**

	1	2	3	4	5	6	7
						Price per Ounce	
DATE (Thursdays)	London on Paris	London on Berlin	London on Calcutta	New York on London	London on Hong Kong	Gold Bars (fine)	Silver Standard Bars (cash)
	Cables (middle rate)	Cables (middle rate)	Demand (middle rate)	Cables (closing rate)	T.T.		
1939.	*f. c.*	*Reichmarks.*	*s. d.*	*$ c.*	*s. d.*	*s. d.*	*s. d.*
Jan. 12......	177 3/16	11·65	1 5 31/32	4·67½	1 2 15/16	148 9½	1 8⅜
„ 26......	177 15/32	11·70	1 5 31/32	4·67½	1 2 15/16	148 0	1 8⅛
Feb. 9......	177 1/64	11·68	1 5 31/32	4·68¾	1 2 15/16	148 3½	1 8¼
„ 23......	177 1/64	11·69½	1 5 31/32	4·68 15/16	1 2 15/16	148 4	1 8 9/16
Mar. 9......	176 57/64	11·69	1 6	4·69 1/16	1 2 15/16	148 3	1 8½
„ 23......	176 53/64	11·70	1 6	4·68⅜	1 2 15/16	148 5	1 8
Apr. 6......	176 47/64	11·66½	1 5 31/32	4·68⅛	1 2¾	148 6	1 8
„ 20......	176 45/64	11·67	1 5 31/32	4·68 1/16	1 2¾	148 6	1 8 1/16
May 4......	176 47/64	11·66½	1 5 15/16	4·68⅛	1 2¾	148 6	1 8¼
„ 18......	176 47/64	11·67	1 5 15/16	4·68 5/32	1 2⅞	148 6	1 8 1/16
June 1......	176 47/64	11·67½	1 5 15/16	4·68 5/16	1 2 15/16	148 5	1 7 15/16
„ 15......	176 47/64	11·67½	1 5 31/32	4·68⅛	1 2 13/16	148 5½	1 7⅞
„ 29......	176 23/32	11·66½	1 5 31/32	4·68⅛	1 2 13/16	148 6	1 5 15/16
July 13......	176 23/32	11·66½	1 5 15/16	4·68¼	1 2 13/16	148 6	1 4 13/16
„ 27......	176 23/32	11·66½	1 5 15/16	4·68⅛	1 2 11/16	148 6½	1 4½
Aug. 10......	176 23/32	11·66½	1 5 31/32	4·68⅛	1 2 11/16	148 6½	1 5
„ 24......	176 23/32	11·75½	1 5 31/32	4·68⅛	1 2 11/16	148 5	1 7 1/16
Sept. 7......	175½		1 5 31/32	4·36 3/16	1 3	168 0	1 9 3/16
„ 21......	*176½		1 5 31/32	3·99½	*1 3	168 0	1 11½
Oct. 5......	*176½		1 5 31/32	4·04⅛	*1 3	168 0	1 9¼
„ 19......	*176½		1 6 1/32	4·01	*1 3	168 0	1 10 13/16
Nov. 2......	*176½		1 6 1/32	3·99⅞	1 3	168 0	1 11 1/16
„ 16......	*176½		1 6 1/32	3·93⅛	1 3	168 0	1 11 1/16
„ 30......	*176½		1 6 1/32	3·88¼	1 3	168 0	1 11½
Dec. 14......	*176½		1 6 1/32	3·93 13/16	1 3	168 0	1 11⅜
„ 28......	*176½		1 6 1/32	3·95⅛	1 3	168 0	1 10 15/16

* Rate fixed by Bank of England.

BANK OF ENGLAND

***Pursuant to the Act 7th and 8th Victoria, cap.* 32 (1844),**

(000's omitted)

1	2	3	4	5	6	7	8
		Issue Department				Collateral Columns	
Liabilities	Dates (Wednesdays)	Assets					
Notes Issued		Govt. Debt (£11,015) and Govt. Securities	Other Securities	Gold Coin and Bullion	Silver Coin	Notes in Hands of Public	Minimum Discount Rate
£		£	£	£	£	£	Per cent.
556,416	Jan. 4.........	229,645	69	326,416	286	488,071	2
526,414	,, 11.........	399,626	91	126,414	283	475,644	
526,414	,, 18.........	399,623	66	126,414	311	467,953	
526,414	,, 25.........	399,657	34	126,414	309	463,845	
526,414	Feb. 1.........	399,587	100	126,414	313	471,948	
526,414	,, 8.........	399,603	60	126,414	337	474,998	
526,414	,, 15.........	399,618	21	126,414	361	473,221	
526,414	,, 22.........	399,464	50	126,414	486	472,734	
526,414	Mar. 1.........	299,402	12	226,414	586	478,449	
526,033	,, 8.........	299,330	7	226,033	663	479,178	
526,033	,, 15.........	299,260	3	226,033	737	477,248	
526,033	,, 22.........	298,807	431	226,033	762	477,412	
526,160	,, 29.........	298,900	317	226,160	783	481,987	
526,160	Apr. 5.........	299,099	88	226,160	813	491,451	
526,160	,, 12.........	298,932	255	226,160	813	489,703	
526,160	,, 19.........	298,478	707	226,160	815	485,004	
526,160	,, 26.........	298,484	676	226,160	840	489,080	
526,160	May 3.........	298,665	471	226,160	864	495,456	
526,160	,, 10.........	298,595	514	226,160	891	495,894	
526,160	,, 17.........	298,083	1,028	226,160	889	493,620	
526,160	,, 24.........	298,095	1,017	226,160	888	494,565	
526,160	,, 31.........	298,120	967	226,160	913	499,776	
526,160	June 7.........	299,117	68	226,160	815	498,371	
526,287	,, 14.........	299,146	43	226,287	811	494,952	
526,287	,, 21.........	299,181	98	226,287	711	494,671	
526,414	,, 28.........	299,209	78	226,414	713	498,980	
526,414	July 5.........	299,252	35	226,414	713	505,722	
546,417	,, 12.........	299,129	162	246,417	709	507,929	
546,417	,, 19.........	299,137	152	246,417	711	507,638	
546,417	,, 26.........	299,259	25	246,417	716	510,898	
546,417	Aug. 2.........	299,268	20	246,417	712	520,570	
546,417	,, 9.........	299,170	120	246,417	710	521,877	
546,555	,, 16.........	297,204	2,085	246,555	711	512,389	
546,417	,, 23.........	295,235	4,052	246,417	713	508,063	
563,011	,, 30.........	295,816	3,471	263,011	713	529,499	4 (Aug. 20)
580,102	Sept. 6.........	575,837	3,451	102	712	549,886	
580,127	,, 13.........	578,032	1,459	127	509	553,474	
580,143	,, 20.........	576,748	2,942	143	310	546,498	
580,148	,, 27.........	576,462	3,334	148	204	541,833	
580,165	Oct. 4.........	576,656	3,135	165	209	538,749	3 (Sept. 28)
580,165	,, 11.........	576,854	2,937	165	209	535,033	
580,165	,, 18.........	576,776	2,962	165	262	530,316	
580,185	,, 25.........	576,708	2,933	185	359	527,137	
580,185	Nov. 1.........	576,613	2,931	185	455	527,966	2 (Oct. 26)
580,193	,, 8.........	576,612	2,929	193	458	528,372	
580,213	,, 15.........	576,519	2,972	213	508	527,644	
580,219	,, 22.........	576,497	2,093	219	510	527,004	
580,219	,, 29.........	576,285	3,155	220	559	528,660	
580,219	Dec. 6.........	576,320	3,019	220	660	533,876	
580,219	,, 13.........	576,362	3,075	219	663	545,120	
580,219	,, 20.........	576,264	3,073	219	663	552,340	
580,219	,, 27.........	576,340	2,997	220	662	554,616	

WEEKLY RETURNS

for Wednesday in each Week, during the Year **1939**

(000's omitted)

9	10	11	12	13	14	15	16	17	18
BANKING DEPARTMENT									
Liabilities				Dates	Assets				
Capital (£14,553) and Rest	Public Deposits	Banker's Deposits	Other Deposits	(Wednesdays)	Govt. Securities	Discounts and Advances	Other Securities	Reserve (Notes and Coin)	Totals of Liabilities and Assets
£	£	£	£		£	£	£	£	£
17,997	20,503	135,955	37,242	Jan. 4	71,381	48,906	22,380	69,030	211,697
18,030	19,214	118,139	36,653	,, 11	96,171	22,121	22,321	51,422	192,036
18,050	12,251	119,935	36,693	,, 18	85,751	18,491	23,606	59,081	186,929
18,074	12,918	118,188	36,714	,, 25	82,006	18,784	21,862	63,242	185,894
18,119	12,262	108,306	37,137	Feb. 1	77,901	21,314	21,360	55,248	175,824
18,157	15,352	103,950	35,490	,, 8	77,636	20,649	22,357	52,307	172,949
18,179	12,477	110,529	35,498	,, 15	82,491	18,123	21,866	53,203	176,683
18,190	16,348	103,135	35,133	,, 22	78,941	17,529	21,646	54,689	172,806
18,107	11,643	110,361	36,550	Mar. 1	99,046	6,349	22,398	48,967	176,761
18,220	11,449	97,424	35,804	,, 8	89,246	3,360	22,395	47,896	162,897
18,234	10,525	106,470	35,549	,, 15	97,531	1,707	21,704	49,836	170,778
18,241	28,597	92,205	35,708	,, 22	99,406	3,179	22,434	49,733	174,752
18,242	21,775	98,509	36,268	,, 29	102,301	4,776	22,472	45,245	174,794
17,642	16,042	91,946	40,529	Apr. 5	100,351	8,485	21,509	35,814	166,159
17,678	17,739	97,646	40,936	,, 12	108,666	6,276	21,522	37,535	173,999
17,686	22,129	96,173	38,830	,, 19	101,976	6,742	24,225	42,175	175,118
17,709	26,957	91,425	37,589	,, 26	107,156	6,171	22,344	38,009	173,680
17,733	10,489	108,214	37,041	May 3	111,906	8,175	21,942	31,453	173,477
17,718	19,059	101,356	36,279	,, 10	114,126	8,102	21,245	30,969	174,442
17,759	16,803	107,315	36,151	,, 17	113,511	10,076	21,270	33,171	178,028
17,777	25,487	95,347	36,366	,, 24	113,966	7,097	21,715	32,199	174,977
17,807	38,340	82,391	37,085	,, 31	118,966	7,999	21,715	26,943	175,623
17,833	18,401	100,623	36,104	June 7	114,176	7,801	22,607	28,377	172,961
17,871	22,079	100,297	36,399	,, 14	116,261	5,632	22,952	31,801	176,646
17,912	25,237	97,235	36,128	,, 21	112,631	6,185	25,198	32,198	176,512
17,914	15,385	101,442	36,955	,, 28	113,856	6,837	22,877	28,155	171,726
17,985	19,301	96,477	37,560	July 5	118,706	8,789	22,464	21,364	171,323
18,004	29,524	87,111	36,782	,, 12	100,441	9,233	22,610	39,137	171,421
18,029	28,381	95,520	36,423	,, 19	105,051	9,120	24,744	39,438	178,353
18,055	26,010	91,378	36,742	,, 26	106,491	8,049	21,502	36,143	172,185
18,087	23,279	89,578	36,182	Aug. 2	112,611	6,857	21,168	26,490	167,126
18,116	31,846	81,654	35,929	,, 9	114,831	5,955	21,532	25,227	167,545
18,144	27,457	94,728	35,717	,, 16	111,636	5,909	23,550	34,951	176,046
18,178	22,372	92,132	36,229	,, 23	99,666	5,711	24,335	39,199	168,911
18,205	31,068	90,144	38,976	,, 30	113,126	6,388	24,629	34,250	178,393
18,221	15,029	111,267	41,236	Sept. 6	123,721	6,017	25,213	30,802	185,753
18,229	12,001	109,609	40,815	,, 13	123,671	3,858	25,938	27,187	180,654
18,236	21,450	107,899	39,934	,, 20	121,866	2,539	28,886	34,228	187,519
18,251	19,841	107,131	40,468	,, 27	120,336	2,481	23,883	38,995	185,694
18,247	15,346	106,034	40,867	Oct. 4	113,616	2,392	22,335	42,151	180,494
17,695	10,910	116,716	39,673	,, 11	114,311	3,046	21,698	45,939	184,994
17,683	12,010	116,445	38,535	,, 18	107,536	3,224	23,250	50,693	184,703
17,737	12,574	116,761	39,242	,, 25	106,041	4,639	21,699	53,935	186,314
17,757	10,540	114,802	42,992	Nov. 1	105,336	5,633	22,033	53,089	186,092
17,774	21,267	101,859	41,477	,, 8	102,246	5,152	21,923	52,756	182,377
17,792	17,844	107,084	40,580	,, 15	103,196	5,261	21,292	53,551	183,300
17,818	36,392	92,945	39,349	,, 22	103,946	5,285	22,930	54,343	186,504
17,835	27,822	103,503	40,185	,, 29	109,816	4,530	22,310	52,689	189,345
17,868	47,141	86,288	39,823	Dec. 6	114,886	5,400	23,362	47,472	191,120
17,878	45,804	87,739	40,657	,, 13	126,106	5,379	24,427	36,166	192,078
17,880	34,498	114,851	40,376	,, 22	145,231	5,446	28,016	28,912	207,605
17,915	29,725	117,329	42,027	,, 29	151,466	4,269	24,620	26,641	206,996

JOURNAL

OF THE ROYAL STATISTICAL SOCIETY

PART II, 1940.

Flexibility of the Yield of Taxation—Some Econometric Investigations

By Victor Edelberg.

Mere facts are blind, mere theories—empty.

The need for quantitative estimates of the effect on the yield of taxes resulting from changes in the rates of taxation, from variations in the National Income and other factors, has been recognized—indeed, emphasized—by recent writers on the theory of taxation. Little has been done, however, to achieve that desideratum. This has been due to difficulties inherent in any attempt to put a qualitative economic theory on to a quantitative basis. The obstacles are largely overcome by quite simple but careful analysis of (*a*) the nature and imperfections of statistical data, and (*b*) the sort of causal connections to look for. The latter includes the choice of relevant variables and of suitable functions to express their relations.

Statistical answers will be sought to three questions: (1) what is the quantitative effect of short-period fluctuations in the national income, normally associated with the trade cycle, upon the yield of a tax? (2) what are the effects of trend forces? and (3) what is the quantitative effect of changes in the rates of tax on the amount of revenue raised by taxation?

The periods for which the different taxes are studied commence from 1920 to 1924 and terminate with the year 1937 or 1938.

Five important British taxes have been selected for investigation. Together they are responsible for over half the total yield of taxation in this country. They do not present any great statistical difficulties arising either from defects in the data or from "inter-correlations." These five represent three important types of tax: tax on consumption, tax on turnover, and tax on income earned; and are

(1) Tobacco duties,
(2) Spirits duties,
(3) Beer duty,

(4) Stamp duties,
(5) Income tax,

and they will be taken up in turn.*

1. *Yield of tobacco duties*

In spite of increases in rates of duty, consumption of tobacco has been rising. During the period covered by the study, 1924–37, the rates of duty have been increased twice, in 1927 and 1931. These changes are too few to be taken into account explicitly, and will be taken account of implicitly by the trend.

The yield of tobacco duty (X_1) is correlated (logarithmically) with time (X_2 with 1900 as origin) and percentage employment (X_3). Time is taken as an independent variable to express the upward trend in the yield due to the steady rise in consumption of tobacco and to the two increases in the rates of duty.

The second independent variable (X_3), or percentage employment, is the cyclical index. The percentage employed for the period is obtained by deducting from one hundred the percentage unemployed. It is thus a good index of cyclical fluctuations in employment. Income being a function of employment, we are able, by correlating yield with employment, to obtain indirectly a measure of the income elasticity of demand—*i.e.*, elasticity with respect to the national income.

Obviously, the bigger the trend value of consumption of a commodity, the greater generally will be the fluctuation in that consumption due to the trade cycle. It is clear that in the case of tobacco duties the upward trend must be accompanied by cyclic fluctuations of increasing amplitude. If we expressed the yield as a linear function of time and percentage employment, the cyclic fluctuation of the yield would be independent of the trend level and so we should fail to express the true relation. As an approximation to reality we can assume that the amplitude of cyclical fluctuations (strictly the effect of changes in percentage employment (X_3)) is directly proportional to trend value. This can be done by expressing percentage deviation of the yield from its trend as a linear function of employment (X_3). The most elegant way, however, is to make the yield (X_1), a " power function " of X_2 (time) and X_3 (percentage employment). Then

$$X_1 = KX_2^bX_3^c \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

* The table of data will be found in the appendix. Valuable assistance has been rendered by Mr. R. N. Poduval in collecting source-materials, in the supervision of the computers, and by way of discussion of problems encountered at the various stages of the inquiry. Anyone with experience of empirical research will appreciate the value of such assistance.

where K, b, c are constants. The influence of X_3 on X_1 is proportional to the trend component (KX_2^b), since

$$\frac{\partial X_1}{\partial X_3} = cKX_2^b X_3^{c-1}.$$

Writing $Z_1 = \log X_1$, $Z_2 = \log X_2$, $Z_3 = \log X_3$, and $a = \log K$, and putting the equation in logarithmic form, we have a linear equation :

$$Z_1 = bZ_2 + cZ_3 + a \quad . \quad . \quad . \quad . \quad . \quad (2)$$

the constants of which are found by the usual method of least squares.

An advantage of form (1) is that the trend and the regression on Z_3 in (2) are determined simultaneously instead of by stages, as when trend is eliminated first and the deviations from trend are then correlated with the independent variable, or variables, other than time. Our method has the advantage of allowing us to measure easily the errors of sampling and to adjust the correlation coefficient.

Calculating a, b, c, equation (2) becomes :

$$Z_1 = 0{\cdot}943Z_2 + 0{\cdot}426Z_3 - 0{\cdot}413. \quad . \quad . \quad . \quad (2)'$$

In equation (1) the constants b and c are, of course, partial elasticities—*e.g.*, $c = \dfrac{\partial X_1}{\partial X_3} \cdot \dfrac{X_3}{X_1}$.

From the value of $c = 0{\cdot}426$ we conclude that cycle elasticity of demand is nearly one half. Thus if percentage employment increases by 10 per cent., the revenue from tobacco duties will increase by about 4 per cent. The cycle elasticity of demand approximately equals income elasticity of demand when the trend of income is eliminated.

The coefficient of multiple correlation showing the influence of both factors, trend and employment, is :

$$R = 0{\cdot}87$$

The coefficient can be "adjusted" for the number of observations in the sample and for the number of constants in the equation, and the adjusted coefficient is :

$$\bar{R} = 0{\cdot}84^*$$

Although regression coefficients in a logarithmic regression are independent of the units in which the original variables are expressed,

* $(\bar{R})^2 = 1 - (1 - R^2)\left(\dfrac{n-1}{n-m}\right)$, where R^2 in the square of the Correlation Coefficient, n the number of observations of the set of variables correlated (in the present case the set consists of three variables, and $n = 14$) and m is the number of constants in the equation of regression.

they indicate only the elasticity or sensitivity of dependent variable X_1, with respect to the independent variables, but they do not indicate the relative share of the independent factors in the observed variation of the dependent variable. This is done, approximately, by " β " coefficients. They are defined and evaluated as follows :

$$\beta_2 = b\frac{\sigma_2}{\sigma_1} = 0{\cdot}92$$

$$\beta_3 = c\frac{\sigma_3}{\sigma_1} = 0{\cdot}15$$

where σ_1 is the standard deviation of Z_1, σ_2 is that of Z_2, and σ_3 that of Z_3. If the observed variation of the dependent variable Z_1 is expressed by its standard deviation σ_1, the " β " coefficients serve as indices of relative shares of the independent variables in the observed variation of the dependent variable. Thus β_2 measures the relative importance of Z_2 and β_3 that of Z_3. In this way the relative influence of trend forces and percentage employment (the trade cycle) on the yield of the duty is indicated. It is seen from the values of β's that the influence of the trend has been about six times as great as the effect of variations in employment.

It may be noted that equation (2) may be expressed in terms of " β " coefficients :

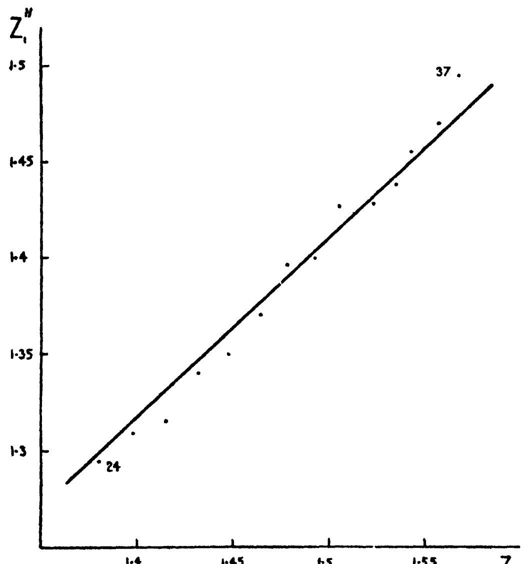
$$\frac{Z_1}{\sigma_1} = \beta_2\frac{Z_2}{\sigma_2} + \beta_3\frac{Z_3}{\sigma_3} + \frac{a}{\sigma_1}.$$

Chart I shows the partial regression of Z_1 on Z_2. The co-ordinates are Z_2 and Z_1''. From the regression line $Z_1'' = bZ_2 + a$ are plotted " residuals " or deviations of the observed values of Z_1 from the corresponding values of Z_1 as estimated from the regression equation (2)′. If by Z_1 be understood the observed and by Z_1' the estimated values, the residuals are differences $Z_1 - Z_1'$, which are attributable to random causes. All the deviations lie close to the regression line, some on the line itself.

Chart II shows the partial regression of Z_1 on Z_3. The graph of partial regression equation $Z_1''' = cZ_3$ is drawn and the residuals $Z_1 - Z_1'$ are plotted about the line. The cyclical effect, not so great as that of trend, is still substantial. Consumption was probably reinforced during depression by the substitution of cheaper varieties of tobacco.

The influence of trend-forces on the yield of tobacco duties has been very great. Changes in rates of duty have been absorbed in the trend, the effect of alterations in rates being evened out gradually through a period of years. The effect of changes in the rates on consumption is largely unknown, chiefly because the depressive effect of duties on con-

sumption is masked by the growing tobacco habit. The cyclical effect, though small compared to the trend-effect, is by no means negligible in itself. In fact, the cyclical effect would have been greater but for the use of cheaper substitutes in the shape of Empire tobacco. As incomes fall many people substitute Empire tobacco for American, and actually



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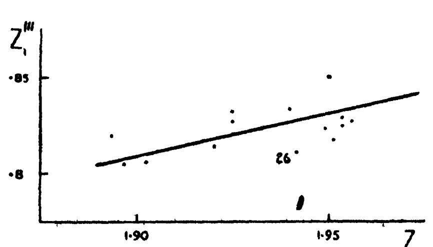


II

increase their consumption of Empire tobacco as compared with the preceding boom. This is shown by the statistics of tobacco-consumption. So, Empire grade has behaved as an "inferior good" for a large number of consumers, and this feature is an anti-cyclic factor in the total demand for tobacco. This anti-cyclic factor acts also on the revenue from the duty, but is counteracted to some extent by the fact that the Empire growth enjoys an excise preference.

The cycle elasticity of the revenue from tobacco duties has been estimated as $c = 0{\cdot}426$—*i.e.*, a 10 per cent. change in percentage employment produces approximately a 4 per cent. change in the yield. The cycle-elasticity of the aggregate demand for tobacco in this country is somewhat smaller than this, in so far as the Empire grade behaves as an "inferior good."

2. *Spirits duties*

Common-sense observation suggests that spirits are luxury goods, on account of the high spirits duties. Consequently the income elasticities of demand for them should be considerable, and so must be the cycle sensitivity of their yields. Ordinary observation further suggests that there has been a great secular decline in consumption of spirits. The decline may have been partly the result of persistence of high duties and prices over a long period, or it may have been due to moral and social progress, or to the rivalry of other stimulants or pastimes.

During the period 1922–37 investigated there have been no changes in spirits duties, and so they do not have to be taken into account in the correlation. Also, it follows that consumption of spirits is approximately proportional to the yield of spirits duties, so that our investigation studies simultaneously the empirical elasticity of demand for spirits and the behaviour of the yield of the duties.

We take three variables as before: X_1 the annual yield, and the two independent variables X_2 (time) and X_3 (% employment), our indicator of the trade cycle.

There is a strong downward trend in the yield X_1, and, for the same reasons as before, a power function is employed

$$X_1 = KX_2^bX_3^c \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

For purposes of computation this formula is employed in its logarithmic form. As before, putting $Z_1 = \log X_1$, $Z_2 = \log X_2$, $Z_3 = \log X_3$, a regression equation is obtained:

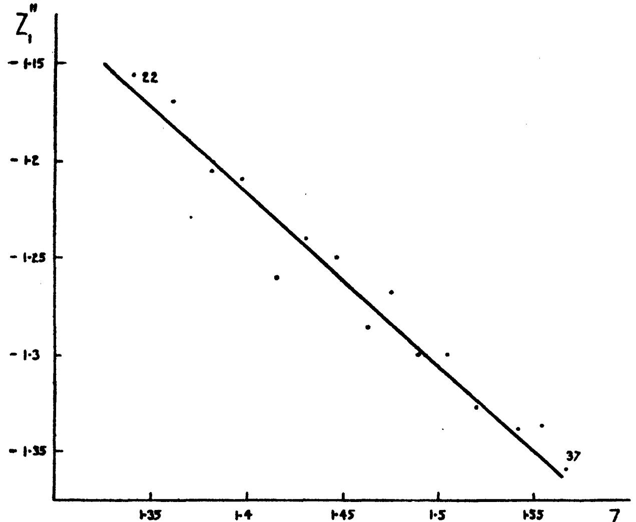
$$Z_1 = 0{\cdot}393 - 0{\cdot}869Z_2 + 1{\cdot}29Z_3 \quad . \quad . \quad . \quad (2)$$

together with an adjusted multiple correlation coefficient $\bar{R} = 0{\cdot}98$, which is very high.

The estimate of cycle sensitivity $c = 1{\cdot}29$ is quite high, showing that the most probable effect on the yield of spirits duties and on the consumption of spirits is more than proportional to the fluctuations in employment (and the national income). Thus statistical analysis has corroborated the impression derived from common observation that spirits have the behaviour of a luxury good. Mrs. U. K. Hicks in her important work *The Finance of British Govern-*

ment (page 256) has argued *a priori* that there is a " slight " income elasticity of demand for spirits, and consequently only a " slight " cycle sensitivity in the yield of spirits duties. The meaning of " slight " is necessarily vague, and Mrs. Hicks may easily have meant what others may describe as a " considerable " degree of cycle sensitivity. But if Mrs. Hicks implies that the cycle sensitivity of the yield is less than that of employment (= 1), she disagrees with an alternative theory *a priori*—namely, that since spirits are a luxury good for most consumers, the income elasticity of demand for spirits must be greater than + 1.

A divergence of conclusions from plausible *a priori* arguments



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can be usefully referred to the Court of Correlation Analysis. In the present case the verdict is in favour of " substantial " cycle sensitivity of the yield of spirits duty. Of course, Mrs. Hicks might appeal against the decision to a higher court—that of criticism of significance of correlation results. And the possibility will have to be examined that certain factors correlated with employment X_3 have conspired to manufacture a spurious regression relation between the yield and employment. With this end in view, it will be useful to examine the partial regressions of Z_1 on Z_2 and Z_3. On Chart III is shown the partial regression of Z_1 on Z_2 revealing a strong downward trend in Z_1, and so in the yield X_1, residuals are plotted about the line and the regression line makes a good fit and there is no suggestion of curvature.

The partial regression of Z_1 on Z_3 is presented in Chart IV. The regression line is steep in relation to the residual scatter about it, and there are only two important deviations. That for 1926 is evidently due to the General Strike, and that for 1934 may be the result of the public having lost the spirits habit in the great depression of 1931–33 in combination with the reduction in 1933–34 in the duty on beer, which naturally competes with the stronger liquors on which the spirits duties are levied. The two observations 1926 and 1934 have no appreciable effect on the slope of the regression line in Chart IV, and so cannot affect the conclusion that the cycle sensitivity of spirits tax is high.

The other possibility that the cycle sensitivity is exaggerated (or under-estimated) can arise from inter-correlation between Z_2 and Z_3. Accordingly, correlation coefficient r_{23} is calculated, and it is found that $r_{23} = -0{\cdot}395$. There is therefore a material inter-correlation. The effect of the trade cycle therefore may have been either over-estimated or under-estimated. However, there is a method of estimating the minimum effect of the trade cycle. We eliminate the trend from the logarithm of the yield Z_1 by correlating it with logarithm of time alone and obtaining a regression equation

$$ {}_tZ_1 = 3{\cdot}1072 - 1{\cdot}084Z_2 \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where, using a well-known notation for coefficients of regression, the regression coefficient on Z_2 is $b_{12} = -1{\cdot}084$. The corresponding partial regression coefficient in the preceding multiple regression equation (2) was $b_{12\cdot3} = -0{\cdot}869$, and therefore nearly the whole of the gross trend in the yield has been attributed to the net trend effect $b_{12\cdot3}\, Z_2$ in equation (2). We now take deviations of the observed values from the gross trend value ${}_tZ_1$ of equation (3), and correlate the deviations $\Delta = Z_1 - {}_tZ_1$ with the logarithm of employment (Z_3). The correlation coefficient is $r_{\Delta 3} = 0{\cdot}90$, and the regression equation obtained is

$$\Delta = -2{\cdot}35 + 1{\cdot}223Z_3,$$

where $b_{\Delta 3} = 1{\cdot}223$. This latter is a minimum estimate of the elasticity of the yield of spirits duty X_1 with respect to percentage employment X_3. The estimate obtained by multiple regression $b_{13\cdot2} = 1{\cdot}29$ in equation (2) exceeds only slightly the minimum estimate $b_{\Delta 3} = 1{\cdot}22$, and can therefore be used as an estimate of the full effect of the trade cycle on the yield of spirits duty without incurring any material danger of exaggeration. In Section 6, therefore, we shall employ the estimate $b_{13\cdot2} = 1{\cdot}29$. A maximum estimate can be obtained by correlating Z_1 with Z_3 alone.

The example of spirits duties suggests that the presence of a

slight downward trend in employment cannot seriously influence the partial regression coefficients in the case of the other taxes investigated.*

To complete the account of the behaviour of spirits duties, it remains to add the measurements of the relative importance of trend and of the fluctuations in employment.

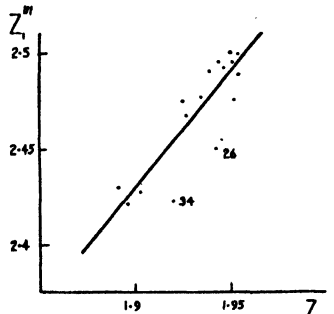
The β coefficient expressing the importance of Z_2 in equation (2) is

$$\beta_2 = -0{\cdot}79$$

and that expressing the importance of Z_3 is

$$\beta_3 = 0{\cdot}35$$

The absolute trend effect is thus over twice as great as cyclical variation in Z_1. That does not mean that the cyclical variation



IV

is in itself small—because the trend effect with which it is compared is very considerable. The slope of the net trend in Z_1 presented in Chart IV is $b = -0{\cdot}87$, so that the trend component of the yield X_1 has been falling at a rapid though diminishing rate and fell by 36 per cent. in the period 1922–37.

There is no doubt that the strong trend in spirits duties can obscure the cylical variation on a cursory examination of the time series, and that may explain why the full importance of the cyclical component may have escaped the notice of so discerning an investigator as Mrs. Hicks. Actually, according to the best probability estimate $b = 1{\cdot}29$, the cyclical amplitude of percentage fluctuation in the yield of spirits duties X_1 exceeds the relative amplitude of cyclical fluctuation in percentage employment X_3 by about 30 per cent.

* The trouble due to the presence of trends in the variables can be got rid of altogether by correlating only deviations of the variables from their trends. The analyses of the paper could be repeated in this way with some advantage and with no great harm.

3. *Beer duty and beer consumption* *

(a) *Consumption*

The effect of an upward or a downward shift in the scale of duties on different kinds of beer upon the total yield of the duty depends on the reaction of the aggregate consumption of beer. A rise in the rates of duty may depress consumption so much that the yield falls instead of rising. It will be of interest, therefore, to investigate the reaction of consumption.

For the period 1920–38 studied, the available series express the aggregate annual consumption of different beers in terms of equivalent "standard barrels" of beer of fixed specific gravity (1055°).

Since there is only a moderate trend in the series, a simple linear (non-logarithmic) regression equation is used, of the form

$$X_1 = a + b_2X_2 + b_3X_3 + b_4X_4,$$

where X_1 is annual consumption in millions of standard barrels, X_2 is the average rate of duty in £ per barrel, X_3 is percentage employment, and X_4 is time with 1900 as origin. The b's and a are constants.

The average rate of duty X_2 is calculated by dividing the yield by consumption. Some inaccuracy results from the fact that consumption data relate to calendar years and revenue relates to financial years.

Correlating consumption X_1 with co-temporaneous values of the independent variables, the regression equation

$$X_1 = 3{\cdot}85 - 0{\cdot}026X_2 + 0{\cdot}297X_3 - 0{\cdot}334X_4. \quad . \quad (1)$$

is obtained, with an adjusted multiple correlation coefficient of $R = 0{\cdot}88$. The coefficient $b_2 = -0{\cdot}026$ suggests that the influence of the average rate of duty on consumption is negligible.

Equation (1) is unsatisfactory. A partial regression diagram, of the type used in this paper, showed clearly that consumption X_1 lags behind the cyclical movements in employment X_3 by about one year.

Accordingly, taking X_3 with a "lead" of one year and correlating the set $X_1(t)$, $X_2(t)$, $X_3(t-1)$, $X_4(t)$, a new regression equation is obtained by the method of least squares :

$$X_1 = -0{\cdot}385 - 1{\cdot}113X_2 + 0{\cdot}378X_3 - 0{\cdot}287X_4 \quad . \quad (2)$$

with a correlation coefficient $\bar{R} = 0{\cdot}98$.

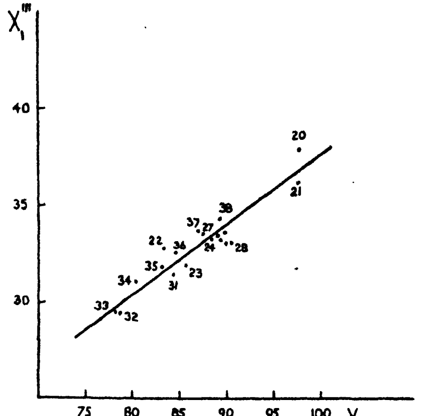
* An accumulation of minor difficulties (to do with comparability of data over time, with time-lags and with inter-correlations) has slowed up the work of analysing these variates. The obstacles have been overcome in time for a brief statement of the results attained to be inserted in the paper before it goes to press. I take the liberty of presenting the brief statement because its statistical material appears to be of unusual interest, both from the practical and the theoretical points of view.—V. E.

Taking the lead in X_3 into account has raised the correlation coefficient materially. The absolute size of the regression coefficients b_2 and b_3 has also increased. The small size of b_2 in equation (1) was simply due to errors created by neglect of the lead in X_3. In (2), $b_2 = -1{\cdot}113$ and the influence of the average duty no longer appears to be negligible.

The relative importance of the factors X_2, X_3, X_4 in equation (2) is respectively indicated by " β " coefficients :

$$\beta_2 = -0{\cdot}18$$
$$\beta_3 = 0{\cdot}65$$
$$\beta_4 = -0{\cdot}53.$$

The correlation coefficient $R = 0{\cdot}98$ is unusually high for eco-



V

nomic material. Its high value is chiefly due to the close association between beer consumption and employment, which is the most important factor in the regression. The high correlation coefficient is no freak result of trend correlation, since the principal component in the variation of the employment percentage series is cyclical fluctuation. Chart V gives a visual impression of the dependence of beer consumption on previous year's employment. The residual scatter is very small relatively to the systematic variation along the line of regression b_3X_3. No doubt the intimacy of the relation arises from the fact that the majority of consumers of beer consist of persons appearing on the Ministry of Labour's employment register from which the figure of percentage employment is compiled.

The explanation of the time-lag between employment and consumption may be that the desire for beer is an acquired taste. Beer consumption is only gradually adjusted to fluctuations of personal incomes in course of the trade cycle. It is interesting

that no similar lag is observed in relation to changes in the average rate of duty. Whilst the income-elasticity effect is delayed, the price-elasticity effect is more immediate.

Diagrammatic analysis indicates a relaxation (curvilinearity) of the net downward trend of consumption, although the residual scatter is very small relatively to the variation along the trend line b_4X_4.

The residual scatter becomes important when considered in relation to the partial regression b_2X_2 on the average rate of duty. The reliability of the regression coefficient $b_2 = -1{\cdot}11$ is therefore best investigated not graphically, but algebraically.

The residual scatter is measured by the "standard error of estimate"—*i.e.*, by the standard deviation of the residuals $X_1 - X_1'$, where X_1 is the observed value of consumption and X_1' is the value estimated from the regression equation (2). Writing σ_1 for the standard deviation of X_1, the "adjusted" standard error of estimate is

$$\begin{aligned}\bar{S} &= \sigma_1\sqrt{1-(\bar{R})^2}\\ &= 0{\cdot}61.\end{aligned}$$

Writing n as the number of observation-sets (19), and $\sigma_{2\cdot34}$ as the standard error of estimate of X_2 from the equation of regression of X_2 on X_3 and X_4, the standard error of the coefficient b_2 of equation (2) is

$$\begin{aligned}\sigma_{b_2} &= \frac{\bar{S}}{\sigma_{2\cdot34}\sqrt{n}}\\ &= 0{\cdot}34.\end{aligned}$$

This is a measure of uncertainty of the coefficient b_2 arising from "sampling errors"—*i.e.* from random disturbances affecting consumption X_1 which do not quite cancel out because the number of observations n is small. Evidently, intercorrelation among the independent variables of (2) is taken into account by the term $\sigma_{2\cdot34}$. But for this correlation, $\sigma_{2\cdot34}$ would be larger and the error of b_2 smaller.*

* Quite generally, measuring variables from their means, the standard error of a coefficient b_{i+1} $(i = 1, 2, \ldots, p-1)$ in a regression equation

$$x_1 = b_2x_2 + b_3x_3 + \ldots + b_px_p$$

is

$$\sigma_{b_{i+1}} = \bar{S}\sqrt{\frac{\Delta_{ii}}{\Delta}},$$

where

$$\Delta = \begin{vmatrix} \Sigma(x_2)^2 & \Sigma(x_2x_3) & \ldots & \Sigma(x_2x_p) \\ \Sigma(x_2x_3) & \Sigma(x_3)^2 & \Sigma(x_3x_4) & \ldots \\ \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots \\ \Sigma(x_2x_p) & \ldots & \ldots & \Sigma(x_p)^2 \end{vmatrix}$$

summation being taken over the observed n values and Δ_{ii} is the cofactor of $\Sigma(x_{i+1})^2$.

See *Applications of Student's Distribution* by R. A. Fisher, *Metron*, 1925.

Assuming normal distribution of b_2, there is a 99·7 per cent. probability that the true value of the regression coefficient lies within a distance of 3 standard deviations $\sigma_b = 0{\cdot}34$ from the estimate $b_2 = -1{\cdot}11$. That is, it is practically certain that the true coefficient b_2 lies between 0 and + 2, say. Similarly, it is almost certain that the coefficient β_2 lies between 0 and + 0·35, say. It follows that the average duty X_2 could not have been an important factor. Application of the "*t*" test, especially adapted to small samples, gives similar results.

(b) *Revenue*

The estimate of consumption X_1 from equation (2) is reliable—the standard error of estimate $S = 0{\cdot}6$ is only about 3 per cent of the average consumption ($M_1 = 19{\cdot}8$). Hence the yield Y can be reliably estimated by multiplying equation (2) by X_2—*i.e.*, from the equation :

$$Y = b_2X_2^2 + (a + b_3X_3 + b_4X_4)X_2 \quad . \quad . \quad . \quad (3)$$

where the constants are the same as in (2).

The function reduces to a simple parabola for a given pair of values of X_3 and X_4, such as their means M_3 and M_4. In the latter case, the form is economically plausible : as X_2 increases from O, Y rises from O, reaches a maximum and then declines passing through a O value again. Actually, negative returns set in sooner, because the duty becomes a more important proportion of price as the rate of duty rises.

Writing $K = a + b_3M_3 + b_4M_4$, the rate of duty which maximises revenue is

$$X_2 = \frac{K}{2b_2} \qquad (K = 24{\cdot}3) \quad . \quad . \quad . \quad (4)$$

For brevity K can be treated as a constant since its standard error is negligible for the argument in hand.*

It is perhaps a pseudo-problem to attempt to estimate the maximal X_2 (which is about 12 if $b_2 = -1$). That involves extrapolation of a linear regression b_2X_2. A legitimate problem is whether the highest observed rate of duty, $X_2 = 5{\cdot}2$, is in danger of approaching or exceeding the maximal level. That is a question which has been debated in Parliament and elsewhere. Substituting the value $X_2 = 5{\cdot}2$ in (4), b_2 becomes equal to $-2{\cdot}3$.

* The error is $\sigma_K = \sqrt{\sigma_a^2 + (M_3\sigma_{b_3})^2 + (M_4\sigma_{b_4})^2 + 2(\bar{S})^2 M_3M_4 \frac{\Delta_{23}}{\Delta}}$
$= 2{\cdot}03.$

Here M_3 and M_4 are treated as fixed values, and—of the new symbols—σ_a, σ_{b_3} and σ_{b_4} are the standard errors of a, b_3 and b_4; while Δ_{23} is the minor of $\Sigma(x_3x_4)$ in row 2, column 3 of Δ in the preceding footnote.

Now, taking the errors of b_2 into account (arising both from sampling errors and from the probable curvilinear character of the true regression on X_2), such a large negative value of the regression coefficient is extremely unlikely. Hence, the maximal level of the rate of duty has not been reached in practice.

(c) *Revenue. Logarithmic regression*

The same conclusion is obtained by a more direct attack.

Since there is a considerable trend in the yield, due to the downward trends in consumption and the rate of duty, a logarithmic regression is employed. Writing $Z_1 = Y$ and Z_2, Z_3, Z_4 for the logarithms of the X's with the same subscripts, a regression equation is obtained by the method of least squares :

$$Z_1 = -0{\cdot}789 + 0{\cdot}636Z_2 + 1{\cdot}542Z_3 - 0{\cdot}476Z_4 \quad . \quad . \quad (5)$$

The adjusted correlation coefficient is $R = 0{\cdot}994$ and "β" coefficients are :

$$\beta_2 = 0{\cdot}36$$
$$\beta_3 = 0{\cdot}44$$
$$\beta_4 = -0{\cdot}45.$$

The logarithm of percentage employment Z_3, has been taken with a lead of one year. When no lead is taken, the correlation coefficient drops to 0·95.

The logarithmic standard error of estimate is $\bar{S} = 0{\cdot}010$ and its antilogarithm is 1·023. That means : the actual values of the yield Y fall within approximately 2·3 per cent of the estimate with about 2/3 probability. The smallness of sampling errors of the yield of Beer Duty is astonishing. In his speech, the Chancellor need only make a negligible error of estimate, seeing especially that he has easy access to better statistical material.

In (5), the elasticity of the yield Y with respect to the rate of duty X_2 is 0·636. The standard error of the elasticity is 0·056. Therefore, it is pragmatically certain that the true elasticity is between 0·45 and 0·8, say. Diagrammatic analysis indicates no significant tendency for the elasticity to fall as the rate of duty increases. So the true elasticity must be positive in the observed range of the rate of duty. Again we have the conclusion that the maximal rate of duty has not been reached.

It is to be noted that equation (5) is incompatible with a maximum, although it "fits" the observations well.

The elasticity of the yield of beer duty with respect to percentage employment is given in (5) as 1·54 and is considerable. The corresponding "mean elasticity" of consumption with respect to employ-

ment, calculated from constants (M_1 and M_3 being the means of X_1 and X_3) relating to equation (2), is

$$b_3 \frac{M_3}{M_1} = 1{\cdot}66.$$

4. *Stamp duties*

From the standpoint of trade-cycle discussion it will be of interest to isolate the yield of stamp duties on securities from that of other stamp duties. For brevity, stamp duties on securities will be referred to simply as " stamp duties."

Stamp duty is roughly an *ad valorem* tax, except that British Government securities are exempt. The yield depends on the turnover of securities and on their price-level.

The yield is correlated with an index of the price-level of industrial shares and with the rate of change of the latter.* Writing $X_1(t)$ as the yield of stamp duties, $X_2(t)$ as the index of prices of shares, and

$$X_3(t) = X_2(t) - X_2(t-1)$$

as the change in the index from the preceding year, the regression equation is obtained :

$$X_1 = 2{\cdot}653 + 0{\cdot}019X_2 + 0{\cdot}031X_3$$

together with a multiple correlation coefficient $\bar{R} = 0{\cdot}91$ and β coefficients

$$\beta_2 = 0{\cdot}52$$
$$\beta_3 = 0{\cdot}57.$$

The difference in the relative importance of the two factors is not significant.

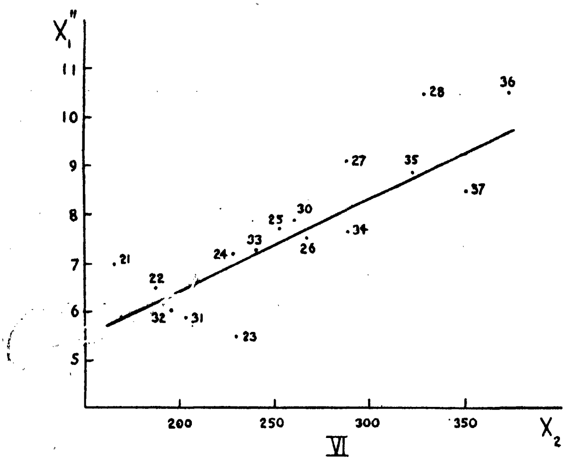
X_3 expresses the speculative factor, *i.e.* the fact that the expectations of buyers and sellers of securities are conditioned by the behaviour of security-prices in the recent past. Evidently, the yield of stamp duty should increase with X_2 and X_3, and this is confirmed by the equation of regression.

There is a possible objection that the rate of change X_3 is already taken account of in the price-level of securities, and that X_3 is not strictly an independent variable. When two variables are closely correlated, the joint effect on the dependent variable may be ascribed to either, or divided between them arbitrarily. The correlation coefficient r_{32} is only 0·4.

The price-level of securities X_2 and its rate of change X_3 are

* " A New Index of Prices of Securities," by Prof. Bowley, G. L. Schwartz and K. C. Smith. Special Memorandum No. 33. Acknowledgment is due to the London and Cambridge Economic Service for permission to use the Index.

taken jointly to represent the action of the trade cycle. In this instance the trade cycle cannot be represented satisfactorily by the percentage employment, for while the movements in security



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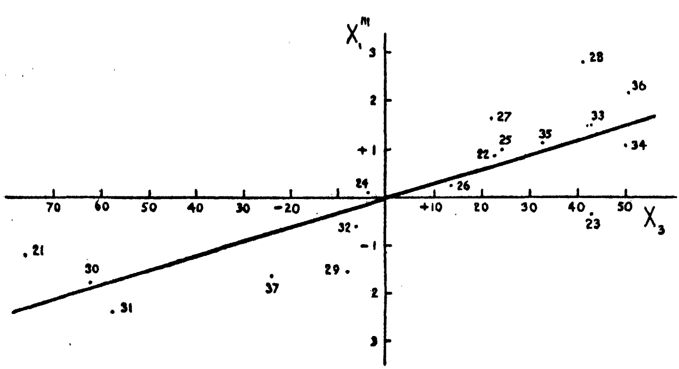
transactions are essentially a part of trade-cycle mechanism, no close relation is to be expected between these transactions and the level of percentage employment. The matter is investigated closer in section 6, where the question of percentage employment as an indicator of the trade cycle is especially considered.



VII

Chart VI shows the partial regression of X_1 on X_2. There is a close correspondence between high stock prices and increased revenue and low stock prices and diminished revenue.

The partial regression of X_1 on X_3 is shown in Chart VII. Examining the residual scatter, we find that the effect of turning

points in prices of securities is much greater than the average effect as shown by the slope of the regression line. In years of collapse of Stock Exchange boom we have negative residuals, and in periods of recovery from trough, positive residuals. Drawing a line through that class of dots corresponding to turning points, we should get a line much steeper than the average regression line. In this way the psychological effect of the collapse of prices can be seen from the Chart. People tend to become unduly cautious and fear that speculation might bring about a crash as before. Thus the recovery of prices from 1930 is very gradual, and it is only from 1933 that an upward tendency is noticeable. It is an effect of the psychology of fear and of increased liquidity preference.

Stamp duties have a very strong cyclical effect via (1) share prices, (2) the rate of change of share prices. Both represent trade-cycle factors, and appear to be of about equal importance.

5. *The yield of income tax*

The yield from income tax depends, of course, on the size of the "taxable" income and the graduation of the rate of taxation. The derivation of the yield from these data is a matter of arithmetic and does not call for verification by correlation analysis. A simplified form of the relation is

$$y = k.s.I$$

where y is the yield, s the standard rate of tax, I "taxable income" and k a constant. The relation is obvious, and does not need to be established statistically. The constant k can be estimated, but the gain in knowledge from such an estimate is negligible. We want to go deeper than to establish an arithmetic identity. An important question is how the yield of income tax is affected by trade-cycle fluctuations and how it is influenced by changes in the standard rate, taking into account the possible effect of such changes on taxable income itself. For much economic controversy centres on the possible effects of the rate of income tax on the national income.

As before, we can express the cyclical factor by the figure of percentage employment and, besides the standard rate, we shall have to take a possible trend factor into account. Now, the quantitative dependence of the yield of income tax on these three factors is not a matter of mere arithmetic, but it is a matter of actual working out of economic forces. The quantitative relation cannot in this case be established *a priori*, and if correlation analysis can throw light on it, that would represent a gain in our knowledge of the importance of some of the leading factors determining the yield from income taxation. If no inter-correlation difficulties are encountered, we

should be able actually to measure the cyclical component of the yield and the net effect of changes in the standard rate of tax.

It may be mentioned that we could not use " actual income-tax income " as an index of cyclical variation, because it has an anti-cyclical component. The exemption limit has been lowered in times of depression and raised again in recovery, thus tending to stabilise " actual income-tax income." That tends also to stabilize the yield of income tax, but trade cycle can have an effect on yield even if " actual income-tax income " is stabilized. To estimate this effect we obviously cannot use actual income-tax income as a variable, but the employment index might enable us to do so.

Four variables are used in the correlation study :

Income tax receipts X_1 (which exclude sur-tax) are correlated with the standard rate of tax X_2, with percentage employed X_3 and with time X_4. The percentage employed X_3 is taken with a lead of one year and three months, and the standard rate X_2 with a lead of exactly one year relatively to the yield X_1.

This is done to allow for the fact that there is an interval between income assessed in a given year and the time when the tax is actually paid to the Exchequer. The income tax year runs from April 6th in one calendar year to April 5th the next. Payment of the full amount of the tax becomes due on January 1st in the year of assessment, but payment of one-half may be postponed to the following July (that is to the succeeding financial year) in the case of assessments under Schedule B, Schedule D and Schedule E when made on individuals as distinct from companies. Thus there is a time-lag of eight months to one year and four months. Moreover, the profits of companies under Schedule D are assessed on the profits of the preceding year. We take an average lag of about one year, and correlate the current yield with employment of one year and three months before ($t - 1\frac{1}{4}$) and standard rate of the preceding year ($t - 1$). The extra three months in the case of employment is due to the difference between calendar year and financial year; the employment figures are given for the calendar year, while the income-tax receipts are for the financial year.

The multiple correlation coefficient $\bar{R} = 0{\cdot}876$ and the adjusted coefficient $\bar{\bar{R}} = 0{\cdot}85$. Thus quite a high degree of correlation is observed. The regression equation obtained by the method of least squares is

$$X_1 = -\ 197{\cdot}32 + 36{\cdot}36\ X_2 + 3{\cdot}305\ X_3 + 0{\cdot}45\ X_4.$$

To measure the relative importance of the three independent variable, " β " coefficients are calculated showing the proportions

of total variation in X_1, due to X_2, X_3 and X_4 respectively. The coefficients are :

$$\beta_2 = 0{\cdot}705$$
$$\beta_3 = 0{\cdot}506$$
$$\beta_4 = 0{\cdot}07.$$

It is seen that the standard rate X_2 has been the most important factor, with X_3 (employment) not much less important, whilst the influence of the trend has been very small.

To make sure that we have isolated the separate effects of the standard rate of tax X_2 and employment X_3, we must check whether there has been any correlation between the two. Actually the correlation coefficient works out as $r_{23} = 0{\cdot}08$ and is negligible. Thus isolation of the separate effects of X_2 and X_4 has been assured.

The trend effect (β_4) is negligible, and time X_4 need not have been taken into account.

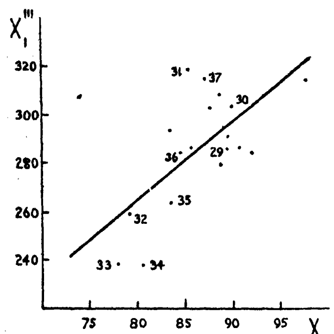
The regression line on Chart VIII shows the average dependence of the yield of income tax on percentage employment X_3 in the period 1920–37. Examining the scatter, it is seen that the regression line for the points up to 1929 would have approximately the slope or regression coefficient as the regression line for the whole period. But for the period 1930–37 the regression line would have a slope or regression coefficient approximately double that of the line for the whole period 1920–37. Thus the cycle sensitivity appears to have doubled between the two periods 1920–29, 1930–37. The regression analysis suggests that we should look for some explanation of this. The explanation, or the main explanation, lies undoubtedly in the fact that up to 1926–27 profits were assessed by taking an average of three preceding years, and this had the effect of diminishing the cyclic fluctuation in taxable income—that is, had an anti-cyclical effect. But from 1927 previous year's profits were assessed, there was no averaging, and so no anti-cyclic effect. The " damping " having been removed, the yield of income tax became more sensitive to the trade cycle. We should have expected the cycle sensitivity of income-tax yield to increase in consequence of the revision in the method of assessment of profits, and if we have not done so, our correlation analysis has brought it to our notice. Correlation analysis frequently brings to notice obvious and important facts which are otherwise apt to be overlooked. The removal of the averaging of profits under Schedule D was bound to increase the cyclical variation of taxable income, for these profits represent about 35 per cent. of the total " gross " income liable to income tax, and are especially sensitive to the trade cycle.

To show the approximate effect of the removal of "damping" in 1927 we calculate by least squares method the b_{13} coefficient for the period 1927–37. We fit a regression line for the points of Chart VIII for the period 1927–37 only. It is found that for 1927–37,

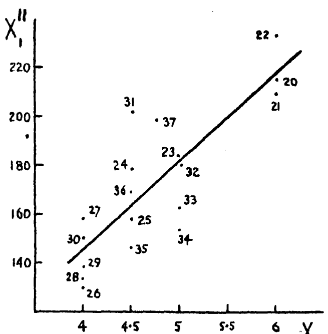
$$b_{13} = 4{\cdot}7903.$$

The previous estimate $b_3 = 3{\cdot}3046$, as the diagram suggests, may be used as a rough estimate of the regression coefficient b_3 for the earlier period 1920–26. The rise from $b_3 = 3{\cdot}30$ to $b_{13} = 4{\cdot}79$ indicates the increase in the sensitivity of the yield of income tax to the trade cycle following the removal of "damping."

If the dots of Chart VIII from 1930 to 1937 are joined consecutively, a closed anti-clockwise loop is formed. That indicates that as employment X_3 goes through a cycle, the movements of the



VIII



IX

yield X_1 lag somewhat behind the cycle of employment. But X_3 has already been taken with a lead so that the chart indicates merely that the lag of $1\frac{1}{4}$ years taken has been insufficient. The correct lag between the yield and employment is slightly greater than this. But there is no similar suggestion that the one-year lead in the standard rate X_2 is too short, as will be seen from Chart IX. This difference is natural, for there is a lag between employment and taxable income in addition to the lag between the income and the payment of income tax.

The standard rate of tax X_2 has been responsible for a considerable proportion of variation of the yield X_1 in the period 1920–37, as we note from $\beta_3 = 0{\cdot}705$, which is greater than that corresponding to X_3. The partial regression of X_1 on X_2 is shown in Chart IX and presents visually the importance of the standard rate X_2. The residual scatter is not considerable in relation to the variation along the line. There is no evidence of curvature

or saturation of the yield as the rate of tax is increased. This is as it should be, for the regression line purports to show the effect of the standard rate with given employment and national income. It is clear that in such a case the yield would not show "diminishing returns" to increments in the standard rate in the range of variation shown in the chart.

The usual objection to a high standard rate of income tax is the alleged unfavourable effect on enterprise and employment. The statistical evidence bearing on this view can be presented as follows. For the whole period 1919–36 the correlation between employment and the standard rate is $r_{24} = 0{\cdot}08$, and is negligible, suggesting that the above view is incorrect. We often hear of "spurious correlations," but the danger of "spurious absence of correlation" also has to be looked for. In the present case we find that the low correlation of 0·08 is the result of cancellation of two distinct relationships: (1) the association of high income tax with a high level of employment in periods immediately preceding or succeeding the time of war, and (2) the opposite relation in normal times when the exchequer tends to raise the rate of income tax in depression and to lower it in prosperity. The years 1919, 1920 and 1936–38 are war-affected or war-infected, and excluding them we do observe the normal relation between cyclical variations in employment and the standard rate of income tax in the period, say, of 1922–35. For that period the correlation coefficient between X_3 and X_2 is $r_{23} = -0{\cdot}56$, a significant change from $r_{23} = 0{\cdot}08$ for the 1919–36 period. All this can be roughly learnt by simple inspection of the table of data in the appendix.

The usual reaction of the Chancellor of the Exchequer to the cycle can be looked upon as one of the effects of the trade cycle. Then, the total action of the cycle on the yield of income tax is the sum of the net effects of employment b_3X_3 and the effects b_2X_2 of consequential changes in the standard rate X_2. The total effect is smaller than the net effect of the fluctuations of employment—because the latter is normally deliberately counteracted by opposite adjustments in the standard rate of tax. In the present paper the separate effect of fluctuations of employment on the yield of income tax is alone taken to represent the effect of the trade cycle—and the indirect effect *via* the counteracting adjustments of the standard rate is excluded. This is done in order to reveal that cyclical fluctuation of the yield of income tax which annoys the Chancellor of the Exchequer and which he endeavours to neutralize.

Now, as to the alleged bad effect of high income tax on employment, the negligible correlation $r_{32} = 0{\cdot}08$ (1919–36) expresses the fact that any such bad effect, if it exists, is negligible when

confronted with the stimulus to employment due to war factors. There is, therefore, substantial statistical (and theoretical) evidence in favour of the view that the standard rate can be raised even beyond the present level of 7*s.* 6*d.* without any danger of depressing the level of employment. Opponents of further increases in the standard rate should make it clear that while these increases in income tax would have no influence on the level of activity or employment, the *efficiency* of that activity is likely to be impaired. For by raising income tax further, we shall soon undermine the profit motive, forego the de-centralizing advantages of the "automatic" mechanism of private enterprise, and finally find ourselves saddled with the difficulties of excessive bureaucratic control: lack of contact between the "centre" and the "periphery," confusion at the "centre," and waste at the "periphery." We are in some danger of sinking into the stupefying embrace of bureaucratic excess.

As to the value of the correlation coefficient $r_{23} = -0{\cdot}56$ for the "normal" years 1922–35, the common-sense explanation is that it represents adjustments of the standard rate of income tax to the trade cycle, rather than indicates any effect of taxation of profit on the level of employment. No one seems to have argued that the trade cycle is *caused* by fluctuations in the rate of income tax, but it *has* been argued that booms are exaggerated and depressions deepened by the cyclical adjustments of the rate of income tax. Qualitative theory shows that this effect must be ephemeral, because the reduction in profits following a rise of income tax is largely offset by the increase in sales due to the expenditure by the State of the proceeds of income tax. Hence, the correlation coefficient $r_{23} = -0{\cdot}56$ is to be interpreted as expressing mainly the "normal" reactions of the Chancellor of the Exchequer to the bliss of prosperity and the sting of depression.

6. It would be interesting to compare the five taxes analysed in respect of the degree to which their yields are affected by the trade cycle. The results of foregoing analyses may be conveniently coordinated from that point of view.

Cycle sensitivity is measured by the elasticity of the yield with respect to percentage employment.

In the case of tobacco, spirits and beer duties, the employment elasticities are given in the respective logarithmic regression equations.

In the case of income tax, a mean elasticity can be calculated

$$E = b_3 \frac{M_3}{M_1}$$

where b_3 is the coefficient of regression of the yield X_1 on employment X_3 and M_3 and M_1 are the means of X_3 and X_1. Substituting the estimates $b_3 = 3{\cdot}3$ and M_3 and M_1 obtained for the period 1920–37,

$$E = 1{\cdot}08$$

and this is an approximate estimate really only for the period prior to 1927, in view of what has been said about Chart VIII in Section 5.

For the period 1927–37

$$b_3 = 4{\cdot}79 \text{ and } \frac{M_3}{M_1} = \frac{84{\cdot}9}{252{\cdot}1} \text{ and}$$
$$E = 1{\cdot}613,$$

showing increased cycle-sensitivity following the removal of anticyclic "damping" in 1927.

In 4 the yield of stamp duties from securities has been correlated with the price-level of securities X_2 and the rate of change of that price-level X_3. To find an employment elasticity of the yield, we can correlate the yield with percentage employment which can be designated here by X_p to avoid confusion. The correlation will have some sense, because transactions in securities and employment are both greatly affected by the trade cycle—in different ways. The movements of security transactions generally precede movements of employment because speculation anticipates the trade cycle. The best correlation is obtained by relating the yield of stamp duties $X_1(t)$ of employment $X_p(t+1)$ a year later. This is clear from the concurrent time series in Chart X.

The correlation should be somewhat lax, for speculation looks forward to more than one year ahead.

The correlation coefficient is $r_{1p} = 0{\cdot}60$ and the corresponding regression coefficient is

$$b_{1p} = 0{\cdot}34049$$

and hence the elasticity of the yield with respect to employment is

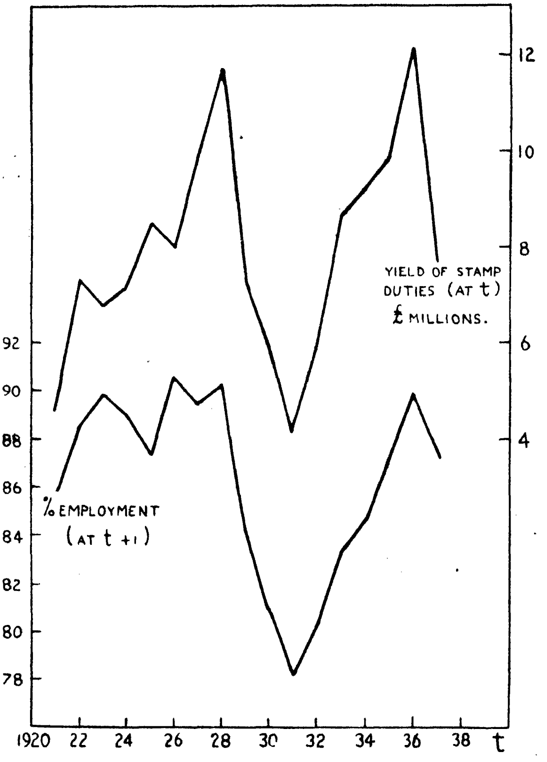
$$E_p = b_{1p} \times \frac{M_p}{M_1} = 3{\cdot}708$$

where $M_p = 86{\cdot}0941$ is the mean of X_p and $M_1 = 7{\cdot}906$ is the mean of X_1.

This estimate E_p is an under-statement (a minimum estimate) of the sensitivity to the trade cycle because the relation between the yield and employment is a loose one: $r_{1p} = 0{\cdot}60$. The full sensitivity to the cycle may be calculated in the following steps. The argument is compressed to save space. First we can take X_3, the

rate of change in security prices, to represent the action of the cycle. The simple correlation coefficient between yield X_1 and the speculative factor X_3 is $r_{13} = 0{\cdot}80$, the corresponding total regression coefficient is $b_{13} = 0{\cdot}04366$ and the elasticity with respect to speculative factor X_3 is

$$E_3 = b_{13} \frac{M_3}{M_1} = 0{\cdot}035$$



X

where $M_3 = 6{\cdot}353$ is mean X_3 and $M_1 = 7{\cdot}906$ is mean X_1. The elasticity E_3 is not comparable to employment elasticities E because the relative standard deviation of the speculative factor $\frac{\sigma_3}{M_3}$ is very much greater than the relative standard deviation of employment $\frac{\sigma_p}{M_p}$ in course of the trade cycle. This illustrates the vast difference in results due to differences in the choice of variables to repre-

sent the trade cycle. The speculation elasticity E_3 is made comparable to employment elasticities by a simple mathematical transformation. The transformed value is

$$(E)' = E_3 \times \frac{\sigma_3}{M_3} \div \frac{\sigma_p}{M_p} = 5\cdot00$$

where σ_3 is the standard deviation of X_3 and σ_p that of X_p, etc.

The formula is equivalent to

$$(E)' = E_p \,.\, \frac{r_{13}}{r_{1p}} = 5\cdot00$$

This $(E)'$ is an under-statement because the effect of price level X_2 has not been taken fully into account because $|r_{23}| < 1$. Extending the above formula, the full elasticity in terms of variations of employment when both X_2 and X_3 are taken fully into account is

$$(E) = E_p \,.\, \frac{R_{123}}{r_{1p}} = 5\cdot69$$

where $R = 0\cdot92$ is the multiple correlation coefficient between the yield X_1 and the factors X_2, X_3.

The estimate (E) is an over-estimate, because there is an upward trend in the price-level series X_2, so that some of the effect on the yield X_1 is due to that trend, and not merely to the cyclical variation in the price-level. But the trend in X_2 is unimportant relatively to the cyclical variation. Hence the error of over-statement in (E) is unimportant.

The following table shows the five yields arranged in order of their cycle sensitivity. The "E" estimates for income tax are shown both for the period before and the period after 1927. The direct employment elasticity of the yield of stamp duties E_p is shown without brackets and the "full" sensitivity (E) is shown in brackets.

Table of cycle sensitivity

Tax	Sensitivity.
Tobacco	0·43
Income (1920–26)	1·08
Spirits	1·29
Beer	1·54
Income (1927–37)	1·61
Stamp	3·71 (5·69)

The above estimates are reliable, especially for purposes of comparison, except for the sensitivity of income tax, a more accurate estimate of which, and its sampling errors, can be calcu-

lated for 1927–38 by correlating the yield with the standard rate and percentage employment for that period. The true sensitivity for the latter period, however, is evidently greater than unity.

The advantage of the elasticity with respect to percentage employment is that all we have to do to estimate approximate relative cyclical fluctuation of a yield is to multiply the elasticity by the relative fluctuation of percentage employment. Thus, if the latter fluctuates by 10 per cent. (*i.e.*, 5 per cent. about trend) the yield of beer duty fluctuates by about 15 per cent. (*i.e.*, 7 or 8 per cent. about trend), etc. Actual cyclical percentage fluctuations of the yields about their trends can be estimated from the equations of regression. The mathematics are somewhat laborious, and are not reproduced. They merely confirm the results obtained in terms of the employment elasticities.

Even on the minimum estimate of $E = 3{\cdot}7$ the yield of stamp duties is by far the most sensitive to the trade cycle of the five tax-yields investigated. The least sensitive is the yield of tobacco duties. The literary economist will be pleased to note that the findings of multiple regression analysis have the advantage of being confirmed by the common observation that smoking is more stable than speculation. To be precise, it is about twelve times as stable—a sort of fact which some literary economists prefer to establish by mental efforts *a priori*.

The results lend themselves to numerous applications. For instance. The sensitivities relating to the important taxes, except on tobacco, are greater than unity. Therefore, so far as the five taxes investigated go, the revenue of the Exchequer would fluctuate in course of the trade cycle more violently than employment, and that creates a problem which the Exchequer has to solve by borrowing or by changing the rates of taxation.

7. APPENDIX

Table of observations

The first column indicates calendar years. These serve also as a parameter for the financial years. A financial year is indicated by the calendar year having with it nine months in common. The yields are expressed in £ millions, and relate to financial years, as does the standard rate. The rest of the data relate to calendar years, except the average rate of beer duty. The latter is the yield divided by consumption and so relates to both kinds of years.

Consumption of beer for 1938 is estimated from a figure given in " bulk barrels " on the assumption that the measurement in " standard barrels " bears the same ratio in 1938 to the measure-

ment in " bulk barrels " as it does in the years immediately proceeding 1938. For 1935–37 the ratio is nearly constant.

Table of Data

Year	Percentage employment	Yield of tobacco duty	Yield of spirits duties	Consumption of beer, millions of standard barrels	Average rate of beer duty, £ per standard-barrel	Yield of beer duty	Yield of stamp duties on securities	Index of the price level of shares	Yield of income tax	Standard rate of income tax, shillings per £1.
1919 ...	97·6	—	—	—	—	—	—	—	—	6
1920 ...	97·6	—	—	26·9	4·6	123·4	—	242	340·7	6
1921 ...	83·4	—	—	24·4	5·0	121·9	4·6	165	334·9	6
1922 ...	85·9	—	53·6	20·9	4·4	92·3	7·2	188	314·4	5
1923 ...	88·4	—	54·0	20·8	3·9	81·7	6·8	231	271·4	4·5
1924 ...	89·8	51·9	51·1	21·9	3·7	82·0	7·1	228	275·5	4·5
1925 ...	89·0	53·5	49·9	22·1	3·7	82·4	8·5	252	258·1	4
1926 ...	87·7	53·9	43·6	21·2	4·0	84·2	8·0	266	230·1	4
1927 ...	90·4	58·1	47·4	20·9	4·0	83·3	9·8	288	253·5	4
1928 ...	89·3	59·1	45·7	20·5	3·7	75·8	11·7	329	237·3	4
1929 ...	89·7	62·8	42·6	20·4	3·8	77·2	7·3	323	237·9	4
1930 ...	84·2	64·1	40·6	20·0	3·8	75·7	5·9	261	255·3	4·5
1931 ...	78·9	63·3	34·9	17·5	4·3	75·2	4·1	203	288·4	5
1932 ...	78·1	67·2	34·5	14·1	5·2	73·7	5·8	197	250·6	5
1933 ...	80·2	67·5	33·5	15·3	3·8	58·9	8·7	240	228·6	5
1934 ...	83·4	70·7	32·4	16·5	3·6	58·7	9·2	290	229·2	4·5
1935 ...	84·7	75·0	34·9	17·3	3·5	60·8	9·9	323	237·4	4·5
1936 ...	87·1	77·3	36·2	17·7	3·5	62·7	12·1	374	257·0	4·75
1937 ...	89·4	82·8	35·8	18·7	3·5	65·7	7·7	350	297·9	5
1938 ...	87·4	—	—	18·9*	3·5	65·6	—	—	—	—

Estimate.

. Most of the arduous computations involved in the regression analyses have been made by Miss Vera Martin of the London and Cambridge Economic Service.

Discussion on Dr. Edelberg's Paper

[The following written comments have been received.]

Mr. R. G. D. Allen : Dr. Edelberg's analysis of the yields of tobacco, beer and spirits duties produces results which appear well-founded and significant, and I have little criticism to offer. On the whole, however, I should prefer to take

$$x_1 = ke^{bx_2}x_3^{\,c}$$

for the relation between x_1 (the yield of tobacco or spirits duties) and x_2 (time) and x_3 (employment percentage)—*i.e.*, a linear regression of the form

$$\log x_1 = a + bx_2 + c \log x_3$$

instead of Dr. Edelberg's (1) and (2) of Section 1. The form I suggest avoids certain obvious difficulties about the arbitrary zero point (*e.g.*, 1·900), from which time is measured, and it appears to

fit the observations quite as well as the form actually adopted. A least-squares calculation (which I have not fully checked) gives, for the yield of tobacco duties,

$$x_1 = \text{constant } e^{0\cdot035x_2}x_3^{0\cdot323}$$

The cycle elasticity of the yield of tobacco duties comes out at about 0·3, somewhat less than Dr. Edelberg's 0·4. The smallness of the cycle sensitivity of this yield is thus emphasized.

When Dr. Edelberg passes to his analysis of the yield of income tax, I am much less satisfied with his method and results. The problem he tackles here is essentially very limited. Apart from the trend term (which turns out to be insignificant), he is attempting to separate the effects on income-tax yield (*a*) of the trade cycle, working through changes in the national income, and (*b*) of administrative changes in the standard rate of tax, in the exemption limit and in reliefs and allowances of all kinds. Changes of type (*b*) are partly of a "sporadic" nature, depending upon short-run political considerations or even upon the whims of the Chancellor of the moment. But they are also, and perhaps largely, anti-cyclical in their effect—*i.e.*, they are designed to render the tax yield less variable over the trade cycle. Dr. Edelberg actually takes only the standard rate of tax to represent (*b*) and other changes, in the exemption limit or in the various allowances, are not covered by his regression equation. In so far as the factors he ignores are anti-cyclical, Dr. Edelberg's cyclical term ($3\cdot3x_3$ or $4\cdot8x_3$) is an under-estimate of the full effect of the trade cycle on tax yield, given a constant standard rate and a constant exemption limit and scale of allowances.

I find Dr. Edelberg's selection of the time-lags for the variables in the income tax regression rather incomprehensible. The years up to 1926–27, in which profits were averaged over a three-year period, should have been rejected at the outset; this particular difficulty is well known to anyone who has made or used estimates of the national income. For subsequent years, and speaking roughly, income declared as obtained in financial year 1 lags somewhat behind the cycle index (employment percentage), is subject to tax at the rate established in financial year 2 and contributes to the tax yield partly in this second year and partly in financial year 3. It seems clear that a lag of one year between yield and the standard rate is excessive, while the interval of one year and three months between yield and the cycle index is insufficient. The latter, at least, is later realized by Dr. Edelberg himself, but it is difficult to see why he chose such a short lag in the first place. The time-lags to be adopted, moreover, would seem to vary with the type of income taken. More illuminating results would, I think, be obtained by establishing relations for the yields under the various Schedules separately.

I cannot conclude without congratulating Dr. Edelberg upon the first stimulating results of a very laborious investigation. I shall look for further developments along this fruitful line of attack.

Professor J. R. Hicks: The five taxes whose yields have been studied by Dr. Edelberg raise decidedly different problems. The first three investigations (into tobacco, spirits and beer duties) are substantially demand studies of the familiar type; the justification for supposing that something significant can be got by multiple correlation methods applied to this sort of data lies in the presumption that most of the factors governing the demands for staple commodities are likely to be constant, or subject only to a steady trend. If this is so, it may be possible to isolate two or three variables to which nearly all oscillations about the trend can reasonably be ascribed. This presumption would seem to be fairly well borne out in the cases of tobacco and spirits by Dr. Edelberg's investigations.

A priori, one would have expected the same to be true of beer; but it is evident that beer has caused a great deal more trouble. In spite of the high coefficients of correlation obtained, one is left feeling rather sceptical about the beer analysis. The very considerable difference made to the results by assuming a lag in the effects of changes in employment is startling and disturbing; although the existence of a lag is not unreasonable in itself, why should there be the same lag when employment is falling as when it is rising? If this possibility were allowed for, would it upset things again? Then there is the queer point that no lag is discovered in the effects of changes in tax, only in the effect of changes in employment. May not the analysis be a victim of the notoriously irregular behaviour of the brewers—sometimes passing on the tax after an interval, sometimes passing it on in anticipation, sometimes doing it through changes in quality, and altogether making a smoke screen of the statistics?

The enquiries into income-tax and stamp duties are on a different footing. I fail to see why the income-tax investigation was worth making at all. The yield of income tax depends upon the size of the national income and upon the regulations which decide how much of the national income is taken in the tax; these regulations are subject to constant and important alterations, and are very inadequately represented by the standard rate of tax. To introduce them into a multiple correlation is to ask for trouble. For the rest, the question seems to boil down to one of the relation between national income and percentage unemployment; a matter which could have been studied directly to much greater advantage, though it has no great relevance to the problem in hand.

It may, indeed, be suggested that the whole investigation into these taxes would have led to more intelligible results if the cyclical movement had been represented by the fluctuations in National Income figures (which are more relevant to the enquiry, and are surely reliable enough for such purposes as these) than by the fluctuations in employment. If this had been done, the case of income tax would have fallen into its proper place, and there would have been less temptation to study the yield of stamp duties in a manner which is so divergent from the rest of the enquiry. At any rate, this is what would seem to the literary economist to be the

natural line of approach. And if this had been done, is it certain that Mrs. Hicks would have been wrong in her guess that the flexibility of the yield of spirit duties (taken against fluctuations in the *national income*) is not greater than 1 ?

Finally, is the literary economist really so powerless in these matters as Dr. Edelberg seems to suppose ? If he wants to compare the flexibilities of the yields of stamp duties and tobacco duties, he can always plot out the figures, draw—by eye—straight line trends through the curves, and compare—by eye—the proportionate fluctuations about these trends. And then (if he has properly studied the histories of the respective taxes) I fancy he will know quite as much about the matter as Dr. Edelberg does.

MR. M. G. KENDALL : In forming his regression equations Dr. Edelberg uses a technique which has recently gained in popularity among econometricians, by taking time as an independent variable. There have always seemed to me to be objections to this procedure in the study of short-term variations in time series, and my doubts are not removed by Dr. Edelberg's investigations. As a general rule I would maintain that in an enquiry into time-variation which includes both trend and short-term fluctuation, it is better to eliminate the former before directing attention on the latter, instead of following Dr. Edelberg's method and trying to consider both simultaneously. One of the advantages which Dr. Edelberg claims for his method is that the separate elimination of trend is unnecessary; but it seems to me that there is no real economy in it. The percentage of employment which he takes as an index of cyclical movements is itself subject to trend. The tax yield is thus not analysed into distinct and independent factors, trend and stationary process. The two are confounded, and accordingly I find it very difficult to interpret his regression equations. Dr. Edelberg seems to feel this disadvantage of the method in the footnote on page 165. Perhaps for his purpose it is not so important to study fluctuations about the trend value as such; but in his method I see no way, for example, of deciding such important points as how much of the fluctuation is attributable to cyclical effects, or whether the fluctuation exhibits the same sort of periodicity as the trade cycle.

There also seems to me to be objection to Dr. Edelberg's treatment of lags in his discussion of beer duties. His equation (1) is, he says, unsatisfactory. Why? Apparently because it does not give the answer which theory demands. A partial regression diagram of the type given in the paper is said to show clearly that one variable lags behind another. Would Dr. Edelberg explain this statement? There are methods of determining lags in time series by correlation analysis, such as that used by Hooker in his study of crop yields and weather, but Dr. Edelberg seems to have something different in mind, and I should be greatly interested to know more about it. I confess to some suspicion of the introduction of lags *ex machina*. Nor am I convinced by his explanation of the lag in this particular case. A taste for beer may take a year to acquire, but it does not take a year to revive.

Anyone who has had to wrestle with economic time series will be less inclined to criticize Dr. Edelberg's work than to sympathize with his efforts. I hope he will accept my comments as constructive. In happier days I should like to have been able to consider his paper in more detail and to examine the merits of alternative methods of approach; but just now there is unfortunately no time for such a thing. I congratulate him on his opportune choice of subject and the precision of his approach to it, and thoroughly endorse his general thesis that economic theories (or, for that matter, any theories) should be founded on numerical facts wherever possible. There are far too many economic theories nowadays and far too few economic facts, and any attempt to restore the balance is to be welcomed.

DR. C. OSWALD GEORGE: As a researcher in the same field, who realizes the obstacles Dr. Edelberg has had to face, I hope that my comments will appear not hypercritical but rather as indicative of the difficulties to be encountered. The paper seems likely to provoke comment at almost every paragraph, but I propose to limit myself mainly to the section on the beer duty, as this subject usually receives less attention than it deserves and is less likely to be dealt with by other commentators. A further reason is that the author declares his beer data to be of unusual interest, both from the practical and theoretical standpoint, and specifically claims that he has overcome the difficulties of comparability. This question of comparability, always of particular importance where data are scanty, appears in the present paper to be of predominant importance, and to overshadow more theoretical and, in other circumstances, more interesting questions.

Let us then examine the data used in the section dealing with beer and consider their comparability. First of all, take the employment data, where it is noticeable that, in a study of United Kingdom revenue, the author has, for some unstated reason, specially chosen employment figures applicable to Great Britain only; the extremely high percentages for 1919 and 1920—more important from both the theoretical and the practical standpoint—belong to an entirely different and by no means comparable series. Secondly, one notices that the beer consumption figures for the years 1924 to 1938 are for Great Britain and Northern Ireland only, while those for 1920 to 1922 are for Great Britain and the whole of Ireland and are uncorrected, but the following year's figure is inexplicably increased by 300,000 standard barrels. Thirdly, the so-called average rate of beer duty calls for special comment. The use of calendar-year consumption as a divisor with the substantially independent financial year's yield as a dividend gives rise to quite unnecessary difficulties, and the admission of "some inaccuracy" in the column seems rather an under-statement seeing that many of the "averages" are inaccurate, and some seriously so. Take, for instance, the increased "average" rates shown for the years 1927 and 1928, when there were in fact no changes at all, the spurious increases being due merely to the increase in revenue arising from reductions in the

period of brewers' credit. Or, take the year 1922, when, owing to an additional error, the "average" rate falls to £4·4, although the £5 rate imposed in the 1920 Budget remained unchanged until 1923. Fourthly, one may wonder why calendar-year consumption figures were ever used, seeing that they bring many difficulties and, apparently, little advantage, whereas the financial year figures, while having none of the disadvantages inherent in the corresponding spirits duty figures, might be expected to be more directly responsive to changes in the rates of beer duty. Fifthly, the figures of beer-duty yield, like those for consumption, relate not to Great Britain and Northern Ireland, but to Great Britain and the whole of Ireland in 1921 and 1922, while a new error appears in the 1923 "yield." Here the figure of £92·3 millions relates to neither area, but is merely an accounting "collected" amount, the correct "attributable" yield being £100 millions.

To pass to the beer-duty equations, the first, with its coefficient of $b_2 = -0{\cdot}026$ is certainly remarkable; and, if it be based on the accompanying data, one may perhaps be forgiven for asking whether the computations were checked. As for the remaining equation and calculations, it must be remembered throughout that they are based on employment, consumption and yield figures which in many cases are inapplicable or not comparable, and on some duty changes which never took place. They do not, therefore, call for detailed comment. It may be added that if Dr. Edelberg re-calculates his regression equation (2), after correcting for changes in area, ascertaining the actual rates of duty, and using employment figures relating to the area studied, he will get a very different equation. There will be corresponding changes in the whole of his dependent equations, and, what is noteworthy, a definite improvement in the standard error of estimate coupled with an appreciable increase in the "unusually high" multiple correlation coefficient.

Similarly, the beer revenue logarithmic regression equation is defective. There are two noticeable and particularly unfortunate additional weaknesses in the data, likely to throw suspicion in particular on his b_2 coefficient; one occurs in 1922, when a spurious fall in the rate of duty is coupled with the low "collected," instead of the much higher correct "attributable" yield for that year, and the other in 1926 and 1927, when the spurious increased rates are coupled with the equally dubious thirteen months' yields, each showing increases over 1925, whereas the actual twelve months' yield showed in each case substantial decreases.

Some of the preceding comments apply, in varying degrees, to the spirits-duty data and calculations. In addition, there is the further objection that the whole of Dr. Edelberg's spirits duty equations and arguments are based on the yields in financial years. But financial year figures, owing to forestalments and postponements which may entirely mask or even reverse actual changes, are notoriously defective criteria of spirits consumption, and every official Annual Report warns its readers of their unreliability. The calendar-year consumption figures are, on the other hand, the only useful guide to annual consumption and, consequently, to the

effective changes in revenue which we are striving to determine and examine. If, then, these annual consumption figures were adopted as basic data, and the necessary area correction made for 1922, a new equation, quite different but less questionable, would be obtained. While securing an increase in the already "very high" multiple correlation coefficient, Dr. Edelberg would find such a difference in the "cycle sensitivity" that, reconsidering the finality of the verdict he himself wrested from the Court of Correlation Analysis, he might be prepared to admit—if he accepts the normal economist's connotation of the word elasticity—that Mrs. Hicks's speculative "slight" was defensible, or even fully vindicated.

Finally, one wonders if a real danger may lurk in a metaphorical reference to a Court of Correlation Analysis. For in this so-called Court there are unfortunately no inflexible laws of evidence and, furthermore, discretion can never be a privilege of the Court but is the bounden duty of the appellant. There is much in Public Finance that may justly be regarded as falling within the jurisdiction of the Court, but the preliminary examination of evidence and causes, while offering exceptional difficulties, is of paramount importance.

MR. A. D. WEBB: I was very much interested in Mr. Edelberg's discussion of the yield of taxes on tobacco, spirits and beer, and of the influences to which the yield is subject. I am not competent to criticize the mathematical analysis contained in the paper, but I should like to offer a few comments born of what Mr. Edelberg would doubtless describe as "mental efforts *a priori*."

As regards tobacco, Mr. Edelberg finds that over the period examined (1924–37) the changes in duty were "too few to be taken into account explicitly" and their effects were merged with those due to the "time" factor. There were two changes of duty—both increases—involved, and it so happens that in each case, for special reasons, the trade itself paid most of the increase out of profits. That is to say, these particular changes were not allowed to exercise their full depressive effect on demand, and it is a fair inference that the revenue showed in consequence a greater expansion (or a greater variation) than would usually occur when duties are raised. It may be, therefore, that Mr. Edelberg's mathematical results regarding the relation between tobacco revenue and other factors might require modification in more normal circumstances—*e.g.*, a 10 per cent. increase in employment might not ordinarily be expected to be accompanied by a 4 per cent. rise in revenue.

As regards spirits, Mr. Edelberg suggests that there may have been in 1934 some turnover to beer "which naturally competes with the stronger liquors on which the spirits duties are levied." I do not wish to deny that there may be some competition between the two liquors, but the expenditure of (say) 1*s*. on each yields such different quantitative results, not to mention qualitative ones, that I should doubt whether a reduction of 1*d*. per pint in the price of beer (which was the change effected in 1933) would result in an appreciable turnover from spirits. Be this as it may, the great drop

in the consumption of spirits occurred in 1931, notwithstanding a *rise* that year in the duty on its alleged competitor beer.

As regards beer, it is perhaps worth while pointing out that Mr. Edelberg might have avoided the inaccuracy resulting "from the fact that consumption data relate to calendar years and revenue relates to financial years" if he had gone for his data to the original sources—the Annual Reports of H.M. Commissioners of Customs and Excise.

The time-lag between employment on the one hand and consumption of beer on the other is an interesting phenomenon. Mr. Edelberg suggests that it may be due to the desire for beer being "an acquired taste." May it not rather be due to the fact that after a spell of good employment consumers have funds to spend and credit is easy to raise, while after a spell of unemployment they have debts to repay?

It is a cheering fact for the Exchequer, though not for the consumer, that Mr. Edelberg's equations indicate that the maximum beer revenue may not be reached until the rate of duty is about double its present level. Of course no wise Chancellor would seek to maximize his revenue by suddenly doubling the existing duty. This would make the price of the cheapest (and weakest) beer about 9*d.* a pint, against 6*d.* to-day. The public would need a long process of education to tolerate that exalted height—which suggests that maximization of revenue is not the sum total of fiscal wisdom!

It is also pleasant to learn that the beer equations are so orderly that the Chancellor of the Exchequer, when forecasting the revenue in his Budget speech, need make only "a negligible error of estimate." I wonder if it is really so certain. Employment may spoil the Chancellor's expectations by taking a large or a sudden turn for the better or the worse; or our capricious weather clerk may inflict on us a prolonged heat wave (which would abnormally stimulate the demand for beer), or a cold and rainy summer (which would depress the demand for beer). No! the Chancellor can do little more than project for a few months ahead the curve of beer revenue as it reveals itself prior to his Budget—and then leave it to the hazards of unpredictable influences. If he should propose to alter the rate of duty, only a genuine prophet of the old sort could be bold enough to be certain of the result!

Mr. E. Grebenik: Dr. Edelberg's calculations are based on the theory that the yield of taxation is subject to cyclical fluctuations which are ultimately related to cyclical fluctuations in income. As an index of the latter he takes variations in employment. The theory is plausible, but what is the value of the elaborate analysis in which Dr. Edelberg indulges? Is it that we are proceeding from qualitative to quantitative reasoning? Regression analysis may be said to give us the size of the quantitative impact of the cycle on the yield of taxation. But is this quantitative theory more than a piece of economic history? If different periods had been investigated, or if the analysis had been applied to different countries, would the beta-coefficients obtained have been the same? Is there

any justification for extrapolating from the regression lines into the future? This would evidently be a very hazardous undertaking. It would seem that the method is useful for the interpretation of historical data, but that it cannot be used to make quantitative forecasts.

Moreover, in computing the multiple correlation coefficients, what are the principles by which the variables are chosen? Employment is not the only quantity that varies cyclically with income, and it is possible that some other variable which is related to income might give us equally high values for the multiple correlation coefficient. In this connection it might be interesting to correlate the yield from taxation with some index of wages.

One more point. When testing the significance of his regression coefficients, Dr. Edelberg uses a formula which is based on the assumption that his observations are independent. But in the case of an economic time series, successive values are probably very rarely independent of their preceding values. If there is any such serial correlation are ordinary significance formulae applicable? But I must leave this point to the mathematical statistician.

Dr. Edelberg, in reply: The foregoing constructive commentary should be of assistance to all future workers in the field surveyed by my paper.

Economic regression analysis extends over an uncomfortably large domain in "logical space." It involves simultaneously economic theory, mathematical theory and even the philosophy of chance. In addition much routine work is involved on mathematical technique and on the observational data. In planning the paper I brushed aside recondite matters on the one hand, and routine detail on the other. The main thing was to visualize approximate economic quantitative relations which could be tested statistically and to guard against elementary pitfalls of intercorrelation and sampling errors.

Omissions on the side of routine can be easily repaired, except in the case of income tax. There, a monograph is badly needed on the statistical material, on the great difficulties of accounting practice and theory, and on the economic consequences of income tax law and administration. A discussion with a specialist accountant convinced me that here are great opportunities for research.

The extensiveness of economic regression analysis in logical space goes against one's inclination for specialization in science, and the difficulty can be overcome either by increasing one's versatility or else by co-operation between the economist, the mathematician and the expert on observational data—*e.g.*, an accountant or a civil servant. An official who lives with time series can render invaluable help (as I have experienced in an industrial inquiry). On the side of routine are required something like factory methods of production. Full facilities of a statistical institute would have greatly strengthened the paper on taxation.

Passing to individual comments, Mr. Allen's suggestion of an exponential trend component hardly offers room for further improve-

ment—such as introduction of a saturation function—at any rate for the comparatively short periods studied. The influence of time-origin with my power functions is unimportant, provided the origin is taken in a fairly distant past. If Mr. Allen wishes to estimate the cycle sensitivity of income tax undisturbed by administrative changes, he can estimate the sensitivity of the " gross income " corrected for changes in the exemption limit and excluding incomes below it. Better still, he can employ the procedure for each separate schedule or sub-group thereof.

I think Professor Hicks's wholesale condemnation of the beer analysis is not justified. It is only the regression coefficient relating to the average duty that is of delicate health. Its health can be improved by taking a better series for the average rate of duty and by a few other repairs of the data. The corresponding coefficient with regard to employment is not much affected by the change in the lag (from 0 to -1). The " lead " of employment can differ as between the up-swing and the down-swing of the cycle, and the difference can be statistically investigated. But there is little reason to suppose that the lead varies much or in any particular direction, because the same kind of causes is responsible for the existence of the lead in both phases of the cycle. These causes have been described in the paper by saying that the taste for beer is " acquired ". I implied not only the inertia of habit, but also the inertia of consumption expenditure. The latter is largely due to the former, so that my emphasis on the habit inertia is excusable. The income elasticity effect is more delayed than the price (or duty) elasticity effect because income changes set heavier machinery in motion, so to speak. It is a slower and a more painful process to adjust one's consumption when one's income drops from £150 to £75 per annum than when the price of beer goes up from 4*d.* to 6*d.* per pint. Professor Hicks will probably agree to this.

Stamp duties were treated differently because their mechanism is different.

I agree with Professor Hicks that graphic correlation is helpful.

Mr. Kendall seems to over-emphasize the difficulty of interpreting my regression equations, and prefers to correlate deviations of variables from their individual trends. As is well known (*e.g.*, Frisch and Waugh, *Econometrica*, Vol. I, pp. 387, etc.), the partial regression coefficients with respect to variables other than time are identical on both methods of trend elimination—*i.e.*, on my method and on the method of correlating deviations from trends. Both methods give mathematically identical estimates of the short-period effects (as measured by regression coefficients) of the independent variables. But I sympathize with Mr. Kendall, for it is illegitimate to attribute the same influence to the long-period (trend) components of the independent variables as to their short-period components (fluctuations). My estimates of cycle sensitivity, etc., are not affected by the point, only my " β " coefficients are affected. The correlation coefficients and standard errors of estimate are not affected. I do not think my estimates of reliability of regression coefficients are affected, but this matter calls for mathematical investigation.

My lag testing method is this. We prepare a partial regression diagram such as are given in the paper. We turn it into an "indicator diagram" by joining the dots consecutive in time by arrows. The dot corresponding to time t is joined by an arrow to the dot for $t + 1$, the arrow pointing to the latter, and so on for the consecutive dots. If the independent variable is *cyclical* in time, and if the arrow-chain runs ellipse-wise round the regression line, we have the necessary (but not sufficient) condition for the existence of a lag between the dependent and the independent variable. It is easy to judge both the sign and the approximate magnitude of the lag from the *sense* of the arrows and other details of the indicator diagram.

Equation (1) of section 5 was unsatisfactory for two reasons. It led to an ellipse-wise chain on the chart of partial regression on employment (X_3), thus violating the strict conditions for the reliability tests. And the regression coefficient b_2 was so small that it was inconsistent with *a priori* theory or *a priori* prejudice. The "ellipse" disappeared when the one-year lag was introduced and equation (2) obtained. Although the lag was introduced *ex machina*, there are good reasons for it. As to acquiring and reviving the taste for beer, the consuming public consists of experts and novices in proportions which remain roughly constant as between the successive upswings of the trade cycle. Consequently, changes in the response of an individual (as he gets older) do not (materially) affect the average lag in the *collective* response.

Dr. George seems to exaggerate the difficulties of the routine order. Thanks to his suggestions, an hour at the Society's library sufficed to put right most of the details he mentions. I am having equation (2) of section 5 recalculated, and the new equation will be given at the end of the reply. But the territorial adjustments for the employment series are not made, as they do not seem to be worth troubling about. Dr. George's remarks on consumption of spirits are not quite relevant to the estimate of cycle sensitivity of the *yield* of the duty. If forestalling and postponements are correlated with the cycle series, they must not be doctored away. If they are random, it follows mathematically that they make little difference to my estimate of cycle elasticity of the yield. It is surprising that Dr. George has not asked for some adjustment from the general strike of 1926.

I congratulate Mr. A. D. Webb on a sense of humour which is a real asset in the exchange of views on economic regression. I can assuage his doubt (*Cartesian* doubt, not that of St. Thomas, as Dr. Bartlett would suggest) about the influence of changes in tobacco duties on my estimate of cycle sensitivity of the yield. There were only two changes. The increase in 1927 was associated with high employment and the increase in 1931 with low employment. The correlation between the rate(s) of duty and employment during 1924–37 is negligible and, consequently, so is the disturbing effect on my sensitivity estimate. The yield fell in 1931 because of the onset of trade depression. The "maximum" investigation was made with a view to ascertaining whether by raising beer duty

further the Exchequer might not be in danger of destroying social utility without financial benefit. "The prophet of old" was fundamentally sound, and his spirit is worth emulating to-day, provided the improvement of technique which has taken place since his days is utilized in the forecast.

With reference to a point of Mr. Grebenik, such serial correlation as is present among the *residuals* in the beer study is not serious enough to upset the reliability tests carried out in that case. The danger can be guarded against by taking a generous margin of safety—*i.e.*, by taking margins of three standard deviations or more from the best estimate of a parameter tested. If serious, the serial dependence can be tackled by "auto-regression." The usual reliability tests are based on rather restrictive mathematical conditions, especially the assumption of a static "universe." Pending developments in the mathematics of an *evolving* universe, we have to be content mainly with static tests, and we can make sensible forecasts for the near future, provided our universe changes without sharp discontinuity.

In this stern hour it is not easy to concentrate on analytical matters. On the other hand, there is less inclination for trivial controversy. A little pure science seems justified, if only for keeping mental powers sharp for greater success in the applied field.

Addendum

Making the main (significant) data adjustments requested, the new equation of beer consumption ("net quantity duty paid") for the financial years from 1922–3 to 1938–9 is

$$X_1(t) = 6{\cdot}77 - 1{\cdot}51X_2(t) + 0{\cdot}315X_3(t - 1\tfrac{1}{4}) - 0{\cdot}295X_4(t)$$

and $\bar{R} = 0{\cdot}97$, $\bar{S} = 0{\cdot}62$, $\sigma_{b_2} = 0{\cdot}38$, $\sigma_{b_3} = 0{\cdot}049$, $K = 24{\cdot}9$, the maximal duty $X_2 = K/ - 2b_2 = 8{\cdot}2$, and the mean cycle sensitivity is $b_3M_3/M_1 = 1{\cdot}42$.

For income tax (1927–8 to 1938–9), taking no "lead" in the standard rate X_2 (so as to meet to some extent a criticism by Mr. Allen), the new equation is

$$X_1(t) = -472{\cdot}6 + 61{\cdot}8X_2(t) + 5{\cdot}22X_3(t - 1\tfrac{1}{4})$$

and $\bar{R} = 0{\cdot}92$, $\bar{S} = 12{\cdot}5$, $\sigma_{b_2} = 8{\cdot}34$, $\sigma_{b_3} = 0{\cdot}92$ and mean cycle sensitivity is $1{\cdot}72$.

The computations are by Dr. H. O. Hartley.

The Unemployment Situation at the Outbreak of War

By R. G. D. Allen

I

A state of " full employment," except under artificial conditions, is not associated with that minimum of recorded unemployment which would result solely from the operation of " frictional " factors. Labour is neither homogeneous nor perfectly mobile; in fact, geographical and occupational immobility is the outstanding feature of any industrial labour market. A steadily rising demand for consumers' and producers' goods is certain, sooner or later, to encounter " bottle-necks "; unemployment tends to disappear amongst some grades of workers, while certain wages, costs and prices begin to rise despite diversions of demand. In different occupational groups, and in other localities, unemployment can continue to exist on a large scale. Pockets of severe unemployment can long survive the attainment of full employment and the onset of an inflationary spiral.

It seems clear that such a position was being approached rapidly in the spring of 1939, as a result of an increasing and inflexible demand for labour in the armament and allied trades and of the attendant indirect demands. It is even more certain that war-time conditions, with ever-growing and insistent calls upon the labour force, can only aggravate the situation. We shall find it easier to face up to the problems of the labour market set by the war if we can discover the nature of the employment and unemployment situation of the months preceding the outbreak of hostilities.

The present study attempts a partial analysis of the position during the summer of 1939 and makes use of published and unpublished data obtained by the Ministry of Labour.* We are on firm ground as long as we confine our attention to industrial and regional distributions of employment. Unemployment figures, on this basis, are obtained for one day in each month, while the numbers of insured workers, in various industries and regions, are now available for July, 1939. But a complete picture cannot be painted in the absence of some of the essential colour—colour which can only be provided by information on the incidence of unemployment in different occupational groups of workers. Occupational data are very

* The study presents some of the material collected for a paper on a rather different topic designed to be read to the Society in the autumn of 1939. Work on the paper was interrupted by the outbreak of war, when the original subject became of little more than academic interest. I have to thank the Ministry of Labour for their kindness in providing me with much unpublished material, and the Economic Research Division of the London School of Economics for a very considerable amount of assistance in the computational work.

scanty and seldom published, and we can go only a little way in our attack upon this aspect of the problem.

II

The summer months of 1929, 1937 and 1939 were periods of boom, with almost identical rates of unemployment, taking all insured workers as a single group. Some illuminating results should be revealed, therefore, by an examination of the incidence of unemployment in different industries and in various regions in these three periods. Table I exhibits the relevant figures for male workers in certain important industrial groups, selected to cover a wide range of activity in the United Kingdom. It should be remembered that June and July are usually the best, or nearly the best, months for employment. The only serious exception is coal-mining, for which the unemployment rate is about one-third higher in the summer than in the winter period of " peak " activity.

TABLE I

Unemployment amongst insured males, aged 16–64, in certain industrial groups, Great Britain and Northern Ireland, 1929–39

Industrial Group *	Estimated number insured at July					Average percentage unemployed, mid-year †		
	Number, 000's			1929 = 100				
	1929	1937	1939	1937	1939	1929	1937	1939
Coal Mining ...	1,069·2	864·5	835·1	80·9	78·1	18·9	18·2	12·9
Brick, Tile, etc., Making ...	75·1	100·1	94·2	133·3	125·4	8·1	6·7	8·1
Glass, Pottery, etc....	71·7	74·9	72·6	104·4	101·2	12·6	13·1	13·2
Engineering and allied trades	883·2	1,069·9	1,217·5	121·1	137·9	8·0	4·6	4·2
Metal Manufacture ...	307·5	318·0	328·5	103·4	106·8	16·5	9·7	8·3
Miscellaneous Metal trades...	181·4	246·7	256·6	136·0	141·5	8·9	6·0	6·1
Shipbuilding and Repairing	201·4	169·9	172·8	84·4	85·8	23·0	22·9	19·2
Cotton ...	200·7	144·2	130·5	71·9	65·0	12·6	11·0	13·2
Woollen and Worsted ...	96·1	92·7	91·0	96·4	94·7	14·4	10·1	6·4
Textile Bleaching, Printing, etc. ...	83·2	72·3	67·1	87·0	80·7	15·6	16·5	16·4
Boots, Shoes, etc. ...	83·9	80·3	80·0	95·8	95·4	15·5	12·8	10·5
Food, Drink and Tobacco...	295·7	340·8	345·7	115·2	116·9	6·7	7·7	6·5
Furniture-making ...	98·4	124·3	117·7	126·3	119·6	6·0	9·1	11·3
Printing, Publishing, etc. ...	168·6	190·3	189·6	112·9	112·4	3·7	6·1	6·2
Building ...	816·8	1,022·9	1,028·2	125·3	125·9	8·7	10·9	11·5
Public Works Contracting...	163·2	292·7	361·6	179·4	221·6	18·7	35·7	27·5
Dock, Harbour, etc., Service	168·9	164·1	161·0	97·0	95·3	31·4	26·7	22·4
Distributive trades...	1,015·4	1,278·5	1,258·9	126·0	124·0	6·6	9·6	8·8
Hotel, Catering, Laundry Service ...	148·9	210·4	212·5	141·4	142·8	7·1	11·6	12·2
All Industries (General Scheme) ...	8,755·3	9,947·3	10,186·1	113·6	116·3	10·8	11·3	10·0

* The grouping follows the classification of the Ministry of Labour. Engineering and allied trades are: General Engineering; Electrical Engineering; Motor Vehicles, Cycles and Aircraft; Electric Cable, Apparatus, etc. Miscellaneous metal trades comprise: Hand Tools, Cutlery, etc.; Bolts, Nuts, etc.; Brass and Allied Metal Wares; Heating and Ventilating Apparatus; Metal Industries not separately specified. The figures given for the Hotel, Catering, Laundry Service, which include Job Dyeing, Dry Cleaning, etc., and for All Industries in 1939 exclude the domestic workers brought into the insurance scheme in 1938.

† The average number recorded unemployed on two days in June and July as a percentage of the number insured at July.

The insured male population associated with the engineering and allied trades increased rapidly (by nearly 40 per cent.) between 1929 and 1939, mainly in the later years of the decade. Employment, however, expanded even more rapidly, so that the rate of unemployment had fallen, by 1939, to little more than 4 per cent. The situation is very similar in the miscellaneous metal trades. The iron and steel and other basic metal industries show a more recent and less remarkable expansion, but, even here, the unemployment percentage had fallen from the high level of 1929 to a very moderate figure in 1939.

Turning to the other expanding trades shown in Table I, we find six groups in which the number of insured males increased by more than 25 per cent. between 1929 and 1937. But it is seen that such rapid growth was not maintained after 1937; in fact, the numbers insured in the building, hotel and catering trades were little greater in 1939 than in 1937, while the numbers in other industries (brick making, furniture making and distributive trades) actually declined over the two years. The insured population, moreover, grew much faster than the opportunities for employment. Whereas unemployment in these trades (apart from the exceptional public works contracting industry) was well below the general level in the summer of 1929, ten years later it was around or above the average. Between 1929 and 1939, unemployment amongst hotel, catering and laundry workers rose from 7 per cent. to over 12 per cent.; for the building industry, in the "peak" summer months, the increase was from under 9 per cent. to 11½ per cent.

The food and printing groups represent "sheltered" trades working for a stable home market. Employment in the food, drink and tobacco industry expanded in a very normal way; the insured population increased by the average amounts and the level of unemployment remained substantially unchanged in the three periods. The printing trade differs only in that a recent falling off in demand has resulted in some decline in the number of insured workers and an increase in unemployment.

An almost constant volume, and hence a relatively declining level, of employment in the period from 1929 to 1939 is found in four rather different industrial groups—glass and pottery, woollen manufacture, boots and shoes and the dock and harbour trades. The number of insured workers in the first industry has also remained steady, so that unemployment persisted at a fairly high rate. In the other groups the insured population fell by about 5 per cent. in the decade 1929–39, with a consequent decrease in the rate of unemployment. Unemployment amongst male hands in the wool industry declined from 14 per cent. to 6 per cent. The level of recorded unemployment among dock workers is naturally much higher, but

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a reduction from over 30 per cent. to a little more than 20 per cent. is by no means negligible.

There remain four industries which declined rapidly in importance during the decade under review. In the shipbuilding and coal trades the number of insured male workers and the volume of employment fell by equal amounts between 1929 and 1937, 15 per cent. in the former and 20 per cent. in the latter trade. Unemployment, therefore, remained at a high level. Since 1937 employment increased and the unemployment rates were reduced. In particular, a fall in the number of insured coal-miners, combined with the increased employment in the mines between 1937 and 1939, reduced the unemployment percentage by nearly one-third. Equivalent decreases in the insured population and in the numbers in employment are also found, but now for the whole decade after 1929, in the cotton and textile finishing industries. Over the ten years, the decrease was 35 per cent. for the cotton section and nearly 20 per cent. for the finishing trade. Unemployment has persisted, therefore, at a high rate in each industry.

Excluding the public-works contracting trade, half the eighteen industries shown in Table I had unemployment rates above the average in 1929. As a result of the movement of workers out of these trades during the subsequent decade, only three of the industries are no better off. The other six show large decreases in their rates of unemployment, in some cases (*i.e.*, woollen manufacture, metal manufacture and, in a good month, coal-mining) to levels below the general average of 1939. Of the nine industry and service groups which have attracted workers in the decade, five show considerably increased unemployment percentages, and only the engineering and miscellaneous metal trades have improved their position. A definite levelling up of unemployment rates has been the result of the large-scale transference of men from the "exposed" export industries to the "sheltered" trades and services. The transference has proceeded so far that some prosperous and depressed trades have changed places. The building, hotel and catering trades, for example, had relatively more men out of work in the "peak" summer months of 1939 than the woollen manufacturing, boots and shoes and heavy metal industries—and almost as many as coal-mining in the off season. The demand for labour kept ahead of the inflow of workers only in the industries most directly affected by re-armament—*i.e.*, the engineering, electrical and finished metal trades. This is partly due to the fact that the re-armament demand came relatively suddenly and late in the decade, and partly the result of the difficulty of transference into trades employing large proportions of specially skilled men.

One set of figures can be quoted to show up the direction of the transference of men out of a particular industry—coal-mining. In the two years of boom between July 1927 and July 1929 the number of men (aged 18–64) insured in coal-mining decreased by over 80,000, partly owing to the small number of new entrants, but also as a result of transferences. In the following eight years, we find:

	Two-year period (July–July)				Total 1929–37
	1929–31	1931–33	1933–35	1935–37	
Estimated decrease in number of men insured in coal-mining...	27,700	9,510	76,920	79,370	193,500
Estimated excess (+) or deficit (−) of retirements over new entrants *	−15,480	−15,050	+32,840	+45,100	+47,410
Estimated net number transferred to other trades †	43,180	24,560	44,080	34,270	146,090

* Retirements from the age-group 18–64 other than to other insured trades *minus* new entrants into the age-group other than from other insured trades.

† Number leaving coal-mining for other insured trades *minus* number returning to coal-mining from other insured trades in the period.

The net number of transferences out of coal-mining thus averaged nearly 20,000 per year. Between 1929 and 1933 the industry attracted many new entrants, but in the later four years half the total loss of insured men in coal-mining was accounted for by retirements not replaced by new entrants. The decrease in the number of insured workers in the coal industry was smaller between July 1937 and July 1939; new entrants appear again and (with increasing prospects of employment) fewer men moved out to other trades.

Table II distinguishes some of the industrial groups into which coal-miners moved in each of the eight years from 1929 to 1937.‡ The numbers of transfers from coal-mining to the building, contracting and brick-making trades are remarkable. On the average, between 1929 and 1937, nearly half the yearly total of transfers are to these industries, and around 20 per cent. of the average yearly increase in the insured male population in these trades are coal-miners transferring in the year. The movement, however, dropped off considerably in and after 1937. The transference from coal-

‡ Though all the figures shown are net, they cannot be added over time. Between July 1929 and July 1930 5,678 more men moved from coal-mining to building and contracting than conversely. If some of these men later return to the coal trade, this movement is allowed for in the subsequent figures. But no record is obtained of those who later move on to a third insured trade—*e.g.*, engineering or transport.

TABLE II

Transference of insured men, aged 18–64, from coal-mining to other insured trades, Great Britain, 1929–37

Transferred to:	Estimated net number transferred in the year (July to July)							
	1929–30	1930–1	1931–2	1932–3	1933–4	1934–5	1935–6	1936–7
Building and Public Works Contracting	5,678	17,836	6,357	6,210	11,427	9,908	9,897	4,890
Brick, Tile, etc., Making ...	364	758	256	661	1,277	1,165	785	459
Engineering, Construction and Repair of Vehicles ...	766	50	– 94	224	807	1,184	1,745	2,268
Metal and Metal Goods Manufacture	645	346	103	297	1,072	396	1,493	2,013
Transport and Communication	1,291	1,451	564	496	842	705	773	464
Hotel and Laundry Services	457	458	302	218	302	356	426	133
Other Industries and Services	5,755	7,321	3,613	5,355	8,704	5,936	6,411	2,515
All Industries and Services	14,956	28,220	11,101	13,461	24,431	19,650	21,530	12,742

mining to the engineering and metal trades almost ceased during the depression years, but it has been growing in volume since 1933, and the number transferred in the year 1936–7 was very large, almost up to the number moving into the building and construction trades in that year. The transport and service trades continued to attract coal-miners even during the depression, though the number transferred has tended to fall generally over the period reviewed. Of other trades, the chemicals and food industries have absorbed considerable numbers of men from coal-mining throughout the period, while the textile trades took coal-miners before 1930 and again between 1933 and 1935. Very few men transferred, at least after 1930, from the coal industry to shipbuilding.

The movement of workers since 1929 has been, generally speaking, from industries largely concentrated in the "depressed" areas to trades more widely dispersed over the country. The industrial transference has been accompanied by a regional movement upon which some light is thrown by the data of Table III. This table relates to men insured in four groups of industries selected to represent the main trades attracting workers from other industries and other areas.

Comparing 1937 with 1929, we see that the engineering and allied trades prospered more in London and the south and Midlands generally. The industries here attracted labour, not only from other local trades, but clearly also from other areas. The numbers insured in general engineering in the southern regions increased at the expense of decreases in all other parts of the country. Again, though all areas in 1937 showed more insured men in the motor and

Table III

Unemployment amongst insured men, aged 18–64, *in certain industries and regions, Great Britain*, 1929–39

Industry and Region *	Estimated number insured at July			Average percentage unemployed, mid-year †		
	1929	1937	1939	1929	1937	1939
General Engineering	Number, 000's					
Great Britain	495·0	505·9	547·3	9·8	5·2	5·2
	Per cent. of Great Britain					
London	42·2	18·6	19·8	4·1	2·7	2·5
Midlands and South ...		29·0	29·2	5·3	2·5	2·6
North	43·9	39·6	39·0	13·7	6·8	7·5
Scotland	12·8	11·8	10·9	12·7	10·0	8·3
Wales	1·0	0·9	1·1	12·8	10·7	9·9
Motors and Electrical	Number, 000's					
Great Britain	311·8	457·1	558·7	6·4	4·4	3·5
	Per cent. of Great Britain					
London	74·6	26·9	24·2	3·7	2·7	2·9
Midlands and South ...		49·6	51·5	7·1	4·6	3·0
North	21·1	19·5	20·1	6·8	5·4	4·9
Scotland	3·6	3·2	3·3	8·6	7·8	5·9
Wales	0·7	0·8	0·9	9·2	16·5	12·4
Building	Number, 000's					
Great Britain	760·9	949·9	964·3	8·9	11·1	11·9
	Per cent. of Great Britain					
London	63·2	24·3	22·2	7·0	11·0	13·6
Midlands and South ...		37·9	38·3	5·8	7·2	8·1
North	24·7	25·4	25·6	13·1	14·3	15·9
Scotland	8·8	9·1	9·9	11·2	14·2	10·3
Wales	3·3	3·4	4·0	23·2	23·7	17·4
Hotel, Catering, Laundry	Number, 000's					
Great Britain	136·1	193·8	196·0	7·3	11·8	12·4
	Per cent. of Great Britain					
London	72·0	44·4	42·4	5·2	8·5	10·4
Midlands and South ...		26·3	26·6	6·6	9·1	9·1
North	18·4	19·0	20·3	11·5	18·7	17·5
Scotland	7·4	7·7	8·1	9·9	18·5	18·2
Wales	2·1	2·6	2·6	13·1	25·8	21·9

* *General Engineering* includes Engineers' Iron and Steel Founding. *Motors and Electrical* comprises Motor Vehicles, Cycles and Aircraft; Electrical Engineering; Electric Cable, Apparatus, etc. *Hotel, Catering, Laundry* includes Job Dyeing, Dry Cleaning, etc. The figures given for 1939 exclude the domestic workers brought into the insurance scheme in 1938.

The regions are composed of Divisions of the Ministry of Labour (as at July, 1939). *Midlands and South* comprises the Midlands, S.E. and S.W. Divisions; *North* the Northern, N.E. and N.W. Divisions. The unemployment percentages for London in 1929 refer to the smaller Division then defined, with corresponding differences for Midlands and South.

† The average number recorded unemployed on two days in June and July (three days in May, June and July for Building) as a percentage of the number insured at July.

electrical trades than in 1929, the increase was proportionately larger in the prosperous areas of the Midlands and south. After 1937, activity in the engineering and related industries increased in the "depressed," as well as in the more flourishing, regions, and the insured population grew rapidly everywhere. Regional differences in the rate of increase were not outstanding. It is true that the drift from the north to the south continued in the general engineering section, but the "depressed" areas had more than their proportionate share of the increase in the number of men attached to the motor and electrical trades. In these latter industries, at least, the geographical movement of workers had become smaller in volume.*

The movement of workers southwards was not in sufficient volume to meet the increased demand for labour, and the unemployment rates in London, the Midlands and the south had become almost nominal as early as 1937. Transference of men out of the northern region had also reduced the rate of unemployment to about 6 per cent. by the middle of 1937. The effect of geographical movement and of increased local activity was seen rather later in Scotland. It is clear that the main need in 1939 was for new workers in all areas, and that there was little scope left for transference of men already established in the industries.

In the building and service trades, on the other hand, the large expansion in the insured population between 1929 and 1937, and the smaller subsequent growth, had proportionately greater incidence in the north, in Scotland and in Wales. The numbers of insured men actually fell between 1937 and 1939 in London. The industrial transference behind the general growth of the trades must have been largely of men originally in the "depressed" areas, so that some considerable movement to the south must have taken place. But it appears that a substantial volume of the transferred labour has become "frozen" in the north and in Scotland and Wales. The increased unemployment also appeared, fairly uniformly, in most areas. There was no general shortage of men in any district, and an increased regional movement, except perhaps of certain specialized operatives, would only have diminished the labour surplus in one area at the expense of another. There is even some evidence that the geographical transference needed in 1939 should have reversed previous movements; for example, the rate of unemployment amongst building workers in London was nearly 14 per cent. in the summer of 1939, as against 10 per cent. in Scotland.

We conclude that transference, both industrial and regional,

* The movement had not ceased entirely. Industrial transference must be concentrated largely in the "depressed" areas, and some regional movement is needed to spread the increase over all areas.

had taken place on a considerable scale during the past decade. The process largely worked itself out, and the picture presented in 1939 was very different from that of ten years before. The "depressed" areas in 1929 contained a vast reservoir of unemployed men—coal-miners, textile operatives, iron and steel workers. This reservoir had fallen to a low level by 1939 and, instead, we find a surplus of building, distributive and service workers, not only in the "depressed" areas, but in the more prosperous districts as well. The industries mainly concerned with armaments have absorbed large numbers of men, and need more. Apart from this, the movement of workers in the decade after 1929 has proved unfortunate from the point of view of the present war economy. The trades that have attracted labour are now faced with a much-reduced demand, while the depleted industries are just those from which we require an increased output to export in payment for our essential imports of raw materials and foodstuffs.

III

Unemployment data classified by industrial groups hide many of the essential features of the employment situation. It may be possible for, say, coal-miners to become unskilled or semi-skilled building or engineering workers. But such transferences are of little avail unless accompanied by a growth in the number of the more skilled hands, either by the upgrading of operatives already in the industry or by giving specialized training to outside workers. We have, here, the problem of occupational "bottle-necks" preventing the expansion of an industry in response to a growing demand. Unfortunately, a classification of workers by occupation presents many difficulties, and little information on occupational unemployment is available.

The Ministry of Labour keeps records of the occupational distribution of unemployed men (excluding those temporarily stopped) on the registers of Employment Exchanges at certain dates in the year. Some of the information provided by these records is set out in Table IV, which relates to a number of engineering occupations selected on the score of reasonable comparability over time.* The numbers of unemployed men shown in the table cannot be converted into unemployment percentages, since the totals of insured workers in the various groups of occupations are unknown.

* The occupational classification of the Ministry of Labour has been recently in process of revision, so that series, comparable for the whole period from 1935 to 1939, cannot be given for such important workers as fitters or tool-makers. Even the series given in Table IV are not perfectly comparable over time; in any case some uncertainty must exist as to the proper classification of particular workers.

TABLE IV

Regional Distribution of unemployed men, aged 18 and over, on the registers of Employment Exchanges, certain engineering occupations, Great Britain,* 1935–39

Occupation and Region †	Number on the register at						
	April 15th, 1935	March 2nd, 1936	March 1st, 1937	March 7th, 1938	Sept. 5th, 1938	March 6th, 1939	June 5th, 1939
Turners							
London			42	141	175	144	96
Midlands and South ...	953	579	212	250	310	208	153
North	2,867	1,805	847	626	779	841	593
Scotland and Wales ...	1,307	737	311	215	252	337	209
Great Britain ...	5,127	3,121	1,412	1,232	1,516	1,530	1,051
Non-ferrous metal workers							
London			66	124	141	133	97
Midlands and South ...	834	698	280	271	361	283	189
North	1,153	771	375	228	234	232	198
Scotland and Wales ...	657	530	259	166	154	158	141
Great Britain ...	2,644	1,999	980	789	890	806	625
Skilled Foundry workers							
London			216	467	523	418	269
Midlands and South ...	2,850	2,113	770	1,338	1,868	1,621	975
North	5,800	4,139	2,127	2,253	3,519	3,868	2,573
Scotland and Wales ...	2,892	1,824	917	1,523	2,162	2,384	1,618
Great Britain ...	11,542	8,076	4,030	5,581	8,072	8,291	5,435
Capstan hands, etc.							
London			225	557	765	460	314
Midlands and South ...	2,109	1,432	729	838	1,388	791	518
North	2,866	2,137	1,279	1,011	1,204	1,374	1,098
Scotland and Wales ...	895	732	490	486	570	578	476
Great Britain ...	5,870	4,301	2,723	2,896	3,927	3,203	2,406
General engineering labourers							
London			862	1,049	953	1,084	794
Midlands and South ...	4,585	3,901	2,328	2,834	3,225	3,056	2,291
North	9,330	8,186	6,546	6,344	6,538	7,274	6,105
Scotland and Wales ...	3,505	3,296	2,912	2,593	2,737	2,843	2,411
Great Britain ...	17,420	15,383	12,648	12,820	13,453	14,257	11,601

Turners and non-ferrous metal-workers are good representatives of highly skilled engineering operatives. The rapid expansion of employment during the upswing from 1935 to 1937 reduced the total number of these workers on the register to quite small proportions by March 1937. Transference of men from other occupations had made little impression, but a considerable geographical movement must have taken place. The reduction in the register was largest in Scotland and Wales for turners, and in the north for the non-ferrous metal workers. The recession of 1938 had relatively little effect on the numbers unemployed in the country as a whole. But the stationary conditions of employment are ideal for showing up the extent of the regional movement; between March 1937 and March 1938 the numbers of turners and non-ferrous metal-workers recorded as unemployed *increased* in London and the south generally and *decreased* in other parts of the country. Regional transference had fallen off by 1939, when other parts of the country were no longer able to supply large numbers of skilled craftsmen to London and the south, and the reduction in the numbers on the register recorded during 1939 was largest in the southern areas. What figures are available for tool-makers and highly skilled fitters tend to show similar variations. Taking these two groups of workers together, we find only 901 recorded as unemployed (apart from those temporarily stopped) in June 1939—176 in London, 317 in the Midlands and south, 285 in the north and 123 in Scotland and Wales. The numbers of skilled foundry workers on the register were greater and more fluctuating, but generally showed similar changes. The boom of 1937 had the strongest effect on the numbers on the register in Scotland and Wales, while the recovery of 1939 affected London and the south more than elsewhere. The movement of these workers southwards had clearly fallen off.

Semi-skilled engineering workers are well represented by our group of capstan hands and automatic machine-workers. The fall in the total number on the register from 1935 to 1937 was quite moderate, a clear indication of a greater influx of men from other occupations and industries into the less skilled classes of engineering operatives. The inflow, moreover, was combined with sufficient

* Wholly unemployed men and non-claimants, excluding temporarily stopped workers.

† *Turners:* engineering turners, excluding non-ferrous metal turners. *Non-ferrous metal workers:* turners, fitters, finishers and goods makers in brass, bronze, aluminium, etc.; coppersmiths and copper goods makers. *Skilled foundry workers:* pattern-makers, founders and moulders in iron, steel and non-ferrous metals. *Capstan hands, etc.:* capstan and automatic machine hands; drillers, grinders, millers and other machinists not able to set up. *General engineering labourers:* including mates to skilled workers. The regions are composed as in Table III.

regional movement to cause the numbers on the register to fall almost in proportion in all areas. The recession of 1938 had a considerable effect on the numbers shown as unemployed, particularly in London and the south, from which we conclude that occupational and geographical transference continued despite the slackness of trade. But, once again, the movement to the south had diminished by 1939, for the southern areas show the greatest reductions in the volume of unemployment during the recovery of the spring of that year. Stronger conclusions of the same nature are reached as regards the inflow and regional movement of unskilled engineering workers.

The transference of men from other trades to the engineering and allied industries has not been sufficient to meet the extraordinary expansion of activity during recent years. And, what is more serious, the deficit has been greatest in the more highly skilled occupational groups; workers entering the less skilled classes have outnumbered those being upgraded into the highly skilled categories. An all-round scarcity of such workers as fully skilled fitters, toolmakers and engineering turners was evident in 1939; the possibilities of regional transference of suitable men had been worked out and recruits from lower grades of skill were difficult to find.

Turning to the building and contracting trade, as an example of an industry in which the insured population had expanded more rapidly than employment, we find reliable figures available on unemployment amongst workers of different occupations.* Table V gives the percentages of workers unemployed in various building occupations, in London and in Scotland, and during four periods of great building activity in the summers of 1929, 1936, 1937 and 1939. The two areas have been selected, since, at least up to 1937, London was the main region attracting labour from elsewhere, while Scotland was an area providing many of the workers absorbed by the building trades of the south.

Building activity in London in 1936–37 was at a far higher level than in 1929, and this growth, moreover, was accompanied by an increased use of craftsmen as opposed to less skilled workers. In Scotland, on the other hand, employment in the building trades increased relatively little during the eight years. As we have seen, the period was characterized by a great inflow of workers from coalmining and other industries, and by a considerable movement from the "depressed" to the prosperous areas. Table V now makes it clear that the inflow was largely of unskilled men remaining in the

* Occupations are most clearly defined in the building trade, and so it is not surprising that this is the single instance of regularly published unemployment percentages by occupation. Cf. the *Ministry of Labour Gazette* (before the outbreak of war) and the *Building Industries Survey* (published by the Building Industries National Council).

TABLE V

Unemployment amongst insured males, aged 16–64, in the building and public works contracting trades, London and Scotland, 1929–39

Occupation	Average percentage unemployed, mid-year *							
	London				Scotland			
	1929	1936	1937	1939	1929	1936	1937	1939
Carpenters	3·7	4·2	7·5	7·9	4·8	5·1	4·0	1·5
Bricklayers	2·2	4·2	5·8	13·2	5·6	1·7	1·2	1·4
Plasterers	3·3	4·0	7·4	19·4	5·0	3·6	3·2	2·5
Masons, Slaters, Tilers ...	2·7	9·9	10·1	17·6	5·6	7·5	5·9	6·2
Plumbers	7·2	7·5	8·9	13·9	8·1	10·1	6·3	3·3
Painters	7·0	9·2	11·5	15·4	3·8	7·7	7·1	5·8
Labourers and miscellaneous building workers ...	7·9	11·4	12·2	13·2	18·5	29·2	25·3	18·7
Public works contracting ...	9·3	11·5	11·9	10·1	18·3	43·7	39·8	28·0

* The average number recorded unemployed on three days in May, June and July as a percentage of the number insured at July. The figures for London in 1929 and 1936 refer to the smaller London Division then used by the Ministry of Labour.

"depressed" areas, while the regional movement was mainly of established craftsmen and of workers in the process of improving their skill. The increase in unemployment, which took place in London between 1929 and 1937 despite the growth of building activity, was particularly large for the main groups of tradesmen—carpenters, bricklayers, plasterers, masons, slaters and tilers. In Scotland, as a result of the southward movement of skilled men, there was much less unemployment amongst these workers, while labourers and the less skilled operatives, many of whom had transferred from other industries, remained unemployed in large numbers. As a consequence of eight years of great change, London had acquired an extensive "casual fringe" of craftsmen obtaining infrequent work, and Scotland, with a scarcity of skilled men, had a huge surplus of unskilled workers loosely attached to the industry.

Between 1937 and 1939 further changes took place which made the situation even more unbalanced. The building trades as a whole ceased expanding, while regional movements of workers almost stopped. Building activity actually declined in London, with a consequent increase in unemployment, particularly among the craftsmen recently recruited from other areas or upgraded from the ranks of the less skilled. Unemployment amongst bricklayers rose to over 13 per cent., and amongst plasterers, a very specialised group, to nearly 20 per cent. in the summer months of great activity. Only the group of carpenters, affected by the military and A.R.P. demands for wooden structures, showed a smaller proportion unemployed than the unskilled workers. In Scotland, partly as a result of the constructional work required by the armament trades, the building industry boomed, unemployment decreased in practically all occu-

pations and the scarcity of skilled men became acute. Only 1·5 per cent. of all carpenters and bricklayers, or about 350 men, were out of work in Scotland in the middle of 1939. The surplus of unskilled men, though diminished, remained very large; nearly 8,000 labourers and miscellaneous building workers were without work, together with over 14,000 men in the public-works contracting industry. At the outbreak of war, therefore, there was an ample supply of building craftsmen very unevenly distributed over the country, and far more unskilled men than could be employed at all regularly in any area.

IV

We are not yet in a position to assess the extent of the interactions of war conditions and the employment situation we have described. No data are available on the reductions of the numbers of men insured in various industries following recruitment to the Forces, neither do we know what redistribution of the remaining insured population has taken place in response to the industrial needs of war. The provision of the relevant material should be one of the first national services to be performed by the Ministry of Labour and National Service. We have only continuing records of the numbers of men unemployed in different industries. With the industrial grouping of insured males, aged 16–64, in the United Kingdom shown in Table I, we find :

Industrial Group	Average number (000's) of males unemployed on 2 days in	
	June and July, 1939	Oct. and Nov., 1939
Coal-mining	107·4	62·9
Brick, Tile, etc., Making	7·6	8·7
Glass, Pottery, etc.	9·6	7·5
Engineering and allied trades	50·7	45·8
Metal Manufacture	27·3	19·6
Miscellaneous Metal trades	15·6	11·4
Shipbuilding and Repairing	33·3	17·7
Cotton	17·2	9·0
Woollen and Worsted	5·8	3·3
Textile Bleaching, Printing, etc.	11·0	5·7
Boots, Shoes, etc.	8·4	4·5
Food, Drink and Tobacco	22·6	20·3
Furniture-making	13·4	13·9
Printing, Publishing, etc.	11·7	25·4
Building	118·4	143·8
Public Works Contracting	99·4	88·1
Dock, Harbour, etc., Service	36·1	41·4
Distributive trades	110·8	107·1
Hotel, Catering, Laundry Service	25·8	34·9
All Industrial (General Scheme)	1,020·0	940·5

After two or three months of hostilities, therefore, there were only some 80,000 fewer men out of work than in the summer before the war. In view of the recruitment of men into the Forces, we can only conclude that there has been no increase in total employment in insured trades. The figures for the separate industries are very illuminating. The effect of seasonal factors must be taken into account; between the summer and the autumn the number of unemployed coal-miners and furniture-makers normally falls considerably, while the volume of unemployment in building, and to a less extent in some other trades, usually increases. Making appropriate allowances, we find a large reduction in unemployment in all the armament and allied trades, in shipbuilding and in the export trades, notably coal-mining, textiles, glass and pottery. On the other hand, the figures make it clear how severely the outbreak of war hit the building and contracting industries, the printing trade and all the services. Recruitment of men and transference to other industries have not prevented a considerable rise in unemployment in these previously expanding trades.

Much attention has been paid to the increase, after the start of the war, in the number of unemployed persons on the registers of Employment Exchanges. This increase, however, was confined to women and juveniles; the number of men (aged 18 and over) registering as out of work fell slightly between August and the end of 1939. The number of wholly unemployed men increased, but was rather more than offset by a decrease in the number of men temporarily stopped. Unfortunately, the analysis of the register according to the period of continuous unemployment was dropped by the Ministry of Labour in September, 1939. The figures for the immediately preceding period were (in thousands):

Date, 1939	Number of men, aged 16–64, applying for insurance benefit or unemployment allowances, on the Registers of Employment Exchanges in Great Britain who had been on the register for					
	Less than 3 months	3 months but less than 6 months	6 months but less than 9 months	9 months but less than 12 months	12 months or more	Total
April 17th...	596·3	159·9	104·9	48·8	258·2	1,168·1
May 15th ...	536·7	128·8	97·4	47·4	250·4	1,060·7
June 12th...	497·8	105·7	82·9	46·6	243·9	976·9
July 10th ...	475·3	81·3	66·5	51·7	235·0	909·9
Aug. 14th...	470·5	73·3	52·1	46·5	223·1	865·5

In April 1939, after many months of trade recession, there was an unusually large number of men with a moderately long period (between three and nine months) of continuous unemployment

behind them. Many of these workers have since found work again. The most striking feature of the table, however, is the persistence of the "solid core" of unemployment, as shown by the numbers on the register out of work for a year or more. The boom of 1936–37 reduced the number in this class, excluding those who either failed to register or did not apply for benefit or allowances, to 300,000 in the spring of 1937 and to 260,000 at the end of the year. During 1938 and the early part of 1939 the number was practically unchanged, and it was only after April 1939 that any reduction was recorded. There were indeed signs, in August, that the "solid core" was beginning to melt away, and the men unemployed for at least twelve months numbered fewer than 140,000 at the beginning of 1940, according to a special count made by the Ministry of Labour.* What are these men, and to what industries are they allotted in the monthly unemployment figures? Few of them can be in the trades, notably building and the services, badly hit by the war, since most men attached to such casual trades obtain at least short spells of work at frequent intervals. It would seem that they must constitute, very largely, the last remnants of chronic unemployment in the "depressed" industries (*e.g.*, coal-mining, ship-building, textiles), men who have neither been absorbed in their old trade nor transferred elsewhere. If this is so, the numbers of (*e.g.*) coal-miners or textile operatives currently recorded as unemployed must include some men who can no longer be described as genuinely attached to the industries concerned. The figures we have given, in fact, must exaggerate, rather than under-estimate, the volume of unemployment in the vital export and heavy industries.

To summarize the position after several months of war, we can put the total number of male workers unemployed in insured trades at more than 900,000. But this scarcely constitutes a surplus of labour which can be set to work at once on producing ammunition for the military machine or coal and cotton goods for export. Almost 400,000 men of the total are in the building, contracting, distributive and service trades, while at least another 100,000 have been out of work so long that they must be counted as lacking special industrial qualifications of any kind. A very modest number remains to provide for the needs of industries concerned with armaments, the export trade and essential home production, or to supply the Services with their technical recruits. The situation is not one to be faced with complacency, even when all efforts have been made

* On January 1st, 1940, 136,669 men (aged 18–64) were recorded as unemployed for 12 months or more. Of these, 36,440 had been out of work continuously for at least five years; further, 64,178 were men over 55 years of age. Cf. *Ministry of Labour Gazette*, Feb. 1940, p. 42.

to transfer the building, distributive and service workers not recruited into the Forces, and to re-train for useful employment all those who have been without work for years. An optimum allocation of man-power can scarcely suffice to maintain, let alone to expand, essential production and to satisfy the needs of the fighting arms. We have to tackle the problem of dilution, of setting women to work in as many branches of trade and industry as possible.

Discussion on Mr. Allen's Paper

Mr. A. Reeder : Mr. Allen's paper presents a useful and interesting analysis of changes in the industrial structure during the past ten years and a statement of inferences with regard to the effect of those changes on the unemployment situation in the middle of 1939. The main outline of the changes in industry and in the regional distribution of the insured population is by now well known; for some years past the November and December issues of the *Ministry of Labour Gazette* have included a survey of the changes over a series of years in the numbers of insured persons in each of over one hundred industries and in each of the Ministry of Labour Divisional areas, and of the numbers of such persons in employment. The figures have shown the decline over a period of years of the mining and heavy metal industries, and of the industries mainly concerned in the export trade, coupled with the growing relative importance of industries and services that cater mostly for the home market. These changes have been accompanied by increases in the proportions of the working population located in the Midland and Southern areas of Great Britain.

In the metal industries and particularly in the iron and steel, general engineering, and shipbuilding groups, this trend has been reversed since 1935, largely as the result of the expansion of employment for the purpose of the defence programme, and this change has been accompanied by a distinct slackening of the rate of expansion in a number of other industries.

Figures brought together in Table I of the paper illustrate these movements, although in some cases the industrial grouping adopted has led to the inclusion, within a single group, of industries with rather widely differing records of prosperity or depression. For example, one of the most important groups in the Table—the engineering and allied trades—includes general engineering, in which the numbers declined steadily from 1929 to 1936, with three other industries, namely, electrical engineering; motor vehicles, cycles and aircraft; and electric cable apparatus, etc., in which there has been remarkable expansion throughout the whole period from 1929 to 1939. These four industries include a wide range of important occupations, and from this point of view also they are not entirely homogeneous.

It might perhaps have been worth while to split the group into two parts, so as to bring together for separate study electrical engineering and the manufacture of electric cable apparatus, etc. on the one hand, and general engineering with motor vehicles, cycles and aircraft on the other.

In the second section of the paper, changes in the numbers of insured persons in particular industries are related to the changes in the rates of unemployment in those same industries. It is suggested that the reduction of the rate of unemployment in an industry which was formerly depressed is very largely the result of the transfer of workers from that industry, and that conversely a rise in the rate of unemployment in an expanding industry may be largely the result of the reception into that industry of transferees from the depressed trades. It may perhaps be questioned whether the relation between these two sets of figures, particularly when based upon unemployment figures for two dates in each of three years, is as simple or as direct as is suggested. While there can be little doubt that transference of workers from declining to expanding industries has had some influence on their respective rates of unemployment, and has probably tended towards greater uniformity, other factors have undoubtedly been at work. Thus, as pointed out in the paper, the rate of unemployment in coalmining, in which the numbers employed have declined steadily throughout the whole of the past ten years, was substantially reduced between 1937 and 1939, largely as the result of an increased demand for coal, rather than through the transference of labour to other industries. On the other hand, in spite of very large transfers from coalmining since 1933 to the expanding engineering and metal trades, the rate of unemployment in these industries has been steadily falling. In the last three months of 1937, unemployment tended to rise in a number of important industries. As pointed out by the representative of the Committee on Economic Information of the Economic Advisory Council in the advice which he gave to the Unemployment Insurance Statutory Committee * towards the end of that year, " there were clear signs of a recession of activity in a number of different directions, such as building, motor car production, and certain export industries. . . ." This recession, which continued through 1938, affected particularly some of the more important industries into which workpeople had transferred from the declining trades, such as the building, hotel and catering trades, and there was no substantial recovery in these industries by the middle of 1939. On the other hand, the woollen and worsted, boot and shoe and heavy metal industries were by that time influenced by the increased demands of the defence programme, and this no doubt accounts for the reduction in their rates of unemployment. This has a bearing upon the point made in the paper that as the result of transference, the building, hotel and catering trades, for example, had relatively more men out of work in the " peak " summer months of 1939 than the woollen manufacturing, boot and shoe and heavy metal industries. It is necessary to exercise the same caution in

* House of Commons Paper No. 68/1938, Para. 20.

drawing inferences from the regional figures of unemployment in industries and occupations. It is probable that the defence programme has had an important influence on these figures. The location of new factories and works of construction has been the subject of careful consideration from many points of view, and this factor, coupled with the expansion of the shipbuilding and ship-repairing industry, has tended to reverse the movement of industry towards the Midlands and South which had been in operation over a number of years.

At the end of seven months of war it is still too early to assess with any precision the effects of the recruitment of men for the Forces and the transference of workpeople to the armament industries. Since the paper was written there have been substantial decreases in the numbers of men unemployed, and at March 11th, 1940 there were over 166,000 fewer men on the registers of Employment Exchanges in Great Britain than there were at August 14th, 1939. Coupled with this reduction in the numbers unemployed, there have been very large increases in the number of new entrants to insurance, and in the number of men returning to insured employment. It will be impossible to come to any definite conclusion as to whether employment in the insured trades has expanded since the outbreak of war, until the results are available of the count of insured persons in July next. It is still more difficult to determine to what extent there has been a movement of workpeople from one industry to another during the last few months. While it is true that the industries which were formerly depressed are now faced with the need for a greatly increased output, it is perhaps not too much to hope that in the search for suitable labour it may be found that a great deal is available in the industries to which men have transferred during the last ten years.

Dr. E. C. Rhodes: Mr. Allen's analysis of published and unpublished official statistics relating to employment and unemployment serves a very useful purpose, presenting as it does a picture of the distribution of the insured workers at the commencement of the war. It is admittedly difficult to convey in a concise form as much detail as one would like to have. Another person might present some other figures and perhaps come to the same conclusions. Still, one is left in some doubt about the constitution of Table I. Mr. Allen says that this table exhibits figures for male workers for certain important industrial groups. The Table certainly does this; it accounts for about 70 per cent. of insured workers, but one wonders why he left out chemicals but included pottery and glass, why he included boots, shoes, etc., and not tailoring. Supposing the Table had been extended and given in more detail, would the general conclusions which he gets still be correct. Some of the constituents of the Table are large groups, which are not homogeneous, but Mr. Allen cannot control those such as "Distributive Trades" which is the classification of the Ministry of Labour. But other groups have been formed (rather arbitrarily perhaps) by Mr. Allen, for instance, "Engineering and Allied Trades." This is

composed of general engineering, engineers, iron and steel founding, electrical engineering, motor vehicles, cycles and aircraft, and electric cable, apparatus, lamps, etc., which the Ministry of Labour distinguish separately. Some people would certainly prefer the motor group to be kept separate from the rest.

One can see differences between the constituents of this group if one examines detailed figures, especially in relation to Mr. Allen's Table in para. IV which gives the numbers unemployed just before and just after the war. Mr. Allen has confined his attention to male figures, presumably with the object of showing the comparatively small amount of male labour in reserve. But if the object is also to show how the outbreak of the war hit various industries, figures relating to females should be included, especially for those industries where the labour force includes a relatively large proportion of females.

The following Table, which is not exactly comparable with Mr. Allen's figures, gives some details which he does not include, and as some time has elapsed since his paper was written one can include a few later figures.

Industry	Number of Insured Workers, July 1939 (000)		Number Unemployed (000)							
			Males				Females			
	Males	Females	July 1939	Nov. 1939	Dec. 1939	Mar. 1940	July 1939	Nov. 1939	Dec. 1939	Mar. 1940
General Engineering	598·8	62·9	27·7	18·9	17·8	15·0	2·6	3·5	3·2	2·7
Electrical Engineering	96·4	26·5	2·7	2·1	2·0	1·7	0·8	1·1	0·9	0·8
Motor Vehicles, Cycles, and Aircraft	415·5	43·7	13·3	19·6	16·8	10·6	1·5	2·6	2·3	1·6
Electric Cable, Apparatus, Lamps, etc. ...	106·8	78·5	3·9	3·7	3·5	2·9	4·4	5·7	5·2	4·7
Explosives	27·9	7·9	0·539	0·549	0·658	0·555	0·275	0·421	0·400	0·392
Distributive Trades	1258·9	832·0	104·8	104·7	97·7	83·8	45·6	75·1	65·7	65·4
Hotel, Catering, etc.	192·9	314·4	22·9	32·6	29·8	22·8	33·4	74·8	69·7	58·3
Laundry	30·8	123·7	1·4	1·7	1·6	1·3	7·1	16·7	15·4	11·8
Pottery, etc. ...	32·0	41·2	4·5	4·3	4·2	3·1	7·7	8·1	8·6	7·4
Glass	40·6	9·9	4·7	3·0	2·9	2·9	0·6	0·8	0·7	0·7

The first four trades are those which he includes in one general group. The detailed figures show the important part in the electric cable, apparatus, etc., trade of women workers. It is to be noted that unemployment among the women of this trade increased after the beginning of the war, while that of men declined. The motor vehicle, etc. group, also had more unemployed males immediately after the war started. The big drop in the number of registered unemployed in the general engineering group, so described by the Ministry of Labour, counteracted the other tendency in the motor vehicle group and accounts for the drop shown in Mr. Allen's Table in para. IV.

An interesting feature of the Table above is the relatively small

explosives trade. The number of unemployed has risen since the outbreak of the war and is higher now than it was last July. As Mr. Allen says, we do not know how many new entrants there are into various industries since last July. It seems incredible that the obvious conclusion from these figures, that the explosives trade is in a rather worse state now than in July 1939, should be true. We assume that there has probably been a good deal of recruitment to this trade since that period. But we are now in some difficulty; if we cannot believe the tale told by the explosive figures, are we entitled to take the other figures at their face value? Should we only accept those figures which appear to be behaving in a manner which agrees with our notions of what is happening in industry, notions which we get from other sources?

Turning to the Table above, we see that the male unemployment in the distributive trades has declined, but the female unemployment is up. In the hotel, etc. group, in March 1940 the number of unemployed males is practically the same as in July 1939, but the number of females has much increased. Of course, as Mr. Allen says, these figures are affected by the seasons.

In Mr. Allen's Table in para. IV he shows glass and pottery grouped together. Actually, as we see from our Table, the unemployment in the pottery trade, where there is much female labour, increased for females and was practically the same for males from July to November 1939, while that for glass (mostly male labour), unemployment declined.

The difficulty about interpreting Mr. Allen's Table in para. IV is that we do not know how many men between mid-1939 and the autumn of 1939 were drafted into the army, and how much this change affected the numbers insured in the various trades, and therefore how much effect this had on the figures of the Table. Everyone, however, will agree with his general conclusions.

DR. C. OSWALD GEORGE: Of the many important questions raised by Mr. Allen's paper, some can be answered only by experts in labour administration who will, it is hoped, contribute to the discussion. Among the statistical problems involved, one of the most interesting is that of the "hard core," with its age, duration, and occupation distribution, and no apology may be required for presenting the following Tables, which, although originally designed for another purpose, answer questions specifically raised by Mr. Allen, and in part analyse the long-awaited but only recently published occupation data of the "hard core." But space is limited, and the Tables must be left, almost without comment, to speak for themselves. They both relate only to "men" over 18 years of age in Great Britain, and my percentages are in every case calculated to the nearer tenth; the basic data appear in the *Labour Gazette* of July 1939 (p. 262), and February 1940 (pp. 42–43).

The first Table gives the totals for each occupation group in which the number of men claimants and applicants unemployed for a year or more exceeded 500, and the percentage distribution of each

Unemployed for a Year or More on January 1*st*, 1940
(Great Britain only)

Occupation-Group	Total Numbed aged 18–64	Percentages in each Age-group					
		18–24	25–34	35–44	45–54	55–64	Total
Colliery worker (not labourer) ...	6,514	1·2	9·3	17·3	27·0	45·3	100·0
Carman, carter, cabman, etc. ...	3,054	1·3	11·3	17·8	20·8	48·7	100·0
Watchman	2,894	0·3	4·3	12·7	23·9	58·7	100·0
Shop assistant	2,281	7·0	19·1	15·2	20·4	38·3	100·0
Sailor, fireman, etc., fisherman ...	1,725	2·4	9·4	18·2	28·5	41·5	100·0
Stationary engine, crane driver, stoker, etc.	1,724	0·9	4·6	14·0	26·5	54·0	100·0
Porter (not dock, railway, hotel), messenger	1,723	12·9	18·2	15·0	20·4	33·6	100·0
Cotton worker	1,424	0·6	7·7	10·0	20·9	60·9	100·0
Motor van, lorry driver	1,231	5·0	27·5	25·6	23·2	18·8	100·0
Canvasser, insurance agent ...	1,120	2·0	9·0	18·0	28·1	42·9	100·0
Painter, decorator	885	1·6	10·4	11·5	21·4	55·1	100·0
Tailor	826	1·7	9·0	14·8	27·2	47·3	100·0
Junior clerk, addresser, sorter, etc.	819	4·0	11·4	14·8	24·8	45·1	100·0
Warehouseman	773	3·0	9·1	11·8	24·7	51·5	100·0
Dock, wharf worker	690	0·7	9·6	19·7	26·2	43·8	100·0
Commercial traveller, salesman, (wholesale), etc. ...	579	1·0	6·0	12·1	26·1	54·7	100·0
Barman, etc.	537	4·1	13·2	16·8	26·8	39·1	100·0
Riveter	510	Nil	4·5	16·5	28·8	50·2	100·0
All other occupations not classified as "labourer"	20,362	3·3	11·3	14·3	22·7	48·3	100·0
All occupations not classified as "labourer"	49,671	2·9	11·0	15·3	23·8	47·1	100·0
General labourer for heavy work	30,359	2·5	11·4	18·2	25·0	43·0	100·0
General labourer for light work	23,688	2·5	6·4	14·1	24·6	52·4	100·0
Colliery labourer	9,626	1·6	8·2	15·8	25·2	49·3	100·0
Builders' labourer, etc.	3,694	2·9	11·4	16·4	22·3	47·1	100·0
Boilershop, shipyard, constructional ironworks labourer ...	2,576	3·0	12·3	18·9	23·6	42·2	100·0
General engineering and metal trades labourer	2,230	1·9	7·2	11·9	25·4	53·7	100·0
Metal manufacture (including rolling and tube drawing) labourer	1,918	2·8	13·2	17·1	21·5	45·4	100·0
Pavior's, asphalter's, road surface layer's labourer	1,653	0·2	7·2	18·0	28·6	46·0	100·0
Ironfoundry labourer, fettler, etc.	917	1 5	9·1	11·9	21·9	55·6	100·0
All other occupations classified as "labourer"	10,337	5·8	12·5	15·5	23·3	42·9	100·0
All occupations classified as "labourer"	86,998	2·7	9·6	16·2	24·5	46·9	100·0
All occupations	136,669	2·8	10·1	15·9	24·3	47·0	100·0

occupation among the five age-groups. Nearly two-thirds of the men's "hard core" consisted of "labourers," and not much more than one-third of men in higher grades. But it will be noticed that in spite of striking variations within the two classes, the proportion of "labourers" and of "non-labourers" in the 55–64 years group were almost identical with the general average of 47·0 per cent.

The implications of this coincidence cannot be pursued here, but the elderly 47·0 per cent. may be followed into the second Table, where the figures in the last column correspond with those in the last row of the first Table (except where the first age-group is sub-divided into 18–20 and 21–24). The Table shows the distribution of the men's "hard core" as it existed on May 1st, 1939, and January 1st,

Table showing the Percentage of the Men's "Hard Core" (i.e., *of* 252,200 *on May* 1*st*, 1939, *and of* 133,669 *on January* 1*st*, 1940) *attributable on those days to the specified Age-groups, Duration-groups, and Sub-groups.*

Age-group	Duration of Unemployment											
	1 year but less than 2 years		2 but less than 3 years		3 but less than 4 years		4 but less than 5 years		5 years or more		Total 1 year or more	
	May 1939	Jan. 1940	May 1939	Jan. 1940	May 1939	Jan. 1940	May 1939	Jan. 1940	May 1939	Jan. 1940	May 1939	Jan. 1940
18–20	0·8	0·4	0·2	0·2	0·0	0·0	0·0	0·0	0·0	0·0	1·0	0·6
21–24	2·6	1·1	0·9	0·6	0·5	0·3	0·2	0·1	0·2	0·1	4·4	2·2
25–34	7·9	3·9	3·3	2·2	2·4	1·9	1·0	0·6	1·9	1·5	16·5	10·1
35–44	8·2	5·2	4·2	3·2	3·4	3·0	1·5	1·0	3·5	3·3	20·7	15·9
45–54	8·4	6·9	4·3	4·6	3·9	4·4	1·9	1·9	5·3	6·5	23·7	24·3
55–59	4·9	12·0	2·7	8·4	2·4	7·5	1·3	3·8	4·1	15·3	15·5	47 0
60–64	5·3		3·2		2·8		1·7		5·3		18·2	
Total 18–64	38·0	29·6	18·8	19·2	15·4	17·3	7·5	7·4	20·4	26·7	100·0	100·0

1940, the only two days in 1939 and 1940 for which such data are available. Between those dates the number of men unemployed for more than a year shrank at a surprisingly rapid and increasing pace from 252,200 to 136,669, but the shrinkage was far from uniform throughout. This Table, showing the different percentages—at the two dates—of the men's "hard core" attributable to each age-group, duration-group, and all the various sub-groups, is designed to emphasize the widely-varying rates of shrinkage. One noticeable point is that the 4–5 years group not only fell faster than the 2–3 or 3–4 years groups, but decreased even more rapidly than the "hard core" itself. Apart from this exceptional case, it will be seen that the shrinkage becomes slower as the age of the group and the duration of unemployment increase.

But the continuance of this not wholly surprising tendency may hinder the maximization of our labour power, and one tentative suggestion may perhaps be made. Admittedly much of the labour transference now inevitable (including the continuing movement into the various defence forces) places a premium on youth and adaptability, but it seems desirable that in every case where no loss of efficiency is thereby involved, older men should be chosen in preference to younger for voluntary or compulsory appointments or transfers.

Dr. H. W. Robinson: Mr. Allen's very interesting and important paper deals with the application of labour to the national effort. The recent measures to bring man-power under the complete control of the State, to direct when, where, in what occupation and for what remuneration a person shall work, show that it has now been realized how urgent and fundamental are the problems discussed in Mr. Allen's paper.

Table I is very instructive. But perhaps the most notable features of the labour market could be seen more clearly if we calculated indices of the net "redistribution" of labour among industries such as I have used in earlier studies. For this purpose we subtract the percentage expansion of industry as a whole from the percentage expansion of the industry concerned. The index resulting shows the percentage net movement to or from each industry. Such indices, which I have actually calculated for 1937 and 1939 from Mr. Allen's figures, show clearly and unmistakably how, between 1929 and 1937, Coal-mining, Shipbuilding and Repairing, and Textiles were the main industries to lose labour to the rest of industry. On the other hand, Building and allied trades, Distributive trades, Personal Service and Miscellaneous Metal trades showed very large expansions. These movements agree well with the unemployment rates shown in Mr. Allen's table. Comparison of the 1937 with the 1939 indices shows clearly the results of the rearmament programme between these two years. Every industry lost labour to the metal industries and Public Works contracting. Individual industries show interesting movements. As Mr. Allen points out, Coal-mining and Shipbuilding lost heavily between 1929 and 1937, and were even unable to stem the loss between 1937 and 1939. The economy is now faced with the problem of rebuilding these essential war industries. The engineering industries, as a result of the rearmament programme, have shown a phenomenal increase between 1937 and 1939. Metal manufacture is perhaps the most remarkable. It declined until 1937, and has not shown much recovery. Yet it is undoubtedly a vitally important industry, and unless we can import considerable and regular supplies of steel its expansion seems to be an urgent necessity.

Much of the large expansion in the "peace-time" industries of building and personal service came through the transference of workers directly from the contracting industries. In particular the practice of employing unemployed persons in depressed areas for short periods on Public Works contracting means that the Public Works industry is in reality simply a storehouse for unemployed men from other industries.

Mr. Allen points out, rightly, that the trades which have attracted labour are now faced with a much-reduced demand, while the depleted industries now require increased man-power. Yet this is not so unfortunate as it might appear at first sight. All that has happened is that the men from the right industries went to the wrong industries, and are still there, in cold storage, waiting to be used. The situation is much more favourable than if we required a further expansion of already expanding industries. The fact that workers from the war industries moved in peace time into peace-time industries is a fact in our favour, and we should be in a better position to bear the strain of expansion of our war effort now than we were in 1914–18. How mobile the labour force would be if we had all worked in ten industries!

As an illustration of this point let us consider Table II, which shows the industries into which coal-miners transferred between 1929 and 1937. Of the 146,000 men transferring, only 20,000 went into

the war industries. Thus 126,000 coal-miners, or thereabouts, are recoverable. No doubt the same holds good for the shipbuilders, and even for the metal manufacture group. A drive to recover such workers from war industries who are not engaged on work of equal national importance is an obvious measure.

The importance of skill is stressed by Mr. Allen, and his statistics and analysis of unemployment by areas and occupation are most valuable. Yet the importance of skill can easily be overrated. Too often we hear the cry that further expansion is prevented by a bottleneck in some skilled occupation. This might well be the case in peace-time, when a skilled man might have to perform 101 different skilled tasks every day, but this is war-time, when mass production rules the roost. We are faced with the problem of producing millions of shells, aeroplanes, tanks, guns, and so on, exactly alike. Mass production means repetition work and the elimination of skill. Thus in the last war women performed every operation required in the making of a shell, as well as a host of other operations in munitions production that had been thought to be the province of the trained man alone. This was no miracle of training, but simply the adaptation of the work to the labour available. Trade Union regulations which require that for a man to do job A he must be trained to do jobs B, C, D and E are a peace-time luxury that must be dispensed with in war-time. The solution to the labour problem is to be found in an energetic reorganization of production methods, the upgrading of semi-skilled workers and large-scale recruitment of unskilled workers to be trained for specific repetition processes. The recent appeal for garage mechanics to become aeronautical engineers is an indication that the right methods are at last being applied in this country.

It is unfortunate that Mr. Allen has confined his analysis to male labour. Many valuable results would have been obtained from a similar analysis for women. During the last war a tremendous flow of female labour into industry occurred. To-day, to obtain a comparable expansion of female employment, 3–4 million women would need to be drawn into industry. Yet to date no such gigantic recruitment is apparent. The war-time labour problem of expansion is essentially a problem of dilution, a problem of utilization of unoccupied labour. Until all our unoccupied men and women are working, we cannot obtain the maximum effort from our resources. And that problem, the problem of dilution, is a problem of adapting the methods of production so as to eliminate the need for skill.

The fact that these comments are supplementary to Mr. Allen's remarks rather than criticisms of his methods and results pays tribute to the stimulating power of his very valuable analysis.

MR. ALLEN, in reply: There is much for me to comment upon in the discussion of my paper and, indeed, much I could add to the paper itself in the light of subsequent developments. But I must be as brief as possible.

Both Mr. Reeder and Dr. Rhodes suggest amplifications and modifications in the grouping of Table I. This table was presented in a

condensed form largely because it was based upon published figures and could be amplified by anyone with access to the *Ministry of Labour Gazette.* I feel, however, that my deliberate grouping together of all the engineering and allied trades is justified. These industries include a large body of men performing, roughly, the same work wherever they may be employed—tool-room operators, skilled fitters, machinists. In addition, there are many workers of widely varying "specific" occupations—foundry hands, electrical instrument makers, motor mechanics, coach painters. The most desirable breakdown, for the purposes I had in mind, would be one based on these broad occupational distinctions. The division ready to hand, by the industries of the Ministry of Labour's classification, is of little use; it is even positively misleading. All the general engineering operators, and even some of the "specific" workers, are represented in each of the industrial groups. The motor vehicles, cycles, and aircraft trade itself is as diverse as the whole group of engineering and allied industries; it covers a very mixed bag of firms from Ford and Handley Page to the garage and repair shop around the corner and it includes the whole gambit of workers from tool-room hands and foundry workers to electricians and motor mechanics. What could be deduced from a recorded transfer of men from (say) general engineering to the motor trade? The moves may be genuine industrial transfers, but they are just as likely to be perfectly normal changes of job by tool-setters or brass turners.

The absence of statistics of female workers is easily explained. I had assembled many of the necessary tabulations, but omitted them since their inclusion would have lengthened the paper out of all proportion to the results achieved. Statistics of employment and unemployment amongst insured women are extremely difficult to handle and can scarcely be expected to produce clear-cut conclusions. Movements of women in and out of the group of insured trades are very frequent at all times—from and to the uninsured occupations, from and to the reserve of married or otherwise "unoccupied" women. These movements obscure, if they do not obliterate, the traces of genuine transferences within the insured trades.

The detailed figures on the "hard core" of unemployment were published too late for consideration in the text of my paper, and I am grateful to Dr. George for his analysis of them. I am still baffled by the occupational grouping adopted in setting out these figures; what is the "occupation" of a man continuously out of work for five years? It is clear that the details do not answer my relatively simple question: to what industries are these men allotted in the monthly unemployment figures?

Mr. Reeder has misunderstood the argument in my second section, from which I must deduce that my exposition lacks in emphasis. I certainly did not imply that a change in an unemployment rate is "very largely" the result of a transference of workers. What I did try to say was that the change reflects the transference in its relation to the growth or decline of employment in the industry

concerned. I pointed out that certain industries, *e.g.* woollen and worsted (1929–39) and coal-mining (1937–39), have been characterized by an outflow of workers during a period when employment and activity were maintained or increasing. Even before the outbreak of war, the movement of workers away from these trades, however motivated, had turned out to be too extensive in the light of actual developments in the industries. War-time conditions have emphasized, but they have not created, the need for reversing many of the main directions of transference of workers. Both Mr. Reeder and Dr. Robinson have, rightly, pointed out that men transferred in the past decade are available for a move back again. Having gone to the wrong industries, coal-miners and textile operators are (in the words of Dr. Robinson) " still there, in cold storage, waiting to be used." Is it not the severest criticism of government labour policy during these past months of war that so many useful, nay essential, men have been left in cold storage?

MISCELLANEA

CONTENTS

Population Mathematics—II

By E. C. Rhodes, D.Sc.

[Population Mathematics—I, containing Chapters I and II, Paras. 1–54, appeared in *J.R.S.S.*, 1940, Part I, pp. 61–89. References to this work are made in the following paper.]

Chapter III

55. We may use the results obtained in Chapter II to get the ultimate form of the birth function when the population is developed from other beginnings. We have, whatever the origins of the population:

$$B(t) = \int_l^L B(t-x)\phi(x)dx.$$

From this we deduce that $B(t)$ ultimately becomes Qe^{rt}, where r is the real root of $\int_l^L e^{-rx}\phi(x)dx = 1$, and where

$$Q = \frac{\int_l^{2l} B(t)e^{-rt}dt + \int_{2l}^{L+l}\left(\int_{t-l}^{L} B(t-x)\phi(x)dx\right)e^{-rt}dt}{\int_l^L xe^{-rx}\phi(x)dx}$$

and where $\int_l^L \phi(x)dx = R_0$ (paras. 34 and 37).

[As we intend for the moment to confine our attention to the real value of r, we will dispense with the suffix previously used.]

56. Let us consider a case where the births are constant for a period of time up to 0, and then constant at a different level from time 0 to time l.

Thus we will assume $B(t) = C_1$ for $t < 0$

and $$B(t) = C_2 \text{ for } 0 < t < l.$$

We shall then have, generally,

$$B(t) = \int_l^L B(t-x)\phi(x)dx,$$

and, for $l < t < 2l$, in the equation

$$B(t) = \int_{t-L}^{t-l} B(u)\phi(t-u)du,$$

(replacing $t - x$ by u), $t - L$ is negative, but $t - l$ is positive. So when we write

$$B(t) = \int_{t-L}^{0} B(u)\phi(t-u)du + \int_0^{t-l} B(u)\phi(t-u)du,$$

$B(u)$, in the first integral, where u is negative, is C_1, and, in the second integral, where u is positive, but does not exceed l (since $t - l < l$), $B(u)$ is C_2.

Therefore for $l < t < 2l$,

$$\begin{aligned} B(t) &= C_1\int_{t-L}^{0} \phi(t-u)du + C_2\int_0^{t-l} \phi(t-u)du \\ &= C_1\int_t^L \phi(x)dx + C_2\int_l^t \phi(x)dx. \end{aligned}$$

If we call $\int_l \phi(x)dx = f(x)$, then we may write, for $l < t < 2l$

$$B(t) = C_1(f(L) - f(t)) + C_2 f(t).$$

57. In the formula for Q we have to deal with $\int_{t-l}^{L} B(t-x)\phi(x)dx$, where t may have any value from $2l$ to $L + l$. This integral may be written $\int_{t-L}^{l} B(u)\phi(t-u)du$, and the lower limit of the integral is negative for $2l < t < L$, but positive for $L < t < L + l$. Therefore we split the second integral in the numerator of Q into two parts, and we have:

$$\int_{2l}^{L}\left(\int_{t-L}^{l} B(u)\phi(t-u)du\right)e^{-rt}dt + \int_L^{L+l}\left(\int_{t-L}^{l} B(u)\phi(t-u)du\right)e^{-rt}dt.$$

In the first of these integrals, where $2l < t < L$, we must write

$$\int_{t-L}^{l} B(u)\phi(t-u)du = \int_{t-L}^{0} B(u)\phi(t-u)du + \int_0^l B(u)\phi(t-u)du,$$

since $t - L$ is negative. This becomes

$$\int_{t-L}^{0} C_1\phi(t-u)du + \int_0^l C_2\phi(t-u)du$$

which may be written

$$C_1\int_t^L \phi(x)dx + C_2\int_{t-l}^t \phi(x)dx$$

or
$$C_1(f(L) - f(t)) + C_2(f(t) - f(t-l)).$$

In the second of the integrals, where $L < t < L + l$, we shall have

$$\int_{t-L}^l B(u)\phi(t-u)du = \int_{t-L}^l C_2\phi(t-u)du,$$

since now u is positive $(t - L > 0)$. This may be written:

$$C_2\int_{t-l}^L \phi(x)du = C_2(f(L) - f(t-l)).$$

Thus the numerator of Q is:

$$\int_l^{2l} \{C_1(f(L) - f(t)) + C_2f(t))e^{-rt}dt +$$
$$\int_{2l}^L [C_1(f(L) - f(t)) + C_2(f(t) - f(t-l))]e^{-rt}dt +$$
$$\int_L^{L+l} (C_2(f(L) - f(t-l))e^{-rt}dt$$

$$= \int_l^L C_1(f(L) - f(t))e^{-rt}dt +$$
$$C_2\int_l^L f(t)e^{-rt}dt - C_2\int_{2l}^{L+l} f(t-l)e^{-rt}dt + C_2\int_L^{L+l} f(L)e^{-rt}dt$$

$$= \frac{C_1}{r}f(L)(e^{-rl} - e^{-rL}) + \frac{C_1}{r}[f(t)e^{-rt}]_l^L - \frac{C_1}{r}\int_l^L f'(t)e^{-rt}dt$$
$$- \frac{C_2}{r}(f(t)e^{-rt})_l^L + \frac{C_2}{r}\int_l^L f'(t)e^{-rt}dt + \frac{C_2}{r}(f(t-l)e^{-rt})_{2l}^{L+l}$$
$$- \frac{C_2}{r}\int_{2l}^{L+l} f'(t-l)e^{-rt}dt - \frac{C_2}{r}f(L)(e^{-rt})_l^{L+l}.$$

But $f'(x) = \phi(x)$ and $\int_l^L e^{-rt}f'(t)dt = 1$.

Also

$$\int_{2l}^{L+l} f'(t-l)e^{-rt}dt = \int_{2l}^{L+l} \phi(t-l)e^{-rt}dt = \int_l^L \phi(x)e^{-r(x+l)}dx = e^{-rl}.$$

So this expression becomes, since $f(L) = R_0$, and after some cancellation:

$$\frac{C_1R_0 - C_2}{r}e^{-rl} - \frac{C_1 - C_2}{r}.$$

Thus the birth function ultimately becomes, in this case,

$$\frac{1}{\bar{m}}\left(\frac{C_1R_0 - C_2}{r} e^{-rl} - \frac{C_1 - C_2}{r}\right) e^{rt},$$

where
$$\bar{m} = \int_l^L xe^{-rx}\phi(x)dx,$$

r being the real root of $\int_l^L e^{-rx}\phi(x)dx = 1$.

58. Let us consider a few special cases.

Suppose R_0 is very near unity, and r is very small. Then approximately we may write

$$\int_l^L e^{-rx}\phi(x)dx = \int_l^L (1 - rx)\phi(x)dx = R_0 - rR_1,$$

where
$$R_1 = \int_l^L x\phi(x)dx.$$

So we have, approximately, $R_0 = 1 + rR_1$.

Also $\bar{m} = \int_l^L xe^{-rx}\phi(x)dx = R_1$ (approximately).

So the birth function will be:

$$\frac{1}{rR_1}((C_1(1 + rR_1) - C_2)(1 - rl) - (C_1 - C_2))e^{rt}$$

$$= \frac{1}{R_1}(C_1(R_1 - l) + C_2 l) \text{ neglecting powers of } r$$

$$= C_1\left(1 - \frac{l}{R_1}\right) + C_2\frac{l}{R_1}.$$

Thus, if $R_0 = 1$, and $r = 0$, we may say that the births ultimately become constant at the value $C_1\left(1 - \frac{l}{R_1}\right) + C_2\frac{l}{R_1}$.

If $C_1 = C_2$—that is to say, if the original constant births are continued—then $B(t) = C_1$ as it should.

If $C_2 = 0$—that is to say, if the original series of constant births ceases suddenly at time 0, and is zero for some reason for l years—then, on resumption, the births ultimately become steady at the level $C_1\left(1 - \frac{l}{R_1}\right)$.

59. Kuczynski has considered the special case when $C_2 = C_1R_0$, in *Fertility and Reproduction*, pp. 65 *et seq.*

In this case the birth function ultimately becomes $-\frac{(C_1 - C_2)}{r\overline{m}}e^{rt}$, from the formula in para. 57.

Kuczynski took the case when $C_1 = 100{,}000$, $C_2 = 90{,}000$, $R_0 = 0{\cdot}9$. Here r is negative having the value $-0{\cdot}0035524$ (p. 86, *loc. cit.*). For $\overline{m}$, we may write

$$\overline{m} = \int_l^L x\left(1 - rx + r^2\frac{x^2}{2} - r^3\frac{x^3}{6} + \ldots\right)\phi(x)dx$$

$$= R_1 - rR_2 + \frac{r^2}{2}R_3 - \frac{r^3}{6}R_4 + \ldots,$$

where $$R_p = \int_l^L x^p\phi(x)dx.$$

Kuczynski gives $R_1 = 26{\cdot}60110$, $R_2 = 837{\cdot}87810$, $R_3 = 27986{\cdot}48818$, $R_4 = 985528{\cdot}38498$.

We get $\overline{m} = 26{\cdot}60110 + 2{\cdot}97648 + 0{\cdot}17660 + 0{\cdot}00736$, taking the first four terms. Thus $\overline{m} = 29{\cdot}762$.

So that, in this case, $B(t)$ ultimately becomes :—

$$\frac{10{,}000}{0{\cdot}0035524 \times 29{\cdot}762}e^{-0{\cdot}0035524t} = 94{,}584e^{-0{\cdot}0035524t}.$$

60. We can compare some results obtained from this formula with those given by Kuczynski on p. 83 *loc. cit.* for years when the oscillatory components in the birth function have negligible importance. In his description of the case which he develops (on p. 65 of his book), Kuczynski states that the annual births are supposed to occur on January 1st each year, that up to year 2000, 100,000 annual births occur, then on January 1st, year 2001 there are 90,000 births, and so on.

In order to link our work with that of Kuczynski, we must adjust the difference between the continuity of the one with the discontinuity of the other. We must regard his 100,000 births occurring on January 1st, year 2000, as equivalent to 100,000 births spread over the period July 1st, year 1999, to July 1st, year 2000, and his 90,000 births occurring on January 1st, year 2001, as equivalent to 90,000 births spread over the period July 1st, year 2000, to July 1st, year 2001. The sudden break in the birth function which we placed at time 0, occurs in his time scheme at time 2000·5. Thus $t = 119\frac{1}{2}$ in our scheme corresponds to year 2120 in his scheme, and so on.

In the table below we give the births computed from our formula and those calculated by Kuczynski.

t	$B(t)$ from Formula *	Year	Kuczynski's Figures
$118\frac{1}{2}$	62,090	2119	62,091
$119\frac{1}{2}$	61,870	2120	61,870
$120\frac{1}{2}$	61,650	2121	61,650
$121\frac{1}{2}$	61,430	2122	61,430
$122\frac{1}{2}$	61,210	2123	61,212
$123\frac{1}{2}$	60,990	2124	60,994
$124\frac{1}{2}$	60,780	2125	60,777

* These values are taken to four significant figures.

The agreement between these two sets of figures is exact.

61. We may consider a further special case. Suppose $C_1 = C_2$—that is, the number of births is constant up to time l. The birth function ultimately becomes:

$$B(t) = \frac{C_1(R_0 - 1)}{rm} e^{r(t-l)}.$$

62. Let us now consider a case where the birth function is changing by a constant amount per unit time up to time l—*i.e.*, we will assume:

$$B(t) = B + Ct \text{ for } t < l.$$

Then for $l < t < 2l$, we have

$$B(t) = \int_l^L B(t-x)\phi(x)dx = \int_{t-L}^{t-l} B(u)\phi(t-u)du,$$

where $t - x = u$.

In this integral since $u < t - l < l$, we can substitute $B(u) = B + Cu$, and we get:

$$B(t) = \int_{t-L}^{t-l}(B + Cu)\phi(t-u)du =$$
$$\int_l^L (B + C(t-x))\phi(x)dx = (B + Ct)R_0 - CR_1.$$

63. We also have, in the numerator of Q, the integral

$$\int_{t-l}^{L} B(t-x)\phi(x)dx,$$

for $2l < t < L + l$. This integral is equal to $\int_{t-L}^{l} B(u)\phi(t-u)du$, and here $u < l$, therefore $B(u)$ can be replaced by $B + Cu$.

The integral therefore becomes :

$$\int_{t-L}^{l}(B+Cu)\phi(t-u)du=\int_{t-l}^{L}(B+C(t-x))\phi(x)dx.$$

Calling $\int_{l}^{x}\phi(x)dx=f(x)$ and $\int_{l}^{x}x\phi(x)dx=F(x)$, the integral under discussion may be written:

$$(B+Ct)(f(L)-f(t-l))-C(F(L)-F(t-l)).$$

Thus the numerator of Q (see para. 55) is

$$\int_{l}^{2l}((B+Ct)R_0-CR_1)e^{-rt}dt+\int_{2l}^{L+l}((B+Ct)(R_0-f(t-l))-C(R_1-F(t-l)))e^{-rt}dt$$

since $f(L)=R_0$ and $F(L)=R_1$.

The numerator of Q becomes :

$$\int_{l}^{L+l}((B+Ct)R_0-CR_1)e^{-rt}dt-\int_{2l}^{L+l}((B+Ct)f(t-l)-CF(t-l))e^{-rt}dt.$$

The second of these integrals is

$$\int_{l}^{L}((B+Cl+Cx)f(x)-CF(x))e^{-rx}e^{-rl}dx,$$

replacing t by $x+l$. This may be expressed as

$$-e^{-rl}\left(\frac{e^{-rx}}{r}((B+Cl+Cx)f(x)-CF(x))\right)_{l}^{L}+e^{-rl}\int_{l}^{L}\frac{e^{-rx}}{r}((B+Cl+Cx)f'(x)+Cf(x)-CF'(x))dx$$

which is:

$$-\frac{e^{-rl}}{r}(e^{-rL}(B+Cl+CL)R_0-CR_1)+\frac{e^{-rl}}{r}\int_{l}^{L}e^{-rx}((B+Cl)\phi(x)+Cf(x))dx$$

$$=-\frac{e^{-r(L+l)}}{r}((B+Cl+CL)R_0-CR_1)+\frac{e^{-rl}}{r}(B+Cl)+\frac{Ce^{-rl}}{r}\int_{l}^{L}e^{-rx}f(x)dx.$$

The first of these integrals is :

$$-(BR_0 - CR_1)\frac{e^{-r(L+l)} - e^{-rl}}{r}$$
$$- \frac{CR_0}{r}\left(e^{-r(L+l)}\left(L + l + \frac{1}{r}\right) - e^{-rl}\left(l + \frac{1}{r}\right)\right).$$

Thus, the numerator of Q is :

$$(BR_0 - CR_1)\frac{e^{-rl}}{r} - \frac{CR_0}{r^2}e^{-r(L+l)} + \frac{CR_0}{r}e^{-rl}\left(l + \frac{1}{r}\right) - \frac{e^{-rl}}{r}(B + Cl)$$
$$+ \frac{Ce^{-rl}}{r^2}\Big[e^{-rx}f(x)\Big]_l^L - \frac{Ce^{-rl}}{r^2}\int_l^L e^{-rx}\phi(x)dx$$
$$= (BR_0 - CR_1)\frac{e^{-rl}}{r} + \frac{CR_0}{r}e^{-rl}\left(l + \frac{1}{r}\right) - \frac{e^{-rl}}{r}(B + Cl) - \frac{Ce^{-rl}}{r^2}$$
$$= \frac{e^{-rl}}{r}\left(\left(B + Cl + \frac{C}{r}\right)(R_0 - 1) - CR_1\right).$$

64. Thus, if $B(t) = B + Ct$ for values of $t < l$, and thereafter if the fertility function ϕ operates, the birth function ultimately becomes :

$$B(t) = \left(\left(B + Cl + \frac{C}{r}\right)(R_0 - 1) - CR_1\right)\frac{e^{r(t-l)}}{rm}.$$

If $C = 0$, this becomes

$$B(R_0 - 1)\frac{e^{r(t-l)}}{rm},$$

which agrees with the case given in para. 61.

We note that the ultimate form of the birth function is the same as if, instead of the births changing in the time up to l according to the formula $B + Ct$, the births had been constant up to that time, the constant value being:

$$B + Cl + \frac{C}{r} - \frac{CR_1}{R_0 - 1}.$$

65. The births in England and Wales declined from about 900,000 to about 600,000 from 1900 to 1935, at a fairly constant rate, interrupted by the Great War and the immediate post-war years. If we take the year 1920 as our zero, and l as 15, then $B + Cl = 600{,}000$, and $C = -8571$, the drop in the number of births in 35 years being 300,000. Let us suppose that the same fertility and mortality, assumed in Kuczynski's example already quoted, operated in the

future, giving $R_0 = 0{\cdot}9$, $r = -0{\cdot}0035524$, $R_1 = 26{\cdot}60110$, $\overline{m} = 29{\cdot}762$, the values given in para. 59. Then:

$$B + Cl + \frac{C}{r} - \frac{CR_1}{R_0 - 1} = 733{,}000.$$

Thus the birth function ultimately becomes the same as if the births had been constant at the level 733,000 during the period 1900–35.

Chapter IV

66. In Chapter III we were concerned with the ultimate form of the birth function, when the damped periodic terms had died down. We must now consider the form of the birth function in the immediate future. We saw, from our examination of the simple case considered in Chapters I and II, that the direct development of the birth function from the origins from which it is derived is preferable to using the exponential expression in the earlier stages.

67. Let us deal with the illustration used by Kuczynski, *loc. cit.* Here the births are constant at a value C_1 up to time 0, then they are constant at a value C_2 up to time l.

We have for all values of t:

$$B(t) = \int_l^L B(t-x)\phi(x)dx = \int_{t-L}^{t-l} B(u)\phi(t-u)du.$$

This may be written, for $l < t < 2l$

$$B(t) = \int_{t-L}^{0} B(u)\phi(t-u)du + \int_0^{t-l} (Bu)\phi(t-u)du,$$

and, remembering that $t - l < l$, we have

$$B(t) = C_1\int_{t-L}^{0} \phi(t-u)du + C_2\int_0^{t-l} \phi(t-u)du = C_1\int_t^L \phi(x)dx + C_2\int_l^t \phi(x)dx$$

$$= C_1(f(L) - f(t)) + C_2 f(t)$$

where $\int_l^x \phi(x)dx = f(x)$. So, $B(t) = C_1R_0 - (C_1 - C_2)f(t)$, for $l < t < 2l$.

68. In Kuczynski's problem, $C_1 = 100{,}000$, $C_2 = 90{,}000$, $R_0 = 0{\cdot}9$ and therefore $B(t) = 90{,}000 - 10{,}000\, f(t)$, for $l < t < 2l$. The

values of $\phi(x)$ are given on pp. 66 and 85 of his book, and from them we may get the values of $f(x)$, and thence the values of $B(t)$.

Year of Age (x)	$\phi(x)$	$f(x)$	$10{,}000\,f(x)$	$90{,}000 - 10{,}000\,f(x)$
15	0·00869	0	0	90,000
16	0·00867	0·00869	87	89,913
17	0·01730	0·01736	174	89,826
18	0·01726	0·03466	347	89,653
19	0·02583	0·05192	519	89,481
20	0·02574	0·07775	778	89,222
21	0·02565	0·10349	1035	88,965
22	0·03408	0·12914	1291	88,709
23	0·03396	0·16322	1632	88,368
24	0·04230	0·19718	1972	88,028
25	0·04215	0·23848	2395	87,605
26	0·05040	0·28163	2816	87,184
27	0·05022	0·33203	3320	86,680
28	0·05838	0·38225	3822	86,178
29	0·04986	0·44063	4406	85,594
30	0·04968	0·49049	4905	85,095

The values in the last column agree with those given in Kuczynski's book, p. 82, being the births in the years 2016, 2017, etc.

69. For the period $2l < t < 3l$, the birth function becomes more complex. We now have, from

$$B(t) = \int_{t-L}^{t-l} B(u)\phi(t-u)du = \int_{t-L}^{0} B(u)\phi(t-u)du + \int_{0}^{l} B(u)\phi(t-u)du + \int_{l}^{t-l} B(u)\phi(t-u)du,$$

where $t - l < 2l$,

$$\begin{aligned}
B(t) &= C_1\int_t^L \phi(x)dx + C_2\int_{t-l}^{t} \phi(x)dx + \int_l^{t-l} (C_1R_0 - (C_1 - C_2)f(u))\phi(t-u)du \\
&= C_1\int_t^L \phi(x)dx + C_2\int_{t-l}^{t} \phi(x)dx + C_1R_0\int_l^{t-l} \phi(x)dx - (C_1 - C_2)\int_l^{t-l} f(u)\phi(t-u)du \\
&= C_1(f(L) - f(t)) + C_2(f(t) - f(t-l)) + C_1R_0(f(t-l) - (C_1 - C_2)\int_l^{t-l} f(u)\phi(t-u)du \\
&= C_1R_0 - (C_1 - C_2)f(t) - (C_2 - C_1R_0)f(t-l) - (C_1 - C_2)\int_l^{t-l} f(u)\phi(t-u)du.
\end{aligned}$$

It is therefore necessary to obtain $\int_{l}^{t-l} f(u)\phi(t-u)du$.

70. In Kuczynski's case, we have, for $2l < t < 3l$

$$B(t) = 90{,}000 - 10{,}000f(t) - 10{,}000f_1(t),$$

where $$f_1(t) = \int_l f(u)\phi(t-u)du.$$

We may take :

$$f_1(31) = \int_{15}^{16} f(u)\phi(31-u)du = f(16)\,.\,\phi(15) = 0{\cdot}0000755$$

$$f_1(32) = \int_{15}^{17} f(u)\phi(32-u)du = f(16)\,.\,\phi(16) + f(17)\,.\,\phi(15) = 0{\cdot}0002262$$

$$f_1(33) = \int_{15}^{18} f(u)\phi(33-u)du = f(16)\,.\,\phi(17) + f(17)\phi(16) + f(18)\,.\,\phi(15) = 0{\cdot}0006020$$

$$f_1(34) = f(16)\,.\,\phi(18) + f(17)\phi(17) + f(18)\phi(16) + f(19)\,.\,\phi(15) = 0{\cdot}0012020.$$

Continuing in this way, we have the following figures :—

Year of Age (x)	$\phi(x)$	$f(x)$	$f_1(x)$	$(f(x) + f_1(x))$ 10,000.	$90{,}000 - 10{,}000 \times (f(x) + f_1(x))$
30	0·04968	0·49049			
31	0·04125	0·54017	0·0000755	5402	84,598
32	0·03284	0·58142	0·0002262	5816	84,184
33	0·03268	0·61426	0·0006020	6149	83,851
34	0·03252	0·64694	0·0012020	6481	83,519
35	0·03236	0·67946	0·0022495	6817	83,183
36	0·02415	0·71182	0·0037420	7156	82,844
37	0·02403	0·73597	0·0058257	7418	82,582
38	0·01594	0·76000	0·0086450	7686	82,314
39	0·01586	0·77594	0·0122694	7882	82,118
40	0·01578	0·79180	0·0169878	8088	81,912
41	0·01570	0·80758	0·0227930	8304	81,696
42	0·01562	0·82328	0·0301167	8534	81,466
43	0·01554	0·83890	0·0389488	8778	81,222
44	0·00772	0·85444	0·0497149	9042	80,958
45	0·00767	0·86216	0·0623298	9245	80,755

The values in the last column agree with those given in Kuczynski's book, p. 82, being the births in the years 2031, 2032, etc.

Chapter V

71. In practice, the problem of solving the equation for r, $\int_l^L e^{-rx}\phi(x)dx = 1$, presents difficulties owing to lack of knowledge of the form of the function ϕ. In Chapter I we assumed, for purposes

of illustration, that ϕ was a sine function, but, in general, the only information respecting ϕ is that given by the values of $\phi(x)$ for certain values of x. We can, however, use moments or semi-invariants derived from the net-fertility schedule—*i.e.*, from the table giving the values of $\phi(x)$ for certain values of x, to replace the unknown constants of $\phi(x)$.

72. The simplest method is to expand e^{-rx} in the integral, and we get

$$\int_l^L \left(1 - rx + \frac{r^2x^2}{2} - \frac{r^3x^3}{6} + \frac{r^4x^4}{24} - \ldots\right)\phi(x)dx = 1$$

which gives

$$R_0 - rR_1 + \frac{r^2}{2}R_2 - \frac{r^3}{6}R_3 + \frac{r^4}{24}R_4 - \ldots = 1,$$

where $R_p = \int_l^L x^p\phi(x)dx$, the successive values of R_p being obtained from the given net-fertility schedule.

We have previously given values of R quoted from Kuczynski. The R's increase in size very considerably, and it would seem preferable to obtain an expression, involving multipliers of the powers of r, which converges more rapidly.

73. An obvious way out of the difficulty is provided by assuming that the function $\phi(x)$ can be replaced by a series of functions developed by the normal function and involving the semi-invariants or momental constants of the net-fertility schedule. This method is given in detail in Arne Fisher, *The Mathematical Theory of Probabilities* and is outlined in Elderton, *Frequency Curves and Correlation.*

We assume

$$\phi(x) = R_0\left(\frac{1}{\sqrt{2\pi}\,.\,\sigma}e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2} - \frac{1}{6}\lambda_3\frac{d^3}{dx^3}\left(\frac{1}{\sqrt{2\pi}\,.\,\sigma}e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2}\right) + \frac{1}{24}\lambda_4\frac{d^4}{dx^4}\left(\frac{1}{\sqrt{2\pi}\,.\,\sigma}e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2}\right) - \ldots\right)$$

where λ_3, λ_4, . . . are the semi-invariants, and $\lambda_3 = \mu_3$, $\lambda_4 = \mu_4 - 3\sigma^4$, . . . the μ's being the moments. m is the average of the ϕ distribution and σ its standard deviation.

We may write simply

$$\phi(x) = R_0(N(x) - \frac{\lambda_3}{6}\frac{d^3}{dx^3}(N(x)) + \frac{\lambda^4}{24}\frac{d^4}{dx_4}(N(x)) - \ldots),$$

where $$N(x) = \frac{1}{\sqrt{2\pi}\,.\,\sigma}e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2}.$$

74. The integral $\int e^{-rx}\phi(x)dx$ must now be taken between $-\infty$ and $+\infty$. We have

$$\int_{-\infty}^{\infty} e^{-rx}N(x)dx = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2 - r(\sigma u + m)} du$$

putting $\dfrac{x - m}{\sigma} = u.$

This is $e^{-rm + \frac{1}{2}r^2\sigma^2}$.

Again

$$\int_{-\infty}^{\infty} e^{-rx} \frac{d^3}{dx^3}(N(x))dx$$

after some reduction, becomes $r^3\displaystyle\int_{-\infty}^{\infty} e^{-rx}N(x)dx,$

and $$\int_{-\infty}^{\infty} e^{-rx} \frac{d^4}{dx^4}(N(x))dx = r^4\int_{-\infty}^{\infty} e^{-rx}N(x)dx.$$

Thus our equation

$$\int_{-\infty}^{\infty} e^{-rx}R_0\left(N(x) - \frac{\lambda_3}{6}\frac{d^3}{dx^3}(N(x)) + \frac{\lambda_4}{24}\frac{d^4}{dx^4}(N(x)) - \ . \ . \ .\right) = 1$$

becomes $$R_0 e^{-rm + \frac{1}{2}r^2\sigma^2}\left(1 - \lambda_3\frac{r^3}{6} + \lambda_4\frac{r^4}{24} - \ . \ . \ .\right) = 1.$$

If we take this to powers of r^4 only, we may replace it by the equation

$$-rm + \tfrac{1}{2}r^2\sigma^2 - \lambda_3\frac{r^3}{6} + \lambda_4\frac{r^4}{24} = -\log_e R_0.$$

(In para. 22, we saw, $r_0 m = \log R_0$, approximately.)

Using momental constants β_1, β_2, this equation may be written

$$-rm + \tfrac{1}{2}r^2\sigma^2 - \frac{\sqrt{\beta_1}}{6}r^3\sigma^3 + \frac{\beta_2 - 3}{24}r^4\sigma^4 = -\log_e R_0,$$

where $\beta_1 = \dfrac{\mu_3^2}{\mu_2^3}$, $\beta_2 = \dfrac{\mu_4}{\mu_2^2}$, the μ's being moments.

If we write the equation involving semi-invariants only, we have:

$$-\lambda_1 r + \frac{\lambda_2}{2}r^2 - \frac{\lambda_3}{6}r^3 + \frac{\lambda_4}{24}r_4 = -\log_e R_0$$

$$(\lambda_1 = m, \ \lambda_2 = \sigma^2 \text{ or } \mu_2).$$

75. This equation is used by Lotka in "The Progency of a Population Element," *American Journal of Hygiene*, Vol. VIII,

No. 6, Nov. 1928 and in " On the True Rate of Natural Increase," *Journal of the American Statistical Association*, Sept. 1925.

In order to solve the equation

$$-\lambda_1 r+\frac{\lambda_2 r^2}{2}-\frac{\lambda_3 r^3}{6}+\frac{\lambda_4 r^4}{24}=-\log_e R_0$$

completely, we put $r=-u+iv$, and add $-2n\pi i$ to the right-hand side. We then get the two equations :

$$\lambda_1 u+\frac{\lambda_2}{2}(u^2-v^2)+\frac{\lambda_3}{6}(u^3-3uv^2)+\frac{\lambda_4}{24}(u^4-6u^2v^2+v^4)=-\log_e R_0$$

$$-\lambda_1 v-\lambda_2 uv-\frac{\lambda_3}{6}(3u^2v-v^3)-\frac{\lambda_4}{6}(u^3v-uv^3)=-2n\pi.$$

With $n=0$, we have, of course, $v=0$, and the solution is a real one. If u and v are supposed to be small, Lotka recommends (in " The Progency of a Population Element " referred to above) a process of successive substitution of approximate values in the two equations, starting with $v=\dfrac{2n\pi}{\lambda_1}$.

We may test the usefulness of these equations by substituting values of u and v already obtained in para. 47.

76. In our illustration $\phi(x)$ was taken as $\frac{\mu}{2}\sin\mu(x-l)$, where $\mu=\frac{\pi}{36}$ (para. 39). We have, since $\phi(x)$ is symmetrical about the average, $\lambda_3=0$. We have also, $\lambda_1=\dfrac{L+l}{2}$, $\lambda_2=\dfrac{1}{\mu^2}\left(\dfrac{\pi^2}{4}-2\right)$, $\lambda_4=\dfrac{1}{\mu_4}\left(12-\dfrac{\pi^4}{8}\right)$. Thus $\lambda_1=33$, $\dfrac{\lambda_2}{2}=30{\cdot}68775$, $\dfrac{\lambda_4}{24}=-126{\cdot}544$.

The equations for u and v are therefore :

$$33u+30{\cdot}68775(u^2-v^2)-126{\cdot}544((u^2-v^2)^2-4u^2v^2)=0$$
$$33v+61{\cdot}3755uv-506{\cdot}18uv(u^2-v^2)=2n\pi.$$

77. We had, corresponding to $n=1$, $u=0{\cdot}031407$, $v=0{\cdot}17607$ (para. 47). For these values $uv=0{\cdot}00552983$, $u^2-v^2=-0{\cdot}030014$, $uv(u^2-v^2)=-0{\cdot}00016597$, $(u^2-v^2)^2-4u^2v^2=0{\cdot}00077854$. We get $33u=1{\cdot}03643$, $30{\cdot}68775\,(u^2-v^2)=-0{\cdot}92107$,

$$126{\cdot}544((u^2-v^2)^2-4u^2v^2)=0{\cdot}09852,$$

and the first expression is $+\,0{\cdot}0168$ instead of zero.

We also have $33v=5{\cdot}8103$, $61{\cdot}3755uv=0{\cdot}3394$,

$$506{\cdot}18uv(u^2-v^2)=-0{\cdot}0840,$$

and the second expression is $6{\cdot}2337$ instead of $6{\cdot}2832$.

Corresponding to $n=2$, $u=0{\cdot}061442$, $v=0{\cdot}29711$ (para. 47).

For these values the first expression is $-1{\cdot}3004$ instead of zero, and the second expression is $11{\cdot}7058$ instead of $12{\cdot}5664$.

78. Thus, it is not sufficient to take the equation for r to the fourth semi-invariant in this particular case. Actually, this equation for r will give reasonably good values of u and v for the case when $n = 1$, but in order to get good approximations when $n = 2$, we need higher semi-invariants than the fourth. With the other solutions, involving larger values of u and v, the equations for u and v would require at each stage the introduction of more of the higher semi-invariants.

79. On the other hand, if only the real value of r is required, this can quite readily be obtained from the equation involving only a few of the semi-invariants so long as R_0 is not greatly different from 1.

For instance, the case used by Kuczynski (*loc. cit.*, p. 85) gives $R_0 = 0{\cdot}9$, $R_1 = 26{\cdot}6011$, $R_2 = 837{\cdot}87810$, $R_3 = 27986{\cdot}48818$, $R_4 = 985528{\cdot}38498$.

Thus $\lambda_1 = 29{\cdot}5568$, $\lambda_2 = 57{\cdot}3736$, $\lambda_3 = 187{\cdot}99$, $\lambda_4 = -975{\cdot}62$, $\log_e R_0 = -0{\cdot}1053605$.

The equation for the real value of r is, up to the fourth semi-invariant:

$$-29{\cdot}5568r + 28{\cdot}6868r^2 - 31{\cdot}365r^3 - 40{\cdot}65r^4 = -0{\cdot}1053605.$$

The value given by Kuczynski is $r = -0{\cdot}0035524$.

When we substitute this value in the left-hand side of the equation above, we have:

$$0{\cdot}10499758 + 0{\cdot}00036201 + 0{\cdot}00000141 - 0{\cdot}00000001 = -0{\cdot}10536099.$$

The minor part played by the r^3 and r^4 terms is apparent, and to the five significant figures obtained by Kuczynski, the value of r is rightly obtained by neglecting these higher powers of r in the equation.

Obviously, the number of terms used in the equation for r depends upon the nearness of R_0 to unity and the values of the λ's. In some cases it is probably necessary to use more than the first two terms in this equation.

80. Another way out of the difficulty of finding the values of r satisfying the equation $\int_l^L e^{-rx}\phi(x)dx = 1$ is to utilize the Pearson system of frequency curves. The simplest form of asymmetrical curve is that known as Type III,

$$y = y_0 e^{-\overline{p+1}\frac{x}{a}}\left(1 + \frac{x}{a}\right)^p,$$

the origin being the mean (m), and

$$p+1=\frac{4}{\beta_1},\ a=\frac{2\sigma}{\sqrt{\beta_1}},$$

σ being the standard deviation and

$$\beta_1=\frac{\mu_3{}^2}{\mu_2{}^3}.$$

This curve extends from $-a$ to ∞, so in our integral we must replace L by ∞. It is possible that, in particular cases, $\phi(x)$ may be replaced by a curve of this type. If we assume that $\phi(x)$ has this form, then, the distance from the start of the curve l to the mean m being a, we have $a=m-l$.

81. Our integral becomes

$$\int_l^\infty e^{-rx}y_0e^{-\frac{p+1}{a}(x-m)}\left(1+\frac{x-m}{a}\right)^p dx$$

or, putting $x+a-m=x'$ we get eventually

$$\frac{y_0e^{-rl+p+1}}{a^p}\cdot\frac{\Gamma(p+1)}{\left(r+\frac{p+1}{a}\right)^{p+1}}=1.$$

But we also have, for y_0, $\int_l^L\phi(x)dx=R_0$, from which we obtain:

$$\frac{y_0e^{p+1}\Gamma(p+1)}{a^p\left(\frac{p+1}{a}\right)^{p+1}}=R_0.$$

Thus we get

$$\frac{e^{-rl}}{\left(1+\frac{ar}{p+1}\right)^{p+1}}=R_0{}^{-1},$$

the equation for r.

This may be written:

$$e^{rl}\left(1+\frac{r\sigma\sqrt{\beta_1}}{2}\right)^{\frac{4}{\beta_1}}=R_0.$$

(This is equivalent to

$$rm-\tfrac{1}{2}r^2\sigma^2+\tfrac{1}{6}r^3\sigma^3\sqrt{\beta_1}-\tfrac{1}{16}r^4\sigma^4\beta_1-\ \ldots=\log R_0,$$

which agrees with the equation in para. 74, when $2(\beta_2-3)=3\beta_1$.)

82. In order to solve this equation for r with complex roots, it is preferable to write it in the form:

$$1+\frac{r\sigma\sqrt{\beta_1}}{2}=R_0^{\frac{\beta_1}{4}}e^{-rl\frac{\beta_1}{4}}.$$

Putting $r = u + iv$, we get:

$$1 + \frac{u\sigma\sqrt{\beta_1}}{2} = R_0^{\frac{\beta_1}{4}} e^{-\frac{ul\beta_1}{4}} \cos\frac{vl\beta_1}{4}$$

$$-\frac{v\sigma\sqrt{\beta_1}}{2} = R_0^{\frac{\beta_1}{4}} e^{-\frac{ul\beta_1}{4}} \sin\frac{vl\beta_1}{4}.$$

We observe from the second equation that, since R_0 and $e^{-\frac{ul\beta_1}{4}}$ are positive, and assuming v is positive, $\frac{vl\beta_1}{4}$ must lie between $(2n+1)\pi$ and $(2n+2)\pi$.

83. We can use these equations to get some idea of the changes in the values of u and v with changes in R_0 and β_1.

Let us assume $l = 15$ (years) and $\sigma = 6$ (years).

Let us find for what values of R_0 and β_1 we shall have $u = 0$. Put $u = 0$ in the equations. We have:

$$1 = R_0^{\frac{\beta_1}{4}} \cos\frac{15\beta_1 v}{4}$$

$$-3v\sqrt{\beta_1} = R_0^{\frac{\beta_1}{4}} \sin\frac{15\beta_1 v}{4}.$$

We get $$\tan\frac{15\beta_1 v}{4} = -3v\sqrt{\beta_1}.$$

We saw that $\frac{15\beta_1 v}{4}$ lies between $(2n+1)\pi$ and $(2n+2)\pi$. But the first equation, giving $\cos\frac{15\beta_1 v}{4}$ as a positive quantity, shows that $\frac{15\beta_1 v}{4}$ must lie between $(2n+1)\pi + \frac{\pi}{2}$ and $(2n+2)\pi$.

Let us therefore call:

$$\frac{15\beta_1 v}{4} = 2n\pi + \frac{3\pi}{2} + \theta.$$

Then the equations are:

$$R_0^{\frac{\beta_1}{4}} \sin\theta = 1$$

$$\sqrt{\beta_1} R_0^{\frac{\beta_1}{4}} \cos\theta = 0{\cdot}8\left(2n\pi + \frac{3\pi}{2} + \theta\right).$$

Giving $$\cot\theta = 0{\cdot}8\frac{\left(2n\pi + \frac{3\pi}{2} + \theta\right)}{\sqrt{\beta_1}}.$$

Case (*i*). Assume $\beta_1 = 1$ (corresponding to a moderately asymmetrical distribution of $\phi(x)$).

We have $\quad \cot\theta = 0{\cdot}8(2n\pi + \frac{3\pi}{2} + \theta)$.

When $n = 0$, $\quad \cot\theta = 0{\cdot}8(\frac{3\pi}{2} + \theta)$, and $\theta = 14^\circ\, 9'$.

Then $\quad R_0^{\frac{1}{4}} = \operatorname{cosec}\theta$, and $R_0 = 280$.

When $n = 1$, $\quad \cot\theta = 0{\cdot}8(2\pi + \frac{3\pi}{2} + \theta)$, and $\theta = 6^\circ\, 26'$.

Then $\quad \frac{1}{4}\log_{10} R_0 = 0{\cdot}95060$ and $R_0 = 6345$.

With larger values of n, θ becomes smaller and R_0 larger. The least value of R_0 consistent with $u = 0$, when $\beta_1 = 1$ is 280, an impossible value from our point of view.

Case (ii). Assume $\beta_1 = 4$ (corresponding to a definitely skew distribution of $\phi(x)$).

We have now $\quad \cot\theta = 0{\cdot}4(2n\pi + \frac{3\pi}{2} + \theta)$.

When $n = 0$, $\quad \cot\theta = 0{\cdot}4(\frac{3\pi}{2} + \theta)$, and $\theta = 25^\circ\, 50'$.

Then $\quad R_0 = \operatorname{cosec}\theta = 2{\cdot}2949$.

This value of R_0 is more nearly like those which have been computed from actual fertility schedules. On the other hand, $\beta_1 = 4$ corresponds to a distribution which is more unsymmetrical than those usually encountered in this kind of work.

84. We have previously shown that when $R_0 > 1$, the real value of r is positive, and although there is a possibility that the real part of a complex root may be positive or zero in this case, since we only showed that this real part was less than the real root always, it seems unlikely that in a practical case a value of u greater than or equal to zero would occur.

85. Let us consider the case when $u = -0{\cdot}05$, again taking $l = 15$, $\sigma = 6$. Our equations (para. 82) become:

$$1 - 0{\cdot}15\sqrt{\beta_1} = R_0^{\frac{\beta_1}{4}} e^{0\cdot1875\beta_1} \cos 3{\cdot}75v\beta_1$$

$$-3v\sqrt{\beta_1} = R_0^{\frac{\beta_1}{4}} e^{0\cdot1875\beta_1} \sin 3{\cdot}75v\beta_1.$$

Then $$\cot 3{\cdot}75v\sqrt{\beta_1} = \frac{1 - 0{\cdot}15\sqrt{\beta_1}}{-3v\sqrt{\beta_1}}.$$

The right-hand side of this equation is negative, unless β_1 has a very large value.

We can put $\quad 3{\cdot}75v\sqrt{\beta_1} = 2n\pi + \frac{3\pi}{2} + \theta$, and

$$\cot\theta = 0{\cdot}8\frac{(2n\pi + \frac{3\pi}{2} + \theta)}{\sqrt{\beta_1}(1 - 0{\cdot}15\sqrt{\beta_1})}.$$

Assume $\beta_1 = 1$, we have :

$$\cot\theta = 0{\cdot}8\frac{(2n\pi + \frac{3\pi}{2} + \theta)}{0{\cdot}85}.$$

When $\quad n = 0,\ \theta = 12^\circ\ 10'$ and $R_0 = 125.$

86. If we take other values for u, we have the following results. For $\beta_1 = 1$, and $3{\cdot}75v = \frac{3\pi}{2} + \theta$, we get :

when $\quad u = -\ 0{\cdot}10,\ \theta = 10^\circ\ 9',\ R_0 = 55{\cdot}5.$
when $\quad u = -\ 0{\cdot}20,\ \theta = 5^\circ\ 56',\ R_0 = 11{\cdot}2.$
when $\quad u = -\ 0{\cdot}30,\ \theta = 1^\circ\ 31',\ R_0 = 2{\cdot}264.$
when $\quad u = -\ \frac{1}{3},\ \theta = 0,\ R_0 = 1{\cdot}361.$
when $\quad u = -\ 0{\cdot}40,\ \theta = -\ 3^\circ\ 4',\ R_0 = 0{\cdot}484.$

87. If now we consider a case when the distribution of $\phi(x)$ is only slightly skew, where $\beta_1 = 0{\cdot}25$, our equations become :

$$1 + 1{\cdot}5u = R_0^{0{\cdot}0625}e^{-0{\cdot}9375u}\cos 0{\cdot}9375v$$
$$-\ 1{\cdot}5v = R_0^{0{\cdot}0625}e^{0{\cdot}9375u}\sin 0{\cdot}9375v,$$

and we have the following results, replacing $0{\cdot}9375v$ by $\frac{3\pi}{2} + \theta$,

u	θ	R_0
$-\ 2{\cdot}0$	$-\ 15^\circ\ 44'$	$7{\cdot}18$
$-\ 2{\cdot}1$	$-\ 16^\circ\ 55'$	$1{\cdot}65$
$-\ 2{\cdot}2$	$-\ 18^\circ\ 6'$	$0{\cdot}379$

88. If the Pearson Type III curve adequately described the fertility schedule, which actually does not appear to be the case for certain schedules which have been examined, it is a fairly easy matter to compute the appropriate values of u and v for the complex values of r associated with a given R_0. Lotka in the paper already referred to, " The Progency of a Population Element," inclines to the view that the fertility schedule which he used for illustrative purposes is made up of two simple frequency curves. He found it necessary, when computing u and v, to replace the integral $\int_l^L e^{-rx}\phi(x)dx = 1$ by the equations

$$\Sigma e^{-ux_s}\cos(vx_s)\,.\,\phi(x_s) = 1$$
$$\Sigma e^{-ux_s}\sin(vx_s)\,.\,\phi(x_s) = 0,$$

where the x_s's stand for each year from l to L of the reproduction period, and to evaluate u and v by means of successive approximations.

Chapter VI

89. So far, we have dealt exclusively with the birth function. In order to derive the *population function*, which shows the changes, with time, of the population, we need the information contained in the life table, giving the number of survivors at each year of age of a number of children born at the same time. Let us suppose that 100,000 $l(x)$ represents the number of females who attain age x, out of a total of 100,000 female children born at the same time. As before, $B(t)$ represents the number of female births at time t, $B(t-x)$ represents the number of female births at time $t-x$. Of these children, the number who survive x years will be x years old at time t. Thus the number of females aged x to $x+dx$ at time t is $B(t-x)l(x)dx$.

Let us call
$$p(x, t) = B(t-x)l(x).$$

Then at time t, for different values of x, $p(x, t)$ will give the age distribution of the population.

If we consider the ultimate form to which the birth function tends, $B(t) = Q_0e^{r_0t}$, then:

$$p(x, t) = Q_0e^{r_0(t-x)}l(x) = Q_0e^{r_0t} \,.\, e^{-r_0x}l(x).$$

90. The total population is the sum of such terms as $p(x, t)dx$ throughout the whole range of ages (x) from 0 to A, where A is the upper limit of age, which may be taken as 100 for purposes of computation.

Let us call the population at time t, $P(t)$.

Then
$$P(t) = \int_0^A B(t-x)l(x)dx.$$

If we consider the ultimate form which the birth function attains we have, when t is large,

$$P(t) = \int_0^A Q_0e^{r_0t} \,.\, e^{-r_0x}l(x)dx = Q_0e^{r_0t}\int_0^A e^{-r_0x}l(x)dx.$$

91. Then, when t is large, the ratio:

$$\frac{p(x\,.\,t)}{P(t)} = \frac{e^{-r_0x}l(x)}{\int_0^A e^{-r_0x}l(x)dx}$$

This expression, being independent of t, shows that ultimately the population attains a stable age distribution.

Further, the birth-rate when t is large is:

$$\frac{B(t)}{P(t)} = \frac{1}{\int_0^A e^{-r_0 x} l(x)dx}.$$

If we call this birth-rate, which is independent of time, b, then

$$\frac{1}{b} = \int_0^A e^{-r_0 x} l(x)dx,$$

and we may write

$$\frac{p(x,t)}{P(t)} = be^{-r_0 x} l(x),$$

giving the stable age-distribution.

92. A population whose age distribution is indicated by $l(x)$—*i.e.*, a population consisting of $Bl(x)$ persons aged x, where x takes all values from 0 to A—is called a *stationary population* or *life-table population*. The total number of such a population is:

$$B\int_0^A l(x)dx = P \text{ (say)}.$$

But $Bl(o) = B$, since $l(o) = 1$, and B represents the number of births in such a population. Thus

$$\frac{1}{\int_0^A l(x)dx}$$

is the birth-rate in a stationary population.

We observe that, if $r_0 = 0$ with $R_0 = 1$, the stable age distribution becomes $bl(x)$, where b is now the same as the birth-rate in the stationary population.

Thus, if $R_0 = 1$, the stationary or life-table population, which is assumed to exist in the early stages of the population development, is the final form which the population ultimately attains.

93. $l(x)$ may be called the chance, at birth, of surviving to age x, and $1 - l(x)$ is the chance of dying before age x, so $-\frac{dl(x)}{dx} \cdot dx$ is the chance of dying when aged between x and $x + dx$. The average age at death is therefore

$$-\int_0^A x\frac{dl(x)}{dx}dx = -\left[xl(x)\right]_0^A + \int_0^A l(x)dx = \int_0^A l(x)dx,$$

since $l(A) = 0$. Instead of the average age at death, we usually refer to the expectation of life at birth, which is therefore $\int_0^A l(x)dx$.

The birth-rate in a stationary population is therefore the inverse of the expectation of life at birth.

94. We saw that ultimately

$$P(t) = \frac{Q_0 e^{r_0 t}}{b}, \text{ and } \frac{1}{P}\frac{dP}{dt} = r_0.$$

r_0 is therefore the ultimate rate of increase in the population.

r_0 is often called the *natural rate of increase*, and this of course refers to the ultimate change with time in $P(t)$. In a short time dt, the population increases by dP, where $dP = P(t)r_0 dt$. In this period the number of births is $B(t)dt$, so the number of deaths $D(t)dt = B(t)dt - dP$. We have therefore:

$$\begin{aligned} D(t) &= B(t) - P(t)r_0 \\ &= P(t)(b - r_0) = \frac{Qe^{r_0 t}}{b}(b - r_0). \end{aligned}$$

The death-rate is therefore $b - r_0$, when the population has settled down to its ultimate form. (This, of course, excludes migration.)

95. We saw that with a certain given net-fertility schedule $100{,}000 R_0$ represented the number of female children born to 100,000 females all born at the same time. This $100{,}000 R_0$ children may be regarded as the next generation to the 100,000 original children. When the birth function has settled down to its ultimate form we may have, at a given time t_0, $B(t_0) = 100{,}000 = Q_0 e^{r_0 t_0}$, and at a later time $t_0 + T$ we may have

$$B(t_0 + T) = 100{,}000 R_0 = Q_0 e^{r_0(t_0 + T)},$$

where T is given by the equation $R_0 = e^{r_0 T}$.

T is the time elapsing for the birth function to change from 100,000 to $100{,}000 R_0$, and T is called the *mean length of a generation*. Associated with a given R_0 and r_0 there is a particular T derived from the equation $R_0 = e^{r_0 T}$. We had the following equation for r_0:

$$r_0\lambda_1 - \tfrac{1}{2}r_0^2\lambda_2 + \tfrac{1}{6}r_0^3\lambda_3 - \tfrac{1}{24}r_0^4\lambda_4 - \ldots = \log_e R_0,$$

where the λ's are the semi-invariants of $\phi(x)$ (para. 74).

Now we have $$r_0 T = \log_e R_0,$$

hence $$T = \lambda_1 - \tfrac{1}{2}r_0\lambda_2 + \tfrac{1}{6}r_0^2\lambda_3 - \tfrac{1}{24}r_0^3\lambda_4 + \ldots$$

If we replace the semi-invariants by the moments, we have

$$T = m - \tfrac{1}{2}r_0\sigma^2 + \tfrac{1}{6}r_0^2\sigma^3\sqrt{\beta_1} - \tfrac{1}{24}r_0^3\sigma^4(\beta_2 - 3) \ldots$$

where m and σ are the average and standard deviation of $\phi(x)$, and so on.

$\phi(x)dx$ gives the chance, at birth, of a female having a female child when she is aged x to $x + dx$, and $m = \int_l^L x\phi(x)dx$ may be called the average age of mothers at confinement. The mean length of a generation is not the same as the average age of mothers at confinement, except when $r_0 = 0$.

For small values of r_0, when R_0 is nearly unity, we have approximately $T = m - \frac{1}{2}r_0\sigma^2$, and $T \gtrless m$, as $r_0 \lessgtr 0$.

96. Let us now consider the mothers of the $B(t)$ female children all born at the same time (t). The number of such mothers with ages between x and $x + dx$ is $B(t - x)\phi(x)dx$. If the population has settled down to its ultimate form, $B(t) = Q_0e^{r_0t}$, and the age distribution of the mothers of children born at time t is $Q_0e^{r_0t}e^{-r_0x}\phi(x)dx$. The average age of these mothers when the births take place at time t is:

$$\frac{\int_l^L xQ_0e^{r_0t}e^{-r_0x}\phi(x)dx}{\int_l^L Q_0e^{r_0t}e^{-r_0x}\phi(x)dx}$$

This is $\int_l^L xe^{-r_0x}\phi(x)dx$, since $\int_l^L e^{-r_0x}\phi(x)dx = 1$.

This gives $\overline{M}$, the average age of Mother, as :

$$R_1 - r_0R_2 + \frac{r_0^2}{2}R_3 - \frac{r_0^3}{6}R_4 + \ldots$$

We may express this differently. If we write :

$$\int_l^L e^{-r_0x}\phi(x)dx = F(r_0), \text{ then } \overline{M} = -\frac{dF(r_0)}{dr_0}.$$

But, (see para. 73), if we assume

$$\phi(x) = R_0\left(N(x) - \frac{\lambda_3}{6}\frac{d^3}{dx^3}(N(x)) + \frac{\lambda_4}{24}\frac{d^4}{dx^4}(N(x)) \ldots\right),$$

where $N(x)$ is the normal function, $\frac{1}{\sqrt{2\pi}\,.\,\sigma}e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2}$, we have

$$F(r_0) = R_0e^{-r_0m + \frac{1}{2}r_0^2\sigma^2}\left(1 - \frac{\lambda_3}{6}r_0^3 + \frac{\lambda_4}{24}r_0^4 \ldots\right),$$

and $F(r_0) = 1$.

Therefore :

$$-\overline{M} = R_0e^{-r_0m + \frac{1}{2}r_0^2\sigma^2}\left(1 - \frac{\lambda_3}{6}r_0^3 + \frac{\lambda_4}{24}r_0^4 \ldots\right)(-m + r_0\sigma^2)$$

$$+ R_0e^{-r_0m + \frac{1}{2}r_0^2\sigma^2}\left(-\frac{\lambda_3}{2}r_0^2 + \frac{\lambda_4}{6}r_0^3 - \ldots\right)$$

Hence

$$\bar{M} = -m + r_0\sigma^2 - \frac{\frac{\lambda_3}{2}r_0^2 - \frac{\lambda_4}{6}r_0^3 + \cdot}{1 - \frac{\lambda_3}{6}r_0^3 + \frac{\lambda_4}{24}r_0^4 - \ldots},$$

giving $$\bar{M} = m - r_0\sigma^2 + \frac{\lambda_3}{2}r_0^2 - \frac{\lambda_4}{6}r_0^3 + \ldots$$

For small values of r_0, we may say $\bar{M} = m - r_0\sigma^2$.

Thus when r_0 is small, $\bar{M}$, T and m are in arithmetical progression, the mean length of a generation being half-way between the average age of mothers of the original children and the average age at confinement of these children.

97. We have been concerned with the population function in its ultimate form, which, we saw, was given by :

$$P(t) = \frac{Q_0}{b}e^{r_0t}.$$

Before the population function and the birth function attain their ultimate forms, we have

$$B(t) = Q_0e^{r_0t} + Se^{u_nt}(C_n \cos v_nt + D_n \sin v_nt)$$

(para. 34). The corresponding value for $P(t)$ is obtained from the equation $P(t) = \int_0^A B(t-x)l(x)dx$, (para. 90). This gives

$$P(t) = \frac{Q_0}{b}e^{r_0t} + Se^{u_nt}\int_0^A e^{-u_nx}(C_n \cos v_n(t-x) + D_n \sin v_n(t-x))l(x)dx,$$

i.e., $$P(t) = \frac{Q_0}{b}e^{r_0t} + Se^{u_nt}(E_n \cos v_nt + F_n \sin v_nt),$$

where $$E_n = \int_0^A e^{-u_nx}(C_n \cos v_nx - D_n \sin v_nx)l(x)dx,$$

and $$F_n = \int_0^A e^{-u_nx}(C_n \sin v_nx + D_n \cos v_nx)l(x)dx,$$

and $$\frac{1}{b} = \int_0^A e^{-r_0x}l(x)dx.$$

Thus, the form of $P(t)$ includes damped periodic terms similar to those in $B(t)$, and, like those periodic terms, they disappear absolutely when $R_0 \leq 1$, and relatively when $R_0 > 1$, when t is large, leaving the ultimate form which has already been discussed.

Since no simple mathematical form for $l(x)$ is available, a further discussion of these formulæ is merely academic.

In order to compute values of $P(t)$ at an early stage of its development where the periodic terms have importance, it is necessary to use a life table giving values of $l(x)$.

We saw that the exponential series for $B(t)$ involved troublesome computations of the values of u_n and v_n if this series were to be developed in full, and that in the earlier stages, at any rate, it is simpler to develope $B(t)$ directly.

Similarly, in the early stages it is simpler to obtain $P(t)$ directly than by using the exponential series.

98. For example, suppose we take a case previously considered, where $B(t) = C_1$ for values of t up to 0, and $B(t) = C_2$ for t ranging from 0 to l, then for values of t from 0 to l we can write

$$P(t) = \int_0^A B(t-x)l(x)dx = \int_{t-A}^t B(u)l(t-u)du$$

writing $$t - x = u.$$

This may be written :

$$P(t) = \int_{t-A}^0 C_1 l(t-u)du + \int_0^t C_2 l(t-u)du$$

$$= C_1\int_t^A l(x)dx + C_2\int_0^t l(x)dx = C_1\int_0^A l(x)dx - (C_1 - C_2)\int_0^t l(x)dx.$$

If T_x represents the number of persons aged x and upwards living in a stationary community where the number aged O is N, then

$$T_x = N\int_0^{A-x} l(x+y)dy,\ T_0 = N\int_0^A l(x)dx, \text{ and } \frac{dT_x}{dx} = -Nl(x),$$

therefore $$N\int_0^t l(x)dx = -\int_0^t \frac{dT_x}{dx}dx = T_0 - T_t.$$

Thus $$P(t) = \frac{C_1}{N}T_0 - \frac{C_1 - C_2}{N}(T_0 - T_t),$$

and $$P_t - P_{t+1} = \frac{C_1 - C_2}{N}(T_t - T_{t+1}).$$

99. We may illustrate with the case considered previously, that given by Kuczynski (*loc. cit.*). He assumed 100,000 births annually occurring on January 1st each year up to year 2000, and 90,000 births on January 1st each year from 2001 to 2015. He gives a table showing the number of females in each yearly age group (*loc. cit.*, p. 66), from which the values of T may be obtained. Here $N = 100,000$.

Year of Age (1)	Number of Females (2)	Year (3)	Values of T_x (4)	Cumulative Totals of Col. (2) (5)
0 —	90,600	0	5,835,400	90,600
1 —	89,300	1	5,744,800	179,900
2 —	88,700	2	5,655,500	268,600
3 —	88,400	3	5,566,800	357,000
4 —	88,200	4	5,478,400	445,200
5 —	88,000	5	5,390,200	533,200
6 —	87,800	6	5,302,200	621,000
7 —	87,600	7	5,214,400	.
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.

As Kuczynski's first change in the number of births occurs on January 1st, 2001, the population at that time is still the same as it had been before the change occurred—viz. 5,835,400. (In our formula $C_2 = C_1$ before the change occurs and $C_1 = 100{,}000$.) Our $t = 1$ therefore corresponds with his year 2002 and so on.

The formula is :

$$P(t) = T_0 - 0{\cdot}1(T_0 - T_t),\ [N = C_1 = 100{,}000,\ C_2 = 90{,}000].$$

We have :

For $t = 1$, $P(1) = T_0 - 0{\cdot}1(T_0 - T_1) = 5{,}835{,}400 - 9060 = 5{,}826{,}340.$

For $t = 2$, $P(2) = 5{,}835{,}400 - 17{,}990 = 5{,}817{,}410.$

For $t = 3$, $P(3) = 5{,}835{,}400 - 26{,}860 = 5{,}808{,}540$, and so on.

So we get, by successive subtraction, the value of $P(t)$, as follows :

Year of Age (1)	Number of Females (2)	Col (2) × $\frac{1}{10}$ (3)	Values of t (4)	Year (5)	Values of P (6)
0 —	90,600	9,060	0	2001	5,835,400
1 —	89,300	8,930	1	2002	5,826,340
2 —	88,700	8,870	2	2003	5,817,410
3 —	88,400	8,840	3	2004	5,808,540
4 —	88,200	8,820	4	2005	5,799,700
5 —	88,000	8,800	5	2006	5,790,880
6 —	87,800	8,780	6	2007	6,782,080
7 —	87,600	8,760	7	2008	5,773,300
8 —	87,500	8,750	8	2009	5,764,540
9 —	87,400	8,740	9	2010	5,755,790
10 —	87,300	8,730	10	2011	5,747,050
11 —	87,200	8,720	11	2012	5,738,320
12 —	87,100	8,710	12	2013	5,729,600
13 —	87,000	8,700	13	2014	5,720,890
14 —	86,900	8,690	14	2015	5,712,190
15 —	86,700	8,670	15	2016	5,703,500
16 —	86,500	8,650	16	2017	
17 —	86,300	8,630	17	2018	

100. Now if we consider the case when $l < t < 2l$, taking l as 15 as before, we have

$$P(t) = \int_{t-A}^{0} C_1 l(t-u)du + \int_{0} C_2 l(t-u)du + \int_{l}^{t} B(u)l(t-u)du,$$

where $B(u) = C_1 R_0 - (C_1 - C_2)f(u)$ (para. 67).

This gives

$$P(t) = C_1 \int_{t}^{A} l(x)dx + C_2 \int_{t-l}^{t} l(x)dx + \int_{0}^{t-l} (C_1 R_0 - (C_1 - C_2)f(t-x))l(x)dx$$

$$= C_1 \int_{0}^{A} l(x)dx - C_1 \int_{0}^{t} l(x)dx + C_2 \int_{t-l}^{t} l(x)dx +$$

$$C_2 \int_{0}^{t-l} l(x)dx - (C_1 - C_2) \int_{0}^{t-l} f(t-x)l(x)du,$$

when $C_1 R_0 = C_2$, as is the case in Kuczynski's problem.

Hence

$$P(t) = C_1 \frac{T_0}{N} - \frac{C_1 - C_2}{N}(T_0 - T_t) - (C_1 - C_2)F(t),$$

where $F(t) = \int_{0}^{t-l} f(t-x)l(x)dx$. When $C_1 = 100{,}000$, $N = 100{,}000$, $C_2 = 90{,}000$, we have $P(t) = T_0 - 0{\cdot}1(T_0 - T_t) - 0{\cdot}1NF(t)$.

But, $f(x) = \int_{l}^{x} \phi(x)dx$, and we can write:

$$NF(t) = N\int_{0}^{t-l} f(t-x)l(x)dx = T_0 f(t) - \int_{0}^{t-l} T_x \phi(t-x)dx.$$

This is the same as $NF(t) = \int_{0}^{t-l} \phi(t-x)(T_0 - T_x)dx$.

When $l = 15$, $t = 16$, we may take this as $\phi(15)(T_0 - T_1)$.
When $t = 17$, we may take it as $\phi(15)(T_0 - T_2) + \phi(16)(T_0 - T_1)$.
When $t = 18$, we may take it as

$$\phi(15)(T_0 - T_3) + \phi(16)(T_0 - T_2) + \phi(17)(T_0 - T_1).$$

And so on.

The values of ϕ are given in para. 68, and:

$\phi(15)(T_0 - T_1) = 0{\cdot}00869 \times 90{,}600 = 787$

$\phi(15)(T_0 - T_2) + \phi(16)(T_0 - T_1) = 0{\cdot}00869 \times 179{,}900 + 0{\cdot}00867 \times 90{,}600 = 2{,}351$

$\phi(15)(T_0 - T_3) + \phi(16)(T_0 - T_2) + \phi(17)(T_0 - T_1) = 0{\cdot}00869 \times 268{,}600 + 0{\cdot}00867 \times 179{,}900 + 0{\cdot}01730 \times 90{,}600 = 5{,}464.$

101. Proceeding with the computation of the values of $P(t)$ we have:

Year of Age (1)	Number of Females (2)	Col. (2) × $\frac{1}{10}$ (3)	Values of t. (4)	Year (5)	$T_0 - 0{\cdot}1 \times (T_0 - T_t)$ (6)	$0{\cdot}1NF(t)$ (7)	Values of $P(t)$ (8)
15 –	86,700	8,670	15	2016	5,703,500		5,703,500
16 –	86,500	8,650	16	2017	5,694,830	79	5,694,751
17 –	86,300	8,630	17	2018	5,686,180	235	5,685,945
.	.	.	18	2019	5,677,550	546	5,677,004
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.

The values in the final column are those given by Kuczynski (*loc. cit.*, p. 82).

References

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A Statistical Mare's Nest?

By Professor M. Greenwood, F.R.S.

More years ago than I care to count, I was faintly interested to see, in Todhunter's *History of the Mathematical Theory of Probability* (p. 37), a quotation from Gouraud who cited a passage from the *Digest*, which was thought to imply that the Romans knew something about life tables. My interest, however, was not enough to lead me to look up the passage. Since then, I have come upon references to Ulpian as the Roman in question, but still not been tempted to verify them.

Recently, however, while utilizing some not too welcome leisure in mild historical research, Ulpian's name once more emerged and I decided to lay the ghost. Mr. Yule, having access to the necessary literature, aided and abetted me. The result is this note.

In the first place what did Ulpian say? The passage from the *Digest* is appended. Perhaps it needs a few words of explanation. By the Falcidian Law, a testator was not allowed to deprive his legal heir of more than three-quarters of the estate; if he had given legacies to a greater amount than three-quarters they must be scaled down *pro rata*. If instead of bequeathing real or personal property absolutely the testator willed an annuity or the use of any property for life or a term of years how were these usufructs to be valued? The particular usufruct considered in the passage cited was an annuity on the life of a person of assigned age.

We are told by the reporter that the usual practice of the Roman Courts (originally for the purpose of a 5 per cent. inheritance tax under an earlier act) was to value an annuity on any life up to the age of 30 years at thirty years' purchase and for any age from 30 to 60 at the difference between the age and 60. Ulpian, we are told, suggested a modification, viz. thirty years' purchase to age 20, then a series of deductions having the effect that at 60 the valuation was five years' purchase.

Such was Ulpian's plan, and two fellows of our Society examined it in the middle of the last century.

The first was F. Hendriks (a name held in honour by us for more reasons than one). In an article in the second volume of the *Assurance Magazine*, Hendriks concludes that Ulpian took no account of interest and that his figures should be regarded as estimates of the Expectation of Life. He writes:—"Ulpian's estimate betokens no mean skill and discrimination in the subject, and seems to bear intrinsic evidence of some careful collection or observation of facts" (*loc. cit.*, p. 224). In support of this commendation, Hendriks quoted the Expectations of Life obtained from a table

calculated by Price from the data of Stockholm at the middle of the eighteenth century, figures which, he thought, agreed reasonably with Ulpian's estimates. He chose the Stockholm table because, whatever may be said of the accuracy of the data, it was not based upon a knowledge of deaths alone.

I cannot say that the concordance is impressive. The expectations at birth are 14·35 years for males and 18·10 for females; 30·00 and 30·89 at 10; 23·85 and 30·01 at 20. To take 30 every time does not seem very good agreement. But no doubt Hendriks was thinking rather of adult ages.

Here are the comparisons from 25 to 60.

Age	Stockholm Males	Stockholm Females	Ulpian
25	21·40	26·80	28
30	19·42	23·98	25
35	17·58	21·62	22
40	15·61	19·25	20
45	13·78	17·17	14
50	11·95	15·12	9
55	10·30	12·89	7
60	8·69	10·45	5

Five years later another fellow of the Society, W. B. Hodge, in the sixth volume of the *Assurance Magazine*, returned to the subject. His view was less complimentary to Ulpian. "Notwithstanding, however, my high respect for his [Hendrik's] judgment, I can hardly concur in his opinion that Ulpian's estimates were based upon accurate observations" (*loc. cit.*, p. 314). Hodge made the significant remark that an annual sum bequeathed to the State was valued at 30 years' purchase. This, I think, is the cue to lead us to the mare upon her nest. In H. J. Roby's edition of *De Usufructu Justiniani Digestorum*, Lib. VII, Tit. I (Cambridge, 1886), kindly lent me by a colleague of Mr. Yule, the subject is discussed in some detail. Roby remarks that, as the object was to protect the interests of the legal heir, it would be natural to place the valuation high. He notes that the maximum value of *any* usufruct was 30 years' purchase. If the usufructuary were a municipality, in which case the legal term of enjoyment was 100 years, the valuation was still 30 years' purchase.

So the choice of 30 years had nothing whatever to do with an estimate, true or false, of expectation of life; it was simply the legal maximum valuation of any usufruct. Indeed, when one recollects that the mortality of little children was a commonplace of Latin poetry (the paper shortage alone prevents me from inflicting a proof of this upon the reader; he may begin with *Æneid* vi, 426 *et seq.*) it is impossible to believe that even a Roman lawyer could have

thought that an annuity on the life of an infant in arms would have been commercially as valuable as an annuity on the life of a child of 10.

But although the object was to protect the interest of the heir-at-law, the practical Roman would recoil from the absurdity of making no allowance at all for age. The reporter (Macer) says the usual practice was to stop at 60. Apparently, no annuity bequeathed to a person older than 60 would have ranked for Falcidian purposes.

Ulpian might well have thought this unfair, so he tried his hand at a system of reductions which would (1) agree pretty well with the usual practice at the earlier ages, (2) not leave zero at 60. This, I suggest, is what he *did* do. He simply interpolated between 30 and 5. We know the origin of the 30; why should we assume that the 5 had a statistically more respectable parentage?

CORPUS IURIS CIVILIS. I. DIGESTA. XXXV. 2.68

68. *Aemilius Macer libro secundo ad legem vicesimam hereditatium.* Computationi in alimentis faciendae hanc formam esse Ulpianus scribit, ut a prima aetate usque ad anum vicesimum quantitas alimentorum triginta annorum computetur eiusque quantitatis Falcidia praestetur, ab annis vero viginti usque ad annum vicesimum quintum annorum viginti octo, ab annis viginti quinque usque ad annos triginta annorum viginti quinque, ab annis triginta usque ad annos triginta quinque annorum viginti duo, ab annis triginta quinque usque ad annos quadraginta annorum viginti. ab annis quadraginta usque ad annos quinquaginta tot annorum computatio fit, quot aetati eius ad annum sexagesimum deerit remisso uno anno: ab anno vero quinquagesimo usque ad annum quinquagesimum quintum annorum novem, ab annis quinquaginta quinque usque ad annum sexagesimum annorum septem, ab annis sexaginta, cuiuscumque aetatis sit, annorum quinque. eoque nos iure uti Ulpianus ait et circa computationem usus fructus faciendam. solitum est tamen a prima aetate usque ad annum trigesimum computationem annorum triginta fieri, ab annis vero triginta tot annorum computationem inire, quot ad annum sexagesimum deesse videntur. numquam ergo amplius quam triginta annorum computatio initur. sic denique et si rei publicae usus fructus legetur, sive simpliciter sive ad ludos, triginta annorum computatio fit. Si quis ex heredibus rem propriam esse contendat, deinde hereditariam esse convincatur, quidam putant eius quoque Falcidiam non posse retineri, quia nihil intersit, subtraxerit an hereditariam esse negaverit: quod Ulpianus recte improbat.

Centenary of the American Statistical Association

On November 27th, 1839, five men met at 15 Cornhill, Boston, Mass., to establish the American Statistical Association, and in their Report for 1841–42 the Council of the Statistical Society of London expressed their gratification "that the constitution of this Society has served as a model for the new body. On November 27th, 1939, the American Association began the celebration of its one-hundredth anniversary. Our Honorary Fellow, R. H. Coats, an ex-President of the Association, was present as delegate from the Dominion Bureau of Statistics and the Canadian Political Science Association. He writes: "In the afternoon of the 27th we proceeded to the building wherein the first meeting of all was held and unveiled a bronze tablet on its wall. It is a smallish shop with a tearoom or some such place overhead. Thence we visited the Boston Public Library, where an exhibit of mementoes of the Association was on view. Later we saw the Statistical Departments of Boston University and Harvard. The wind-up was a dinner in the evening at which, I should say, about 100 were present, with Pearl in the chair. Wilcox read a paper on the early days of the Association, chiefly about Lemuel Shattuck, the moving spirit, a curious but enterprising man. Then Dewey, who was Secretary of the Association fifty years ago, gave reminiscences under the title, 'The Half-way Point.' It was all very interesting."

The celebration of the centenary was concluded at the Annual Meeting of the Association in Philadelphia in December 1939. The two preceding days and the morning of the 29th were filled with the usual business, but the afternoon and evening were given up to the "Centenary Celebration Program." The afternoon session, under the chairmanship of E. Dana Durand, dealt with "Trends in Statistics and the Opportunities of the American Statistical Association." Dr. H. L. Dunn read a paper on "Census—Past and Future," and Dr. W. F. Ogburn one on "Recent Statistical Trends," after which Mr. J. S. Davis of Stanford University seems to have dealt prophetically with "The Next 100 Years of the American Statistical Association." At the Centenary Dinner in the evening, Raymond Pearl, the President-Elect, presided and delivered his Presidential Address.

It was a matter of great regret to the President and Council of the Royal Statistical Society that the state of world affairs made it impossible for any of them to attend the December meeting, but they conveyed in writing their congratulations on the American Association's worthy achievement during a hundred years of activity and their hopes that the future would be one of even greater usefulness. Mr. R. H. Coats was asked by the Council to attend as their representative and to give oral expression to their congratulations, and this he undertook to do.

REVIEWS OF STATISTICAL AND ECONOMIC BOOKS

CONTENTS

1.—*The Statistical Theory of Estimation.* By R. A. Fisher. Calcutta University Readership Lectures, 1938. $8\frac{1}{2}'' \times 5\frac{1}{4}''$. 45 pp.

This paper is a restatement of Prof. Fisher's theory of estimation, but in the early part of it problems of specification and distribution are also discussed. The three most important methods of obtaining exact sampling distributions—the use of Euclidean hyperspace, the method of mathematical induction and the method of characteristic functions are concisely discussed and clearly illustrated. A new proof is given that the method of maxima likelihood leads in large samples to statistics with minimum variance and is very much clearer than the one originally given (*Proc. Camb. Phil. Soc.*, 1925). The notion of frequency space is briefly discussed. The observed and expected values of the frequencies in a grouped sample may be represented by a point in n dimensional space. If the expected frequencies depend on a single parameter, their representative point will describe a curve as the values of the parameter change. This is the curve of expectation. Regions corresponding to samples all of which give the same value of any estimate of the parameter are equistatistical regions. The relation of these to the curve of expectation is discussed. This section could be most profitably expanded.

J. O. I.

2.—*Methods of Statistical Analysis.* By C. H. Goulden. New York: John Wiley. (London: Chapman and Hall.) 1939. $9'' \times 5\frac{3}{4}''$. vii + 277 pp. 17*s.* 6*d.*

This book is intended for research workers in agriculture and biology who have already acquired some knowledge of the principles of statistics and desire further training in the application of these principles. Actually the book is self-contained, and could be

mastered by a student unacquainted with statistical methods. However, the argument proceeds fairly rapidly; most of the space is devoted to moderately difficult examples, and the use of algebraic symbols and formulæ is more frequent than is customary in introductory text-books.

The material covered includes practically all the methods which are now regarded as standard equipment for the would-be statistician. Amongst the topics which are not usually found in elementary text-books may be mentioned the χ^2 correction for continuity, a discussion of the selection of "error" terms in tests of significance, the analysis of covariance with two independent variables (including the formulæ for the standard errors of the adjusted treatment means) and the new methods of designing variety trials with a large number of varieties. The last subject has not previously appeared in text-books. Curiously enough, this part of the book runs the risk of being out-of-date when a second edition is planned, for the most recent method of analysing these designs, so as to utilize the inter-block information, represents a considerable advance on the methods available when the book was written.

Worked examples occupy a large part of the space. These are taken mainly from cereal chemistry or field experiments. Here the author's ability and shrewd insight as a practical statistician are apparent; the discussion of these examples forms an excellent model for the student in handling his own data.

The exposition of the theory is lucid and accurate. Two minor slips were noted. On p. 76, the usual test for the significance of the difference between two independent correlation coefficients is applied where one variate is common to both correlation coefficients. On p. 214, it is stated that the reduction in the sum of squares of x_1, due to its linear regression on x_2 and x_3, may be written

$$b_{12}^2\,\Sigma(x_2-\bar{x}_2)^2+b_{13}^2\,\Sigma(x_3-\bar{x}_3)^2$$

A term in $b_{12}b_{13}$ should be included in the expression.

The book may be confidently recommended to the public which the author has in mind. W. G. COCHRAN.

3.—*Elements of Statistical Reasoning.* By Alan E. Treloar. New York: John Wiley. (London: Chapman and Hall.) 1939. xi + 261 pp. 19*s.* 6*d.* net.

The object of this book is to describe the elementary processes of statistical reasoning with as few arithmetical and mathematical entanglements as possible. "Those who seek a compendium of statistical techniques, an array of formulas from which one or another may be culled to act as a mill for grinding fine flour from crude grain, merely by turning a handle, should not look further through these pages. What is written herein is intended for those who wish to reason carefully, not merely imitate." In carrying out this plan Professor Treloar has necessarily to introduce the usual statistical measures and to touch upon their mathematical properties; but he endeavours throughout to elucidate the ideas which gave rise to them and the processes of reasoning underlying their

use rather than to provide a detailed exposition and exemplification of their properties.

The book develops its subject on familiar lines. The first five chapters deal with frequency distributions, measures of location and dispersion and moments. There follow chapters on the normal curve, correlation and regression. Then comes a chapter on random sampling, and the book concludes with a treatment of the sampling problems of proportions, the correlation coefficient, and measures of frequency discordance. There are also chapters on probability (considered as a frequency) and vital statistics. Professor Treloar does not deal at any length with small samples, intending to complete his treatment of this subject in a further volume; and he avoids the difficulties of inferential reasoning by relying mainly on what Jane Austen might have called the method of Pictures and Plausibility. Not that this is a criticism of the treatment—the intuitive approach is probably the best for an introduction to the subject.

There appear to be a few errors of statement. On p. 41 it is said that "the consistency of means in random samples from the same supply [= population] is found to be considerably greater than that of median or modal values." If this means that the sampling variance of the mean is less than that of the median or the mode it is inaccurate, the mean of samples from a Cauchy universe, for example, being far inferior to the median as a measure of location. On p. 61 the author applies the name "maximum likelihood estimate" to the estimate of the variance $S(x - \bar{x}^2)(n - 1)$. This is not a maximum likelihood estimate for all forms of population and even for the normal form is not regarded as such by some writers when the mean and variance are to be estimated simultaneously. "Unbiased" is a better description; and in this connection it is to be noted that on p. 138 Professor Treloar says that the method of maximum likelihood is urged by R. A. Fisher for the purpose of forming unbiassed estimates. But these are minor blemishes on a very balanced and readable account of the first principles of statistical reasoning, and the book should prove a useful introduction not only to intended statisticians but to all whose work involves the making of inferences from numerical material.

M. G. K.

4.—*The Variate Difference Method.* By Gerhard Tintner. Principia Press, Inc., Bloomington, Indiana. 1940. $9\frac{3}{4}'' \times 6\frac{1}{2}''$. xiii + 175 pp. $2.50. (Cowles Commission for Research in Economics, Monograph No. 5.)

The most general form of economic time series may be considered as the sum of four components of different type, the trend, the short-term variation (or cycle), the seasonal fluctuation and the random element. In practice the latter is often very substantial and may account for more than half the total variation. Professor Tintner's book is concerned entirely with the study of this random element, the assessment of its importance in given cases and the reduction of random variation by smoothing formulæ. His technique is based

on the use of the variate difference method as developed mainly by Professor Anderson.

The fundamental assumption of the method is that the time series can be represented by a smooth curve on which are superimposed the random fluctuations. Successive differencing of the series then gradually increases the importance of the random element at the expense of the remaining part. It is not difficult to show that the variance of the kth differences a random series with variance v is $\binom{2k}{k}v$. Consequently, if we form successive differences, estimate the variance of each series, and divide by $\binom{2k}{k}$, the variance will tend, as k increases, to a constant which is an estimate of the variance of the random element in the original series, the contribution from the continuous part being successively reduced in magnitude. By the use of standard errors or more refined tests of significance it is possible to say at what stage in the differencing process the variances of successive series of differences are sensibly equal, *i.e.* the contribution from the non-random element of the series is negligible. This gives an idea of the order of the parabola which represents the non-random part of the original series, at all events locally.

It is essential to the use of the method that there shall not exist in the original series a periodic component of very short interval, that is to say of a period about equal in length to the interval of the differencing. Professor Tintner remarks that in his experience (and he is thinking mainly of price series) such components never occur. He concedes, however, that it is probably safer to eliminate seasonal movements before starting the difference analysis.

The method supplies a useful technique for estimating the extent of the random variation in a time series, but it does not, of course, offer a means of eliminating the random element at every point of the series. The contribution of the method to this problem lies in the nature of the smoothing formulæ which it indicates, namely, those due to W. F. Sheppard. Sheppard's method is, in effect, the use of a system of moving averages whose extent is at choice, according to the accuracy desired, and whose weights are determined by the degree of the parabola which is taken to represent the series locally (this being indicated by the variate difference method) and by the criterion that the sum of squares of residuals about the smoothed series shall be a minimum.

Professor Tintner's book is a very thorough account of the use of the technique, illustrated by examples, and is supplemented by mathematical notes to explain the methods set out in the text. One of the principal obstacles to the practical use of the variate difference method is the amount of arithmetical labour involved. Professor Tintner is very much alive to this trouble and has provided numerous tables to overcome it. In fact his book will be an invaluable practical aid in the study of random variation in time series by the variate difference method.

Whether the method is quite as useful as some of its advocates seem to think is another matter. If we are merely interested in the

degree of random variation in a given series or in comparing the random variations of different series it is adequate. But it does not throw any light on trends or cyclical movements—nor is any such claim made by Professor Tintner. On the contrary, there are grounds for believing that any process of smoothing may impair investigations into cyclical movements. It is known from the work of Professor Slutsky, for instance, that the taking of moving averages of a random series may generate a series exhibiting all the characteristics of what is usually called the trade cycle, and by applying a smoothing formula to a series containing a random element we are in danger of imposing spurious cycles on the data. For this reason one could have wished that Professor Tintner would have given better grounds for reliance on the Sheppard formulæ than mathematical convenience or the property of reducing random variation, which, after all, belongs in some sense to all methods of fitting by least squares. All this, however, is another and more difficult story. Within the limits he has set himself, Professor Tintner has produced a very useful book.

M. G. K.

5.—*A Study of Accident Proneness among Motor Drivers.* By E. Farmer and E. G. Chambers. Report No. 84 of the Medical Research Council and the Industrial Health Research Board. H.M. Stationery Office. London, 1939. $9\frac{3}{4}'' \times 6''$. iii + 47 pp. 9*d*.

Studies of industrial accidents have shown that workers exposed to the same degree of risk are not equally liable to mishap, some individuals being more prone to accident than others. This proneness appears to be stable under quite widely varying circumstances, manifesting itself in different periods of exposure and in different kinds of accident; and, what is obviously of great practical importance, it has been found possible in some cases to predict by psychological tests which workers will have the greatest number of accidents. The very interesting report by Mr. Farmer and Mr. Chambers extends the previous inquiries into a new field, the proneness to accident of motor drivers. Clearly the subject is of first importance, an unreliable driver being not only a danger to himself but a menace to everyone on the roads.

The present study is based on investigations into the number of accidents sustained by omnibus drivers, supplemented by insurance records of accident claims by owner-drivers of private cars and records of drivers of heavy lorries. The omnibus drivers were in five groups, numbering from 67 to 398 men, two groups of London drivers, two groups of omnibus and trolleybus drivers from other towns, and one group of Army trainees. These men were given tests of various kinds. The records of the groups of owner-drivers, though considerably more numerous, were not so satisfactory for the purposes of the study, since no tests could be applied to the individuals, there was no complete information about their previous experience, their mileages may have been different, and in any case insurance claims are not a very reliable guide to the number of accidents actually occurring. The data for the group of lorry drivers are also unsatisfactory for some of these reasons.

Using the technique employed by Miss Newbold in her well-known study of industrial accidents, the authors show that there are significant differences in liability to accident among motor drivers. From a consideration of groups and sub-groups within which other factors such as exposure to risk, age, and standard of reporting accidents were approximately constant, it appears that such other factors cannot be called upon to explain the differences in liability, which must then be due to individual differences. In other words, accident proneness exists and is a function of the individual's psychic and physical mechanism; this, at least, was true in the cases investigated where the mean accident rate was 3·6 or more and the period of exposure was 4 years or more. The evidence in cases below those limits was not conclusive.

The authors proceed to discuss the stability of accident proneness by considering the correlations between the number of accidents sustained by individuals in successive years. They conclude that such stability exists. "Drivers who sustain multiple accidents in the first year of observation have a higher mean accident rate in subsequent years than drivers who have no accidents in their first year." The general conclusion about the existence of accident proneness is reinforced by an examination of different types of accident, there being positive correlations between one kind of accident and another. Taken by itself, it seems to me, this last analysis would not prove very much, for it is based on a subjective classification into five categories, "Errors of judgment, over-runs, skids, blameless and miscellaneous," and there appear quite as high correlations between the blameless class and the other four classes as among those four classes. In fact this part of the work seems to be the least satisfactory, the allocation of the true causes of an accident being exceedingly difficult if not impossible in the majority of cases.

Certain of the groups of subjects had a wide age-range and sufficient numbers of accidents to make it possible to consider the relationship between accidents and age. The general tendency is for accidents to decrease with age, an interesting feature being that there is a rise about the period of middle age followed by a fall. The authors suggest as an explanation the gradual and imperceptible failure of powers up to a certain point and then the sudden realization that old age is at hand; but the data do not permit of any reliable conclusions. More convincing is the section of the report dealing with the effect of experience, from which it appears that learners have more accidents than experienced drivers and that there is a diminishing return from experience in the reduction of numbers of accidents.

The remainder of the report is concerned with the relation between accidents and the results of tests given to the drivers concerned. The psychological tests were of four kinds, æstheto-kinetic, linguistic intelligence, mechanical aptitude and perseveration (*i.e.* ability to adapt oneself at short notice to altered circumstances). The general conclusions were that in only one out of four groups of London bus drivers (the group for which the largest experience was available) was there any significant relation between accidents and the æstheto-

kinetic tests, that relation being positive. The results of the perseveration and intelligence tests were not satisfactory, the subjects being obviously nervous—"the general trend of their remarks is to the effect that they have left school a long time and cannot be expected to do tests of this kind." One sympathizes with the drivers, particularly when it is learned that a group of apprentices beat them soundly in the intelligence tests. The linguistic gifts of a London bus driver are not at their best with pencil and paper, and the general conclusion (which, one cannot help but feel, might have been foreseen) is that the tests were unsatisfactory.

It is impossible to deal with all the topics discussed in the report, but enough has been said to indicate its breadth of scope and general interest. Mention must, however, be made of the appendices on the findings of the Massachusetts Highway Accident Survey of 1934 and on the brake reaction time. On the average it seems to take about half a second to move one's foot from the accelerator to the brake pedal under laboratory conditions, and there is some evidence that on the road the time is even longer. If a car is moving at 30 miles per hour a driver with a brake reaction time of half a second would have travelled 22 feet before his foot reached the brake pedal.

The authors' general conclusions are that the accident rate can be reduced by eliminating drivers with a strong degree of accident proneness. The question whether an individual has such a strong degree can be determined by examining his previous record and by psychological tests, preferably in conjunction. There is still room for doubt whether the psychological approach is really satisfactory —some further research with better tests and under more natural conditions seems indicated—but the general conclusion will probably be accepted. Its bearings on the training and selection of drivers need no emphasis.

This report is admirably condensed, the authors being content to give their data and indicate their conclusions without any trimmings whatever. This is all the more commendable in view of the emotional atmosphere created around road accidents by the people who still believe that it is more dangerous to be outside a house than inside it. Obviously the report is not the last word on the subject, and it does not pretend to be; but it is a valuable contribution which will be widely studied. M. G. K.

6.—*Statistical Testing of Business-Cycle Theories. I. A Method and its Application to Investment Activity.* By J. Tinbergen. Geneva: League of Nations; London: Allen and Unwin. 1939. $9\frac{1}{2}'' \times 6\frac{1}{4}''$. 164 pp. 3*s*. 6*d*. II. *Business Cycles in the United States of America.* 1919–1932. By J. Tinbergen. Geneva: League of Nations; London: Allen and Unwin. 1939. $9\frac{1}{2}'' \times 6\frac{1}{4}''$. 244 pp. 5*s*.

The first stage of the inquiry undertaken by the Financial and Economic Section of the League of Nations into the nature and causes of the trade cycle was concluded with the publication of Professor Haberler's *Prosperity and Depression* (reviewed in Part IV of the

Journal for 1937) in which the author presented an analysis, criticism and synthesis of the principal theories of the trade cycle. These two pamphlets by Professor Tinbergen of the Central Statistical Bureau of the Netherlands mark the beginning of the second stage of the inquiry, in which an attempt is being made "to confront these various theories with the historical facts—to subject them, in so far as these facts can be quantitatively expressed, to statistical analysis."

The first of the pamphlets describes the statistical method to be employed—the multiple correlation analysis—and illustrates its application with three examples relating to various aspects of the fluctuations in investment activity. After an all-too-brief introductory chapter, the author provides an excellent description in non-technical language of the multiple correlation analysis which, as he points out, has been applied to the field of economic research only in recent years, despite its wide application in other spheres. The task, he says, is two-fold—the *verification* of the expected relationship between two variables and the *measurement* of the influences exerted by the various casual phenomena.

In the ensuing three chapters, the method is applied to the available data regarding investment activity generally in Germany, the United Kingdom, the United States and France in the pre-1914 years and in the United States and the United Kingdom in the post-1920 period; to house building in Germany and Sweden before 1914, and in the United Kingdom, the United States and Sweden after the Great War, and, finally, to the net investment in railway rolling stock in France, Germany, the United Kingdom and the United States in the years preceding 1913.

The second pamphlet is devoted entirely to the 1919–32 period in the United States. The greater wealth of data has permitted a much closer study of the forces which determined the economic activity of America in those years. Moreover, Professor Tinbergen has elaborated and refined his method, and answered some of the questions which the first pamphlet left unanswered. The method of approach has been to seek by the multiple correlation analysis an "explanation" of the fluctuations of those factors which the economist has shown, by *a priori* reasoning, to be of primary importance during the course of the trade cycle. In this way, forty equations are presented for the United States to "explain" the variations in industry and trade during the period under review. These forty equations are finally combined by the so-called "elimination process" into one equation which purports to show the general nature of the trade cycle in the United States during this period.

This brief description of the contents of these pamphlets can do but scant justice to the enormous labour of computation which must have been involved in the calculations of the regression coefficients alone, nor to the mathematical ingenuity employed to overcome the frequent difficulties. Nevertheless, admiration for Professor Tinbergen's pioneer work in this almost untouched field of study should not obscure the fact that both the methodology and the application of the conclusions of the analysis raise a number of difficult and controversial issues, some of which are mentioned below. It should

not be implied that the author is unaware of these difficulties; on the contrary, he mentions most of them specifically, only to sweep them aside in his enthusiasm for his task.

First, under methodology a number of questions may be asked. To what extent can the multiple correlation analysis be logically applied to this new field? Some of the important factors affecting trade cycle activity are incapable of measurement; for example, the psychological reactions of entrepreneurs and the action of Governments; the magnitudes of many of the measurable factors are admittedly direct and, frequently, indirect approximations, especially the measurements of those broad magnitudes which are so important to business cycle research (investment, saving, inventories, for example) and the theories presented by the economist are usually incomplete and concerned mainly with stressing one particular aspect of the trade cycle rather than with a balanced and cohate picture of the cycle as a whole. Next, can the method do more than show the plausibility or otherwise of any particular theory when confronted with such facts as are known? The causal chains are notably complex in economic affairs—to distinguish cause and effect after the cumulative process of expansion or contraction has commenced is virtually impossible, the trade cycle being an essentially organic structure. Accordingly, how far can it be said that the major variables are statistically independent?—although Professor Tinbergen holds that "multi-collinearity" does not necessarily invalidate his results. It is not only that there may well be close correlation between the explanatory variables, but *a priori* reasoning would tend to suggest that the "unexplained" variables *are* correlated with the "explained" factors, owing partly to this essentially organic structure of the cycle. Further, how far does the necessary simplification of certain relationships affect the results? On p. 73 of Part II, for example, one reads with surprise that it is assumed that, as far as individuals are concerned, the holding of money is independent of the holding of bonds or shares. More important still, the assumption that the coefficients are constant, or at best, slowly changing during the course of the cycle overlooks those discontinuous structural changes which are characteristic and possibly the origin of so-called cyclical movements. This assumption of linear functions may well be a most dangerous one unless its limitations are constantly borne in mind during the analysis. The use of rectilinear trends for the post-war period is another example of the danger of over-simplification; the trends thus calculated for the United States during 1919–32 and for the United Kingdom during 1920–36, give a most misleading comparative picture.

The application of the results of the analysis is even more controversial. Ignoring the questions of method, can the results of the inquiry relating to a given period and a given country be applied to another country and another period? In other words, can the analysis be anything more than purely historical? As an historical analysis of the working of a particular factor through time or at a particular point of time, Professor Tinbergen's study of the United States has much to commend it. In support of this view, it may be

pointed out that the extension of the analysis to 1938 might well have resulted in definitely different conclusions since the Great Depression undoubtedly brought large structural changes in the American economy. It would be interesting to see the author's explanation of the collapse of capital goods production in August 1938 in terms of his analysis. Professor Tinbergen, however, rejects the historical aspect of his study, and is much more concerned with the picture of the cycle as a whole and with the conclusions which can be drawn as to economic policy. The chapters in which he proceeds to construction of a final equation are undoubtedly the most interesting in the book—particularly the analysis of the effects of a stock exchange boom and of hoarding. Yet these same chapters are more in the nature of essays in pure theory than in statistical testing.

In conclusion, it may be useful to summarize the author's conclusions concerning the validity of certain theories of the trade cycle in the light of this study. Monetary factors are found to have had but limited effect during the period under review, the fluctuations in interest rates being small (although it will be recalled that the monetary policy of the American authorities in these years aimed specifically at a "neutralizing" influence). In particular, profits appear to have been a far more important factor influencing capital goods production than the long-term interest rates, but this obscures the fact that it is not so much the absolute height of the rate of interest as the difference between the relevant interest rates and the profit rate which is a leading determining factor. Next, profits are shown to be a highly important influence upon investment activity. In view of the relative stability of consumer goods output and profits, the high degree of correlation between fluctuations in total profits and in capital goods activity is, of course, not surprising. Thirdly, changes in production costs seem to have been less significant than is generally thought, a fact must be attributed in part to the high wages philosophy of the "New Era" years.

It is intended to follow up these pamphlets with similar analyses of the post-war experience of other countries, and it may well be that the true worth of Professor Tinbergen's pioneer work will be rightly assessed only on the conclusion of his task. In view of the wide divergencies in the economic experience of such countries as the United Kingdom, France, Germany and Japan in the two decades after the four years' war, the author's method will be subjected to the most rigorous of tests. J. E. W.

7.—*L'Économie politique contemporaine.* Paris: Librairie générale de Droit et de Jurisprudence. 1939. 9″ × 6″.

I. *La Méthode de l'Économie politique.* By Bertrand Nogaro. vi + 275 pp. 50 fr.

IV. *L'Intervention de l'État en Matière économique.* By Henry Laufenburger. 371 pp. 70 fr.

V. *Production agricole et Économie rurale.* By André Garrigou-Lagrange. 211 pp. 40 fr.

XVII. *L'Économie collectiviste.* By Robert Mossé. 210 pp. 40 fr.

XIX. *Initiation à l'Analyse statistique.* By Léo Dugé de Bernonville. 232 pp. 50 fr.

Twenty-three volumes in all have been promised us, but only the above five have so far appeared. The introductory volume is the work of the general editor, Professor Bertrand Nogaro, and in spite of what to some may seem a rather forbidding title, it provides the brightest reading of the five. The author does not hesitate to speak his mind about mathematical economists in general and Walras in particular; about econometricians who jump to conclusions, who know little or nothing of their basic data, or who cannot recognize a crisis even when it comes; and about hyper-theoretical economists whom he refrains from naming—possibly from loyalty to the Entente Cordiale. And most of what he says is well worth reading and pondering over. But there are criticisms to make. In an introductory volume brilliance is not an unmixed advantage, particularly if it involves over-emphasis or suppression. And one does feel that the young or hasty reader may think Professor Nogaro is preaching a holy war against all methods other than his own favourite, the so-called method of observation, whereas it all boils down to little more than a question of emphasis. Even the printer has been dragged in as an ally in the campaign of emphasis; some pages are so italicized as to give the effect of a school-girl's letter, and the author even goes so far as to italicize, without admitting responsibility, parts of well-known passages he quotes from other authors' works. But these things do not seriously detract from what is undoubtedly a useful addition to methodological literature.

In *L'Intervention de l'État en Matière économique*, Professor Laufenburger has been given the most difficult subject of the five. Here scissors and paste offered little help. How should such a subject have been tackled and what should have been the main lines of attack? On this there may be differences of opinion, but as one reads the book, one feels the author has never quite settled—or for some reason has had to change—his plan of attack. This feeling is strengthened when one examines the architecture of the Table of Contents, with its unusual list of *Parties*, *Titres*, *Sections* and *Chapitres*. But if the author has not given us a completely satisfying and definitive work on State economic intervention, he has at least broken fresh ground and given much new and interesting material. And statisticians will give him higher marks than some of his colleagues for his statistics, which are helpful and up to date.

In *Production agricole et Économie rurale* stress is laid on the rural exodus which has taken place in many countries in recent years. The author studies the position in France, where the proportion of the occupied population engaged in agriculture fell from 45·7 per cent. in 1901 to 35·3 per cent. in 1931, and compares the case of Great Britain where, he says, the proportion fell from about 12 per cent. in 1881 to 7 per cent. in 1921. The British figures are obviously neither comparable nor up to date, and although this is

in itself a small point, it is indicative of a general statistical weakness, and one often wishes the author could have supplied figures less out of date and more in keeping with the title of the series. Yet the book has many excellent features. It is likely to make a greater appeal to English readers than any of the other volumes, not because of the relative scarcity of works on agricultural economics, but because it is very largely a study of French agriculture as seen by a Frenchman. To English eyes the picture drawn may not always seem complete or entirely convincing, as, for example, in the listed disadvantages of peasant proprietorship, the advantages of the *fermage* system, or the reasons why agriculture will never attract company promoters, but it is always interesting.

L'Économie collectiviste covers theory and practice, and is based mainly on the Russian experiment. On the practical side, M. Mossé lays stress on the importance of the Plan, and in describing how this is actually carried out he lays bare some of the weaknesses of Soviet accountancy. On the theoretical side there is little new, but he successfully demonstrates his main thesis that while collectivist theory and practice have radically changed in recent times, the problems of collectivism are fundamentally those of capitalism, and have to be solved mainly in a very similar way. Throughout the work one finds traces of the influence, sometimes the undue influence, of English writers. But on the whole the book provides a reliable introduction to the subject.

Last of all we come to *Initiation à l'Analyse statistique*, the author of which may be congratulated at least on his courage in tackling an impossible task. The task he set himself was to explain to those knowing no statistics and little mathematics the technique of modern statistical analysis, right from the elementary ideas of statistical units and series, averages, dispersion, index-numbers, and graphical representation, up to correlation, partial and multiple, linear and non-linear, concluding with an exposition of statistical induction, probability, the binomial and normal distributions, and many other matters. And he sets out to do all this in a little more than two hundred small pages. In their headlong journey to statistical knowledge, readers will find the safest part near the beginning, and in particular in the chapter on Averages. About this the most captious critic will find little to complain, apart from the absence of any mention of the Mode, a rather questionable graphical representation of the Median, and a verbal explanation where the author slips up. He declares that "lorsque la série comprend un nombre pair de termes ($n = 2k$), toute valeur comprise entre le $(k - 1)^{\text{ème}}$ et le $(k + 1)^{\text{ème}}$ terme répond à la définition de la médiane, qui n'est plus alors déterminée." And he similarly trips up again on the next page, thus emphasizing one of the most obvious risks of symbolic explanation. Yet, in spite of these and other small points, one lays down the book with an impression not so much of the impossibility of the task attempted as of the measure of success actually achieved.

C. O. G.

8.—*Profits, Interest and Investment* and other Essays on the Theory of Industrial Fluctuations. By F. A. von Hayek. London: Routledge. 1939. $7\frac{1}{4}'' \times 4\frac{3}{4}''$. viii + 266 pp. 6*s*.

This stimulating collection of essays includes several that have been printed before—"Note on the Development of the Doctrine of 'Forced Saving'" (Quarterly Journal of Economics, Vol. XLVII, 123), "Saving" (Encyclopaedia of the Social Sciences, 1933), and "The Maintenance of Capital" (Economica, 1933). Two others appear in English for the first time—"Price Expectations, Monetary Disturbances and Malinvestments," published in 1935 in German and French, and "The Present State and Immediate Prospects of the Study of Industrial Fluctuations" contributed in 1935 to a volume in honour of Professor Spiethoff. The well-known essay on "The Paradox of Saving" has also been reprinted as an appendix.

Professor Hayek opens this volume with an essay giving his latest views on "Profits, Interest, and Investment," in which he tries to prove that a large increase in the demand for consumption goods leads to a fall rather than to a rise in the demand for investment goods.

When prices and profits in the consumption goods industries rise, he says, real wages in terms of products fall and the entrepreneurs will substitute more labour for previous capital equipment, thus causing gradually a setback in the investment industries which may develop into a general depression as employment falls off in the equipment industries. That effect of substituting labour for capital which constitutes the master-key to the whole theory, he calls "the Ricardo Effect," forgetting that Ricardo was speaking of a rise in the cost of labour as compared with the cost of capital, while in this essay the relative prices of factors do *not* change: hence there can be no inducement to substitute one for the other. While therefore Ricardo's contention is perfectly correct, Professor Hayek's theorem is open to doubt, and one cannot see why he had chosen to call it the "Ricardo Effect." The use of the term "real wages" is somewhat misleading; what Professor Hayek really means, is wages in terms of their own products. But the fact remains that changes in the price of products leave the relations of "real" wages and "real" capital (in the same sense) unchanged.

The alternative presentation of the "Ricardo Effect" given in the essay is in the form of an arithmetical example, where we are asked to accept an equilibrium in which more up-to-date methods of production (using a large proportion of machinery, with which labour is invested for a short time), yield the same rate of profit per year as the more primitive methods in which the turnover of goods "*in one and the same* technical process" is slower.

(An incidental mistake has crept in the table on page 9, where in line 3 it should be $2\frac{1}{2}$ instead of $\frac{1}{2}$.) The complicated relation between the velocity of turnover of goods and the degree of capitalization (or roundaboutness) of the whole production is not clear.

The increase in the rate of output (turnover) of consumption goods can be achieved only through lengthening (increasing) the roundaboutness of the structure of production by the construction

of more complicated machines, able to turn out goods faster. Would the author say that when we replace the old and slow press of Mainz, in which Gutenberg printed his first Psalter, by a modern printing machine, we have introduced less capitalistic methods of production because it takes less time to print in the new way than in the old?

But does Professor Hayek's theory, that a rise in the prices and profits in the consumers' goods industries leads to a depression in the investment goods industries, really fall to ground if we reject his "Ricardo Effect"? It is enough to assume a reversible type of expectations on the part of entrepreneurs in the consumption industries, where the rise in the demand for consumption goods is thought to be only temporary. The industrialists will then refrain from ordering more equipment and will only put more labour to co-operate with the existing equipment. A labour intensification of the whole productive system follows.

But the most likely way in which a rise in the prices of consumption goods will express itself is through a rise in the prices of raw materials which are common to all stages of the productive process. Raw materials can be made to fulfil here the same function which in *Prices and Production* was assigned to labour. The rise in their price will cause a rise in the prices of capital goods, and this will lead to the real Ricardian substitution of labour for capital. If that substitution effect is strong enough to more than counterbalance the stimulating effect of an increase in the volume of output of consumption goods, it may ultimately lead to a depression. And here we plunge again in the sound part of Neo-Hayekian theory.

There is a third way of saving the theory without the need of accepting the "Ricardo Effect," and this is by assuming that higher profits in the consumption-goods industries lead to higher interest rates which will act as a damper on investment activity, with the result that a too high consumption brings about a decrease in investments.

A. M. N.

9.—*The Economics of Building.* By Herbert W. Robinson. London: P. S. King. 1939. $8\frac{3}{4}'' \times 5\frac{3}{4}''$. xii + 162 pp. 10*s.* 6*d.*

This monograph on the economics of building is a condensation of the author's thesis for the degree of Ph.D. (London), written in 1935–37 while he held a Leverhulme Research Studentship at the London School of Economics, and is the first book in English to be devoted entirely to the economic aspects of building. The original thesis was a more detailed study and is available to readers in the University of London Library, but the present volume contains important additional sections worked out in the years 1937–39 at Oxford. The author is to be congratulated on this, his first published work of length.

The first chapter contains a summary of the main economic features and recent economic history of the Building Industry, which, with Works of Construction, accounted for between 8 and 11 per cent. of the total occupied male population of England and Wales in the period 1881–1931.

In Chapter II a stationary state is assumed, where the industry would be concerned only with repair and replacement activity. The functional relation of length of life of a house with initial cost and repair work is then examined and also the relationship between rents, length of life, rate of interest, and capital and maintenance costs. The assumptions necessary to ensure a stationary state with respect to commercial building are also stated, and the chapter concludes with a mathematical note on the length of life of buildings and the rate of interest, in which a novel use of indifference curves simplifies the results of substituting greater initial capital cost for a larger annual repair bill.

In Chapter III the simplifying assumptions are relaxed, and the effect of demographic factors is analysed. The powerful influence on building activity of a small but rapid change in the number of families and the importance of spatial migration are clearly shown. "Building Need" is defined (following C. F. Roos),* on the assumption that the ideal is one family per dwelling. In the following two chapters the conditions of demand and supply for house-ownership, house-occupation, and for building activity are discussed in detail.

Chapters VI–VII will be those of most interest to statisticians. The available data for the statistical testing of the theory and the limitations of this data are reviewed. The periods covered are 1801–1913 and 1924–39. Actual building (minus replacements) is compared with building need for the first period, and after the trends have been eliminated both curves show the long cycles of about forty years associated with residential building and that "building need," although it ignores shifting population, is a dominant factor in the determination of the number of houses built in each decade. The estimates of the future population of England and Wales by Dr. Enid Charles † are then examined, and it is estimated that building need will have sunk to a minus quantity by 1960 when, if slum clearances, etc., are ignored, there will be 67,000 more houses than families to fill them. The statistics for the second period support the view that unless there are very great internal migrations, demolition schemes or (as the building industry may hope) serious errors in estimates of future population, there will soon be a marked drop in the scale of residential building and a serious reduction in the size of the building industry.

The analysis of economic factors in Chapter VIII is of interest to those interested in econometrics. By judiciously combining the series examined the number of variables for residential building is reduced to three and to four for commercial building. Regression equations are formed and tested by confluence analysis and the χ^2 test, and it is concluded that "residential building activity depends on the investment factor (rents divided by unit cost and interest) alone" (page 141). For commercial building the conclusion is that "as long as business activity pursues its trend, cost of building and the rate of interest entirely determine (without any lag) the tempo of building plans. But if business activity rises above its

* C. F. Roos, *Dynamic Economics*, 1934.

† Special Memo No. 40 of the L. and C. Econ. Service.

trend there is a proposed amount of building in excess of that to be expected from the level of building costs and the rate of interest" (pp. 150–1).

The concluding chapter summarizes the main results and mentions again possible future tendencies which may alter the forecast. As residential building is the most important branch of the industry, the author concludes that "failing some unforeseen stimulus, the industry must be prepared for a period of declining activity."

Although modern economic analysis and statistical methods are employed, all theories and conclusions are made easily comprehensible to the layman. The scheme of analysis is lucid, and charts and diagrams are clearly set out and explained in simple terms for the non-technical reader. Persons in the building industry will find helpful explanations of the factors vital to business policy in building. Readers interested only in economic analysis and statistical technique will find much to interest them. The author is to be thanked for making his conclusions available to the public in so digestible a form.

Vested interests in the building industry may receive a shock as to the possible long-term prospects in the absence of government intervention, but a view of housing conditions in our cities and the presence of aerial warfare may give cause to believe that when peace returns there will be sufficient work for the industry to permit a gradual transition to a lower level of activity. A case for centralized planning could hardly be more clearly made.

H. W. G. G.

10.—*Prices and Wages in England* from the Twelfth to the Nineteenth Century. Vol. I. Price Tables: Mercantile Era. By Sir William Beveridge and others. London: Longmans. 1937. $9\frac{3}{4}'' \times 6''$. lx + 756 pp. 31*s*. 6*d*.

This volume defies analysis and criticism. It is a mine of raw material. Six hundred and eighty pages are occupied with collections of prices of sundry commodities extracted from the records of Winchester, Eton, Westminster, and Charterhouse, among schools, St. Bartholomew's, Greenwich, and Chelsea, among hospitals, Westminster Abbey, Lord Steward's Department, Lord Chamberlain's Department, Office of Works, Navy Victualling, and Naval Stores. Sixty-two more pages discuss the digesting of that material into annual price relatives. Then follow a list of sources and an index of commodities. The whole is introduced by a "General Introduction" and a "Reader's Guide" in which the nature of the material and the mode of its handling are set forth. Doubtless this huge volume and its successors will be found invaluable by students of economic history.

H. W. M.

STATISTICAL NOTES

(1) British Official Statistics

On p. 268 we give a table summarizing the oversea trade of the United Kingdom for the twelve months ended April 1939 and 1940. The table relates only to trade in merchandise, no particulars having been published since the outbreak of war relating to bullion and specie or to shipping. The progress of our trade during the war may be seen from the following table :—

Months	Imports	Exports	Re-exports	Excess of imports over exports
	£ mill.	£ mill.	£ mill.	£ mill.
September–October	111·8	47·7	4·4	59·7
November–December ...	170·6	77·5	5·2	87·9
January–February	200·6	78·5	6·1	116·0
March–April	218·5	89·7	8·0	120·8

After the marked disturbance of our trade as a result of the onset of war conditions and the recovery in November and December to more or less normal figures, except as regards re-exports, there has been a continuous expansion, but whereas the main increase in imports took place between November–December and January–February, there was relatively little increase in exports during this period, with the result that the adverse balance of trade rose by about £14 million a month. Subsequently the increase in imports slowed up and there was a marked rise in exports of United Kingdom goods, as a result of the export drive started in January. The adverse balance showed a further small increase.

The rise in exports, as in imports, is in part attributable to higher prices. While no precise measure of the change is practicable in view of the absence of the usual quantitative particulars, an assessment can be made of the position as regards exports. The President of the Board of Trade stated in the House of Commons on May 30th that the volume of exports in March and April was only 2 per cent. lower than in the same period of last year. The value of such exports rose from £76·8 million to £89·7 million, or by 17 per cent. Accordingly average values of exports must have risen by nearly 20 per cent., which is more than sufficient to offset the depreciation of the pound in relation to the dollar. Average values of United Kingdom exports have therefore risen in terms of gold, and while this rise might be due to the changing make-up of exports following the declared policy of endeavouring to export

a greater proportion of more highly finished goods, there has undoubtedly been a far greater rise than this in prices of textiles, as is evidenced by the published wholesale prices index number. The index numbers for both the cotton and the wool group show increases in April of over 60 per cent. in comparison with April 1939, and though these groups include the imported raw material as well as the manufactures which are exported, the rise for the latter was probably of the order of 50 per cent. While this may be the rise in prices, it will not yet be fully reflected in the average values of exports owing to the time lag.

The value of exports during April, amounting to £48·3 million, was the highest monthly total since July 1930. Especially high totals were recorded for cutlery, hardware, etc., electrical goods and apparatus, and chemicals, drugs, etc., these being in each case the highest for 20 years. Textiles continued to show a substantial improvement, assisted by price increases, and exports during April were the highest in value for ten years for woollen and other textiles apart from cotton goods, exports of which exceeded those in any month since October 1937. Machinery, vehicles and manufactured oils were the only groups of manufactured articles exports of which showed a substantial decline in value in the first four months of this year compared with last; miscellaneous food, raw wool, etc., and non-ferrous ores and scrap also declined by over £500,000. The value of the coal exported rose by £1·1 million (10 per cent.) and, among the groups not mentioned above, there was a marked rise of £3·5 million for the beverages group, the United States being a principal market. Total exports of United Kingdom goods during the four months were £168·2 million, being 9 per cent. more than a year earlier.

Imports rose in value over the same period from £288·3 million to £417·5 million, the rise of 45 per cent. being no doubt due to a considerable extent to the increased cost of freight and the additional cost of war-risk insurance. Part is attributable to the depreciation in sterling, but, as we obtained two-fifths of our imports from British countries before the war and are probably obtaining a larger proportion now and as the depreciation will not apply to imports from the franc area, this is not a very large factor. Substantial increases in prices in the exporting countries are shown in the most recent League of Nations Bulletin of Statistics for grain, textiles, rubber and linseed, but increases for metals are in general small, and for some products, *e.g.* lard, prices are lower than they were a year ago. In connection with these price increases it is perhaps significant that imports of cotton, wool, and grain and

flour were all over £10 million larger this year than last, the only other group showing an increase of this magnitude being manufactured oils. For miscellaneous food the increase was nearly £10 million, and for meat, dairy produce and oil seeds, nuts, etc., about £7 million. There was a rise of roughly £5 million for iron and steel, non-ferrous metals, and both miscellaneous textile materials and miscellaneous textile manufactures, the latter including jute sacks and bags for military purposes. Increases were recorded for all the groups of raw materials. In the other classes, the largest decline was £3·3 million (60 per cent.) for tobacco; the effect of import restrictions is shown by much reduced imports of apparel, footwear and textiles (other than the miscellaneous group), pottery, glass, etc., and miscellaneous manufactures.

Re-exports declined from £18·8 million to £14·1 million, as a result mainly of falls of over £1 million for wool, hides and skins,

Movements and Classes	Twelve Months ended April, 1939	Twelve Months ended April, 1940	Increase (+) or Decrease (−)
Imports, c.i.f.—	£'000	£'000	£'000
Food, drink and tobacco	422,607	437,864	(+) 15,257
Raw materials and articles mainly unmanufactured ...	231,949	298,133	(+) 66,184
Articles wholly or mainly manufactured	226,427	272,330	(+) 45,903
Other articles	8,244	6,847	(−) 1,397
Total Imports	889,227	1,015,174	(+)125,947
Exports, f.o.b.—			
United Kingdom Produce and Manufactures—			
Food, drink and tobacco	36,121	38,070	(+) 1,949
Raw materials and articles mainly unmanufactured	56,480	54,102	(−) 2,378
Articles wholly or mainly manufactured	360,981	350,415	(−) 10,566
Other articles	13,288	10,187	(−) 3,101
Imported Merchandise—			
Food, drink and tobacco	12,278	11,042	(−) 1,236
Raw materials and articles mainly unmanufactured ...	30,270	21,313	(−) 8,957
Articles wholly or mainly manufactured	16,273	8,353	(−) 7,920
Other articles	749	502	(−) 247
Total Exports	526,440	493,984	(−) 32,456

and non-ferrous metals. There was, however, a reduction in the value of re-exports over a wide field. Among the relatively few increases those for beverages and cocoa preparations (£786,000), manufactured oils (£219,000) and miscellaneous food (£177,000) may be mentioned.

By the end of 1939 the general level of wholesale prices, as measured by the Board of Trade index-number, had advanced about 23 per cent. since the commencement of the war, and roughly the same percentage since December 1938. During the first four months of 1940 wholesale prices continued to advance, and in April the index-number stood at 132·7 (1930 = 100), as compared with 122·3 in December 1939 and 97·2 in April 1939—an advance of 8·5 per cent. since December 1940 and 36·5 per cent. since April 1939. The general advance in prices since August 1939 is rather more than 35 per cent., prices of food having risen 41·2 per cent. and those of industrial materials and manufactures 32·3 per cent. As was stated in Part I of the *Journal*, maximum prices have been fixed by the Government for some of the principal articles of food. The Government has also taken over control of many of the important industrial materials, stabilizing prices, regulating consumption by licence and in some cases purchasing stocks and future supplies. It is not considered, however, that the general validity of the index-number has as yet been affected to any serious extent. The groups of cereals has shown the greatest increase in price since the commencement of the war, advancing between August 1939 and April 1940 as much as 63·5 per cent. Advances nearly as considerable have been shown by textiles, cotton rising 55·7 per cent., wool 58·8 per cent. and other textiles 49·1 per cent. The advances in coal, iron and steel and other metals have not been nearly so important. These materials are, however, under considerable control, and the prices of iron and steel, which have advanced only about 19 per cent. since the war started, have been on a fairly high level during the last three years, and are as much as 68 per cent. in advance of the low prices at the middle of 1932. The price of tobacco during April was, of course, the result of the increased duty of 4*s*. per lb., and the prices of practically all paper-making materials advanced very considerably during the same month. During the first four months there have been a number of seasonal changes (both advances and reductions) in the prices of articles of food.

The Board of Trade index-numbers for the last five months and for August 1939, the month immediately preceding the war, are given on p. 270.

(Average for the year 1930 = 100)

Date	Total Food	Total not Food	All Articles	Basic * Materials	Inter-mediate Products	Manu-factured Articles	Building Materials
Aug. 1939	90·4	102·2	98·1	94·5	104·0	108·7	104·1
Dec. ,,	118·1	124·3	122·3	135·0	125·0	122·0	110·3
Jan. 1940	122·9	126·7	125·6	137·0	128·1	123·7	112·2
Feb. ,,	126·4	129·4	128·5	136·6	132·7	127·9	114·1
Mar. ,,	125·3	131·4	129·4	140·4	134·9	129·0	114·3
Apr. ,,	127·6	135·2	132·7	152·1	136·0	130·4	114·5
Apr. 1939	*91·4*	*100·1*	*97·2*	*90·4*	*99·6*	*108·6*	*102·4*
,, 1938	*100·3*	*104·4*	*103·1*	*93·8*	*105·2*	*112·7*	*104·5*

* Excluding fuel.

The figures of certain other British index-numbers and the official index-number for the United States (Bureau of Labour) are given below for comparison. The French and German index-numbers are not available for the period of the war.

Date	Board of Trade (1930 = 100)	*Economist* (1927 = 100)	*Statist* (1866–77 = 100)	*The Times* (1913 = 100)	United States (Bureau of Labour) (1926 = 100) *
Aug. 1939	98·1	70·3	90·4	114·5	74·8
Dec. ,,	122·3	91·7	120·1	142·5	79·0
Jan. 1940	125·6	91·9	124·1	143·7	79·4
Feb. ,,	128·5	91·9	124·5	146·1	78·5
Mar. ,,	129·4	93·4	123·4	145·1	78·1
Apl. ,,	132·7	93·7	126·0	146·0	78·3

* Mean of weekly figures.

The monthly index-numbers of the *value* of wholesale textile trading are available for the first quarter of 1940. These numbers are prepared by the Wholesale Textile Association in conjunction with the Bank of England, and distinguish between the home and the export trade. Over the period an appreciable increase in values is shown for each month, both as regards the home and the export trade; the increase being more marked in the case of the home trade. The total index-numbers for January, February and March 1940 are 80, 112 and 152 respectively, compared with 54, 75 and 116 in the corresponding months of 1939.

At May 1st, 1940, the average level of *retail prices* of articles of working-class consumption had risen nearly 3½ per cent. above the prices current at January 1st, and about 16 per cent. above those

current at September 1st, 1939. According to the Ministry of Labour index-number, food prices had advanced 15 per cent., the cost of fuel and light 14 per cent., and clothing 35 per cent. Working-class rents had advanced slightly, owing to increases in local rates, and the cost of other articles of working-class consumption 17 per cent. above the prices prevailing at the commencement of the war. Over the first four months of 1940 the increase in food prices has not been considerable, owing chiefly to the maximum prices of many important articles of consumption having been fixed by the Government. Clothing costs have, however, advanced about 12 per cent. and the prices of tobacco and of matches have risen considerably owing to the imposition of increased duties. The index-numbers for the various classes of expenditure for the undermentioned dates are given below, the prices for July 1914 being taken as 100.

Date	Food	Rent and Rates	Clothing	Fuel and Light	Other Item	All Items
Sept. 1st, 1939	138	162	205–210	180–185	180	155
Jan. 1st, 1940	157	162	250	200	190	174
May 1st, ,,	159	164	280	208	210	180
May 1st, 1939	*134*	*162*	*205–210*	*180–185*	*175–180*	*153*

Apart from the usual seasonal increase during the first few weeks of the year, unemployment since the middle of December 1939 has decreased with considerable rapidity. The rate of unemployment in December in the insured trades (excluding agriculture) in Great Britain and Northern Ireland was 9·1 per cent., a rate somewhat higher than the rate (8·7 per cent.) in August just before the war. An increase to 10·2 in January and February was followed by a sharp fall to 7·8 per cent. in March, with a further fall to 6·8 per cent. at the middle of April 1940. Unemployment in the insured trades is almost always at a higher rate for males than for females, but during most of the period since the commencement of the war the reverse has been the case, and this has been due for the most part to the registration of women for war work in the insured trades, both of those who were formerly in industry and of others volunteering for such work. The rate of 6·8 per cent. is the lowest recorded at any time since the establishment of the complete general scheme in 1920. The previous lowest percentage was that of 8·7 in May 1927. In April 1939 the rate was 11·5 per cent. As compared with April 1939, employment in April 1940 had improved in 100 of the 105

trade groups in which the Ministry of Labour classifies the general insured trades. The five groups where unemployment had increased were, slate-quarrying, dress-making and millinery, printing and book-binding, professional services and laundry service. In slate quarrying and mining 41·4 per cent. were out of work at the middle of April, and the rate of unemployment among dock labourers is still as high as 21·6 per cent. In public works contracting it was 18·5 per cent. The noticeably high rates of unemployment in Northern Ireland still continue. In April 1940 the rate was about three times as high as in Great Britain. It would be interesting to compare, if figures were available, the contributions and benefit per head received by insured persons in Great Britain and in Northern Ireland respectively over a period of say ten years.

Employment in agriculture improved considerably during the first four months of 1940, the rate of unemployment dropping from 7·4 per cent. at December 11th, 1939, to 4·4 per cent. at the middle of April. Part of this decline is, of course, seasonal, but the rate in April 1939 was 6·2 per cent. In this industry also the extremely high rates for Northern Ireland are apparent. In April 1940 the rate was 23·1 per cent., compared with 3·6 per cent. in Great Britain.

The percentages unemployed in the insured trades in Great Britain and Northern Ireland are set out below :—

Date	Percentage Unemployed in Great Britain and Northern Ireland of Workpeople Insured under				
	General Scheme	Agricultural Schemes	General and Agricultural Schemes		
			Males	Females	Total
Aug. 14th, 1939 ...	8·7	3·9	8·9	7·4	8·5
Dec. 11th, ,, ...	9·1	7·4	8·8	9·8	9·1
Jan. 15th, 1940 ...	10·2	9·7	10·2	10·3	10·2
Feb. 12th, ,, ...	10·2	11·1	10·5	9·8	10·3
Mar. 11th, ,, ...	7·8	7·0	7·4	8·5	7·7
Apl. 15th, ,, ...	6·8	4·4	6·3	7·5	6·7
Apr. 17th, 1939 ...	*11·5*	*6·2*	*11·9*	*9·6*	*11·2*

The number of workpeople aged 16 to 64 (insured and uninsured) on the registers of the Employment Offices of the Ministry of Labour in Great Britain are recorded below for certain recent months. The

figure 972,695 recorded for April 15th, 1940, include 10,606 boys and 19,496 girls between the ages of 14 and 16, and 12,117 boys and 18,516 girls between the ages of 16 and 18. As compared with April 1939, the total number of all ages on the registers declined by 671,699. The number of men over 18 decreased by 583,139 and the number of women by 49,102.

Date	Wholly Unemployed	Temporarily Stopped	Persons Normally in Casual Employment	Total
Aug. 14th, 1939	968,108	211,978	51,606	1,231,692
Dec. 11th, ,, ...	1,170,798	143,065	47,662	1,361,525
Jan. 15th, 1940	1,219,503	249,723	49,670	1,518,896
Feb. 12th, ,, ...	1,141,358	315,072	47,670	1,504,100
Mar. 11th, ,, ...	965,667	109,380	46,166	1,121,213
Apl. 15th, ,, ...	840,027	90,182	42,486	972,695
Apl. 17th, 1939	*1,343,295*	*238,729*	*62,370*	*1,644,394*

(2) OTHER STATISTICS

There was an appreciable rise in Stock Exchange Values during the first three months of 1940 and a small reaction during April, and the index-number of the *Bankers' Magazine* (values at December 1921 = 100), which stood at 107·4 in August 1939, and at 108·7 in November and December, rose to 114·6 in March 1940. In April the number fell slightly, and at April 19th it stood at 114·1. Both Fixed Interest Stocks and Variable Dividend Securities shared in the decline, which was more noticeable in the latter class. The Preference Stocks of British Railways, which fell away noticeably in March, continued to fall during April, and among industrial securities oil, rubber and iron and steel shares declined appreciably. The shares of copper-mining companies rose during April over 10 per cent.

The statistics of *retail sales* prepared by the Bank of England in conjunction with various retail distributors' Associations and cooperative societies show that during the three months January to March 1940 sales increased in value compared with the corresponding months in 1939 following on a slight set-back in December 1939. Bearing in mind the almost general advance in food prices, as well as increases in other articles, it seems that there must have been some appreciable decline in the quantities sold. The index-numbers of sales of all articles (average daily sales in 1937 = 100) for the four months December 1939 to March 1940 were 136, 102,

95 and 108, compared with 138, 98, 91 and 96 in the corresponding months a year previously. For food alone the numbers were 125, 104, 107 and 115 in the months December 1939 to March 1940 and 122, 98, 103 and 105 twelve months earlier.

Exact figures as to *shipping freights* can no longer be given, although it is clear that these have advanced considerably. The index-number of the Chamber of Shipping is no longer published.

STATISTICAL AND ECONOMIC ARTICLES IN RECENT PERIODICALS

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December, 1939—Solution to a geometrical problem in probability: *W. L. Stevens.* On the construction of balanced incomplete block designs: *R. C. Bose.* Equilibrium between mutation and random extinction: *J. B. S. Haldane.* Stage of development as a factor influencing the variance in the number of offspring, frequency of mutants and related quantities: *R. A. Fisher.* Distribution in the A, A_2, BO blood groups in England: *E. W. Ikin, A. M. Prior, R. R. Race, G. L. Taylor.*

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February, 1940—British bank accounts for 1939.

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March—The nutrition of indigenous workers.

April—Recent developments in compulsory systems of family allowances : *Claire Hoffner.*

INTERNATIONAL—*Contd.*

Revue de l'Institut International de Statistique, 7^{e}, *année, livr.* 4 —Sur la "méthode des profils" et sur d'autres diagrammes à ordonnées jointes, dans le cas de séries non ordonnées (avec résumé anglais), II : *C. Gini.* L'emploi des indices dans les sciences et spécialement dans les sciences sociales (avec résumé anglais) : *G. Hostelet.* Observations sur le rapport de M. Winkler : "la baisse de la natalité," etc. : *W. Winkler.*

LIST OF ADDITIONS TO THE LIBRARY

Since the issue of Part I, 1940, the Society has received the publications enumerated below :—

I.—OFFICIAL PUBLICATIONS

(a) United Kingdom.

General Register Office. Manual of the international list of causes of death, as adapted for use in England and Wales, Scotland and Northern Ireland. Based on the fifth decennial revision by the International Commission, Paris 1938. London: H.M.S.O., 1940. 9¾″ × 6″. lxiii + 166 pp. 3*s.* 6*d.*

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Emergency report No. 1. Industrial health in war: a summary of research findings capable of immediate application in furtherance of the national effort. London: H.M.S.O., 1940. 9¾″ × 6″. iv + 36 pp. 6*d.*

Report No. 84. A study of accident proneness among motor drivers. By *E. Farmer* and *E. G. Chambers.* London: H.M.S.O., 1939. 9¾″ × 6″. 56 pp. 9*d.*

Medical Research Council.

Special report series, No. 235. The chemical compositions of foods, by *R. A. McCance* and *E. M. Widdowson.* 9¾″ × 6″. 150 pp. 4*s.* No. 236. Medical uses of radium: summary of reports from research centres for 1938. 9¾″ × 6″. 49 pp. 1*s.* No. 239. A comparative study of the seasonal incidence of mortality in England and Wales and in the United States of America, by *E. Lewis-Faning,* 9¾″ × 6″. 150 pp. 4*s.* London: H.M.S.O., 1940.

Meteorological Office. The weather map: an introduction to modern meteorology. 3rd. ed. London: H.M.S.O., 1939. iv + 85 pp. 3*s.*

Office of the Parliamentary Council. Defence Regulations (Being regulations made under the Emergency Powers (Defence) Act 1939, printed as amended up to and including 19th March 1940) . . . London: H.M.S.O., 1940. viii + 167 pp. 2*s.* 6*d.*

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(b) British Empire.

Eire—

Department of Industry and Commerce. Census of population, 1936. Vol. II. Occupations of males and females in each province, county, county borough, urban and rural district. 1940. viii + 233 pp. 2*s.* 6*d.* Vol. III, Part I, Religions, Part II, Birthplaces 1939. viii + 225 pp. 2*s.* 6*d.* Dublin: Stationery Office.

India—

Office of the Economic Adviser. Studies in Indian Economics. First Series, Aspects of the Indian tariff. No. I. The burden of the Indian tariff, by *T. E. Gregory* and *W. R. Natu.* 1939. 48 pp. 7*d.* No. 2. The history of the Indian tariff 1924—39, by *B. N. Adarkar.* 1940. 100 pp. 1*s.* 3*d.* Second Series, Aspects of Indian social economics. No. I.

(b) **British Empire**—*Contd.*

Changes in the occupational distribution of the population, by *B. G. Ghate*. 1940. 46 pp. 7*d*. Delhi. $9\frac{1}{2}'' \times 6\frac{1}{4}''$. (From Dr. Gregory).

Union of South Africa—

Census and Statistics Office. Sixth census of the population of the Union of South Africa enumerated 5th May 1936. Vol. VIII, Dwellings . . . xxiv + 156 pp. 7*s*. 6*d*. Vol. XI, South African life tables . . . xv + 14 pp. 2*s*. Pretoria: 1939. $12'' \times 9\frac{1}{2}''$.

(c) **Foreign Countries.**

Brazil—

Instituto Brasileiro de Geografia e Estatistica. Anuario estatistico do Distrito Federal, Ano VI, 1938. $10'' \times 7''$. xlvii + 541 pp.

Brazil 1938: a new survey of Brazilian life, economic, financial, labour and social conditions from a general point of view. $10\frac{1}{2}'' \times 7''$. xxii + 425 pp. Rio de Janeiro. 1939. (From the institute.)

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Estonia—

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Italy—

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Censimento industriale 1937. Censimento delle trebbiatrici e delle sgranatrici per cereali, leguminose da seme, semi minuti al 1 agosto 1937. (Monografia No. 4). Rome: 1939. $12'' \times 9''$. 71 pp.

Latvia—

Bureau de Statistique. Budgets familiaux 1936/37. Riga: 1940. $10'' \times 7''$. 208 pp.

Luxemburg (Grand Duchy)—

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(d) **International.**

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Series F, No. 17. Silicosis: proceedings of the international conference held in Geneva from 29th August to 9th September, 1938. 1940. iv + 223 pp. 5*s*.

(d) **International**—*Contd.*

Series M, No. 16. The investment of the funds of social insurance institutions. 1939. viii + 196 pp. 5*s*.

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Basu (*Saroj Kumar*). Industrial finance in India: a study in investment banking and state-aid to industry with special reference to India. University of Calcutta, 1939. 9½″ × 6″. xvii + 436 pp.

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Kendall (*M. G.*) and *Babington Smith* (*B.*). On the method of paired comparisons. Reprint from *Biometrika*, March 1940, pp. 324–45. 10¾″ × 7½″. (From Mr. Kendall.)

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Some Aspects of Population in Bristol

By E. Grebenik

Before the Population Statistics Act of 1938 became law, English vital statistics did not record the ages of mothers to whom children were born. This prevented accurate forecasts of population from being made, and such forecasts as were made were approximations. It was one of the purposes of the 1938 Act to provide more complete statistics in this respect.

In the City of Bristol the Medical Officer of Health has kept particulars of the mother's age for all births notified to him. Such figures have been collected since 1915, but when the Public Health Department moved to its new headquarters a few years ago, some of the older records ceased to be available. For the purposes of this study we chose the year 1932, being the earliest year for which these statistics were available, and 1937, the year in which the University of Bristol Social Survey was taken.

From the birth register we could obtain the mother's age, the parity order of the child and the place where the child was born (*i.e.*, whether it was born in a hospital or similar institution, or at home).

In this paper we shall first of all attempt a forecast of the future population of Bristol, assuming fertility and mortality to remain at their present level. We shall then proceed to an analysis of differential fertility between different occupational groups of the population, our information being taken from the records of Bristol's Health Visitors.

1. *Mortality*

In order to obtain Bristol mortality we have constructed a life-table for the city. We have tried to make it conform as closely as possible to English Life-Table No. 10, published by the Registrar-General. But as the life-table was to be used primarily to obtain

estimates of the future population of Bristol, and was not meant to give a meticulously accurate picture of Bristol mortality, complete theoretical accuracy has to some extent been sacrificed for ease of computation. It will be sufficient here to give a short sketch of the method of construction.

In English Life-Table No. 10 the Registrar-General uses the 1931 Census population, tabulated in five-yearly age groups. Using a graduation formula, he proceeds to obtain quinquennial pivotal values of the population, to which he applies the deaths, which have previously been graduated in a similar manner. Thus, the central death rate m_x is obtained. From this q_x, the probability of dying in a given year, may be deduced. A series of q_x's at five-yearly intervals is thus calculated, and the remaining values are filled in by osculatory interpolation. For ages under 22 a different method is used, and for ages higher than 87 a Gompertz graduation is applied.

The Bristol Life-Table is based on the Census population of 1931, adjusted to allow for the interval between the date of the Census and the middle of the year. The average number of deaths occurring in 1930, 1931 and 1932 has been used, and the method described in the preceding paragraph has been applied to give a series of q_x for ages 12 to 72.

For ages over 72 we have extrapolated by using a Makeham formula. This is a very approximate method which exaggerates mortality at late ages. As the number of aged persons in the population is, however, relatively small, this will not affect our forecasts very much. The use of this formula will, however, slightly reduce the values given for the expectation of life. For ages 0 to 12 a number of approximate methods has been used, and the life-table functions have been computed from figures given in the Quarterly Returns of the Registrar-General.

The Bristol Life-Table compares rather favourably with English Life-Table No. 10, with which it is strictly comparable for ages 22 to 72. As regards females, mortality at ages 39 to 55 is worse than for the country as a whole, whereas at other periods of life it is lower than that of the general population. In Table I we show the ratio of q_x—the probability at birthday x of dying in that year—in the Bristol Table, to the q_x given in English Life-Table No. 10. As regards males, this table does not exhibit the same regularity as it does for females. In the case of adult males vitality in Bristol does not seem to differ appreciably from that of England and Wales as a whole, whereas the vitality of women of menopausal and immediate post-menopausal age seems to be lower in Bristol.

TABLE I

Ratio of Bristol q_x to q_x given in English Life-Table No. 10

Age	Males	Females	Age	Males	Females
20	0·97	0·98	50	1·02	1·11
25	1·03	0·96	55	0·96	1·00
30	0·95	0·99	60	0·93	0·91
35	1·05	0·87	65	1·00	0·88
40	0·98	1·10	70	1·01	0·89
45	0·99	1·10			

At other ages the two tables are not strictly comparable, as different methods have been used in their construction. For ages over 72 mortality in Bristol would seem to be definitely heavier than in England and Wales, though this excess may be apparent rather than real, owing to the method of graduation employed. It should be noted that in spite of the graduation, the national mortality rate for females exceeds the rate for Bristol females up to age 79. Male mortality experience, however, is definitely worse.

Bristol mortality in youth and adolescence is, on the whole, lower than that shown in the national table, though females aged 15 to 20 and males aged 2 to 4 experience a mortality very slightly above the national rate. Infantile mortality in Bristol compares very favourably with mortality in the country as a whole.

In Table II we compare the probability of surviving 10 years at birthday x, given by the National Table and by the Bristol Table. This tends to confirm the impression given by the comparison of the series of q_x. The superiority of Bristol female mortality experience is clearly shown, and no appreciable difference in male mortality is indicated.

TABLE II

Comparison of ${}_{10}p_x$ in Bristol Life-Table and English Life-Table No. 10

Age	Males		Females	
	Bristol	E.L.T. No. 10	Bristol	E.L.T. No. 10
0	0·9051	0·8902	0·9270	0·9108
10	0·9824	0·9800	0·9824	0·9814
20	0·9673	0·9676	0·9720	0·9710
30	0·9581	0·9588	0·9668	0·9642
40	0·9249	0·9241	0·9374	0·9435
50	0·8552	0·8506	0·8887	0·8891
60	0·6834	0·6816	0·7818	0·7570
70	0·3716	0·3736	0·4862	0·4680

The expectation of life at birth in Bristol is 59·82 years for males and 64·14 years for females. These figures compare favourably with the corresponding figures given in English Life-Table No. 10, 58·74 years for males and 62·88 years for females.*

The Bristol Table reproduces one of the phenomena commented upon by the Government Actuary in his report on English Life-Table No. 10. The regular progression of the q_x's is interrupted between the ages of 20 and 30. This tendency is actually accentuated in the case of the Bristol table. For males the maximum q_x occurs at a slightly later age than in English Life-Table No. 10, and the consequent depression is more pronounced. In the case of females there are two such interruptions, both small, between ages 19 and 21 and 33 and 35. No explanation can be given for this phenomenon. The Government Actuary comments on p. 6 of the Registrar-General's Decennial Supplement for 1931 Part I:

> "Had this feature obtained only among females there might have been an inclination to assign it to misstatements of age, but the fact, that it is more pronounced among males than among females would appear to indicate that some special factor or factors are operating at these ages to disturb the progressive increase in the rate of mortality from age to age."

2. *Fertility*

In order to compute specific fertility rates we have estimated the 1932 Bristol population by applying the sex-age distribution of the estimated 1931 mid-year population to the Registrar-General's figure of the mid-year 1932 Bristol population. From the birth register we have excluded all births of non-Bristol women, who came to Bristol solely for the purpose of being confined there. This proved possible, as the home address of the mother was shown in the Birth Register. We could not, however, make an allowance

* It may be of interest to record that there was an attempt to calculate a life-table for Bristol about 100 years ago. In 1830 Robert Rankin, the then Secretary of the Bristol Union Fire and Life Insurance Company, published a book entitled "A familiar Treatise on Life Assurances and Annuities comprising a historical Sketch of the Science of Life Assurance Offices with Observations on the Duration of Human Life and other Objects of Interest connected with the Subject, to which are appended Original Tables of the Probabilities and Expectations of Life in the City of Bristol." The table, like most tables of that period, is based solely on burial statistics. Some of the tables are especially interesting, as the author has tried to calculate mortality tables for different sections of the population. Thus the expectation of life at birth for all Bristol inhabitants was 28·78 years, but for members of the Society of Friends it was 40·40 years. It appears that this table has not been previously noticed. It is not mentioned either by Kuczynski or Newsholme, nor does it appear to have been noted in the historical survey of English life-tables published under the auspices of the American Institute of Actuaries.

for Bristol women who left the city for their confinements, but the number of such women is probably small, as there are excellent maternity hospital facilities available in Bristol itself. In 1932 there were 96 births and in 1937 35 births for which the age of the mother was unknown. These births have been allocated to the different age-groups in the same proportion as the births for which the age of the mother was known. In Table III the calculation of the various reproduction rates for 1932 is shown.

TABLE III

Calculation of reproduction rates, Bristol 1932

Age	Estimated female population, 1932	Female live births, 1932	Specific fertility rates	Total births in stat. pop.
15–19	17,654	71	0·0040	1,833
20–24	18,991	561	0·0295	13,345
25–29	17,978	929	0·0517	23,057
30–34	17,072	705	0·0413	18,135
35–39	16,426	450	0·0274	11,830
40–44	14,938	172	0·0115	4,852
45–49	14,098	6	0·0004	163
All	117,157	2,894	0·1658	73,215

As we are working with five-yearly age groups, the sum of the specific fertility rates must be multiplied by five in order to give us the gross reproduction rate. We thus obtain a Gross Reproduction Rate of 0·1658 × 5 = 0·829 and a net reproduction rate of 0·732. The stationary population in each five-yearly age group has been calculated by the formula ${}_5L_x = 2\frac{1}{2}(l_x + l_{x+5})$. An attempt to calculate the rates by individual ages led to almost the same result for the gross reproduction rate and to exactly the same result for the net reproduction rate. As it was felt that there might have been some misstatement of age in the records, the calculation by five-yearly age groups was thought to be the more reliable one.

In order to find the gross reproduction rate necessary to make the net reproduction rate equal to unity—*i.e.*, the rate that would be necessary if the population were just to replace itself—we use the ratio $\frac{0{\cdot}732}{0{\cdot}829} = \frac{1{\cdot}000}{x}$, where x is the required gross reproduction rate. This is found to be 1·133. It is thus seen that the gross reproduction rate must rise by 36·7 per cent. if the population of Bristol is to replace itself. If fertility in each age group were increased in this proportion, there would be 3,956 female births a year, instead of 2,894 in 1932, an increase of 1,062.

The rates we have computed may be compared with similar rates

calculated by Glass * both for Bristol and for England and Wales as a whole. The comparisons are set out in Table IV.

TABLE IV

Comparison of Bristol reproduction rates with Glass's rates

Gross reproduction rate, Bristol, 1932	0·829
Net reproduction rate, Bristol, 1932	0·732
Glass's gross reproduction rate, England and Wales, 1930–32 ...	0·929
Glass's net reproduction rate, England and Wales, 1930–32	0·807
Glass's gross reproduction rate, Bristol, 1931 †	0·860
Bristol, 1932, gross reproduction rate as percentage of England and Wales gross reproduction rate, 1930–32	89·2
Bristol, 1932, net reproduction rate as percentage of England and Wales net reproduction rate, 1930–32	90·7
Bristol, 1932, gross reproduction rate as percentage of Bristol, 1931, gross reproduction rate	96·4

The difference in the ratios of the gross and net reproduction rates reflects the slightly better mortality experience of Bristol females of child-bearing age. This was commented upon in the first section.

In order to compute reproduction rates for 1937, the second year studied, an estimate of the 1937 Bristol population had to be

TABLE V

Estimated population of Bristol, 1937

Age	Males	Females	Total
0–4	15,078	14,269	29,347
5–9	14,344	13,971	28,315
10–14	16,673	16,182	32,855
15–19	15,869	15,713	31,582
20–24	16,441	17,682	34,123
25–29	16,630	18,992	35,622
30–34	15,851	17,955	33,806
35–39	13,857	17,027	30,884
40–44	13,039	16,282	29,321
45–49	12,055	14,671	26,726
50–54	11,179	13,686	24,865
55–59	10,429	12,927	23,356
60–64	8,897	10,470	19,367
65–69	6,427	8,211	14,638
70–74	4,251	6,178	10,429
75–79	2,216	3,854	6,070
80–84	978	1,835	2,813
85–89	279	565	844
90–94	42	95	137
All	194,535	220,565	415,100

* Glass, "Changes in Fertility in England and Wales, 1851–1931," in *Political Arithmetic*, ed. L. Hogben, p. 168. Glass had to use foreign specific fertility rates in his estimates.

† *Ibid.*, p. 183.

made. This was done by applying 1932 fertility rates and the mortality of the life-table to the 1932 population. The method is described in Appendix 1. The application of this method gave us a population which was smaller than the Registrar-General's estimate for 1937 by 5,882 persons; a difference of the order of 1 per cent., which falls well within the error of the method. We have added to each sex age group of the estimated population that proportion of the 5,882 persons that the population of that group bears to the total population. Table V shows the population of Bristol in 1937 estimated in this way.

Births to non-Bristol women were again excluded and gross and net reproduction rates for 1937 were then calculated as shown in Table VI.

TABLE VI

Calculation of reproduction rates, Bristol, 1937

Age	Estimated female population, 1937	Female live births, 1937	Specific fertility rate	Total births in stat. pop.
15–19	15,713	87	0·0055	2,520
20–24	17,682	652	0·0369	16,692
25–29	18,992	930	0·0490	21,853
30–34	17,955	707	0·0394	17,300
35–39	17,027	355	0·0208	8,980
40–44	16,282	125	0·0077	3,249
45–49	14,671	23	0·0016	654
All	118,322	2,879	0·1609	71,248

The gross reproduction rate for 1937 is 0·805 and the net reproduction rate is 0·712. In the five years 1932–37 the gross reproduction rate has thus fallen by 2·9 per cent. and the net reproduction rate by 2·7 per cent. But this decline in fertility is by no means evenly distributed over the different age groups. Changes in specific fertility rates are shown in Table VII. Of these changes

TABLE VII

Changes in specific fertility rates, 1932–37

Age of mother	S.F.R., 1932	S.F.R., 1937	% Change
15–19	0·0040	0·0055	+ 37·5
20–24	0·0295	0·0369	+ 25·1
25–29	0·0517	0·0490	− 5·2
30–34	0·0413	0·0394	− 4·6
35–39	0·0274	0·0208	− 24·1
40–44	0·0115	0·0077	− 33·0
45–49	0·0004	0·0016	+300·0

we may dismiss the very large increase in the age group 45–49 as due to chance fluctuations of the very small numbers involved. The age group 15–19 is known to have fluctuating specific fertility rates, but the 25 per cent. increase in the age group 20–24 merits consideration. In a later section we shall show that the proportion of first-born children is significantly higher in 1937 than in 1932. But this may be due to a sudden increase in the number of marriages. We have accordingly extracted the number of marriages in Bristol from 1929 to 1937 from the *Annual Reports of the Medical Officer of Health for Bristol*. These are shown in Table VIII.

TABLE VIII

Number of marriages in Bristol, 1929–37

1929	3,197	1932	3,098	1935	3,558
1930	3,320	1933	3,183	1936	3,805
1931	3,287	1934	3,435	1937	3,781

Whilst the number of marriages registered or solemnized in Bristol does not, of course, account completely for births in the next year, or even for first births, it is probable that an increase in marriages will later be reflected by an increase in births. Table VIII shows that there was a peak in marriages in 1936. According to the *Registrar-General's Statistical Review* for 1936, 44·5 per cent. of all women marrying in England and Wales in that year were aged 24 or less. If we assume a similar percentage for Bristol, and if we further suppose one half of these 44·5 per cent. to have their first child within a year or so of marriage, then there would be 740 such births in 1932 and 856 in 1937, an increase of 15·7 per cent. in specific fertility rates, if the female population had remained stationary. In fact the population aged 15 to 24 was 36,645 in 1932 and 33,395 in 1937, so that the proportional increase in specific fertility rates would be even larger. These calculations are meant to show that it is possible that a large part of the increase in specific fertility rates may be accounted for by the 1936 marriage peak. Without entering into the vexed controversy whether the birth rate is or is not affected by economic conditions, it may safely be suggested that 1937 was a more suitable year in which to have a child than 1932. Children born in the latter year would have been conceived in the beginning of the slump, when economic insecurity was very pronounced. All this suggests that the rise in specific fertility rates at ages 15 to 24 may not be wholly unconnected with economic revival. It would certainly be unsafe to regard it as a permanent rise of fertility at these ages without adducing much additional evidence.

The rest of the specific fertility rates in 1937 show decreases as compared with 1932, and the decrease seems to be getting larger with advancing age. This aspect of the problem will be dealt with more conveniently in a later section where age and parity are compared.

It remains to deal with relative fertility at various ages. For this problem all births have to be considered, and Table IX gives a comprehensive picture.

TABLE IX

Relative fertility at various ages

Age of mother	Live births, 1932	Live births, 1937	1932, %	1937, %	15–19 = 100	
					1932	1937
15–19	153	171	2·60	2·88	100	100
20–24	1,190	1,368	20·20	23·07	778	800
25–29	1,874	1,928	31·81	32·51	1,225	1,127
30–34	1,436	1,454	24·38	24·52	939	850
35–39	881	718	14·95	12·11	576	420
40–44	335	252	5·69	4·25	219	147
45–49	22	39	0·37	0·66	14	23

It is easily seen that both in 1932 and in 1937 over 50 per cent. of all births are accounted for by women between the ages of 20 and 30 and over 75 per cent. of all births by women between the ages of 20 and 35. More detailed statistics show that the specific fertility rate declines sharply at age 32 or 33.

In computing reproduction rates we have used female births only instead of computing total specific fertility rates by using all births, whose sum would then be multiplied by the sex ratio at birth. Our figures appear to indicate that the sex ratio at birth may differ for different age groups of mothers, and for this reason it seemed desirable to use female births only in the calculations.*

3. *Future population*

In Table X we show the estimated future population of Bristol and its age distribution, on the assumption that fertility remains at the 1937 level and that mortality is measured by our life-table. The method of estimation is described in detail in Appendix 1. It should be noted that the population aged 72 and over will be slightly under-estimated in these tables, as the life-table tends to exaggerate mortality at age 72 and over. In Table XI the estimated total population is shown at five-yearly intervals for the next 65 years.

* I hope to be able to analyse the figures relating to the sex ratio at birth in greater detail later.

TABLE X

Estimated future population of Bristol (in 000's)

Age	Males 1937	Males 1942	Males 1947	Males 1952	Females 1937	Females 1942	Females 1947	Females 1952
0–4	15·1	14·3	13·7	12·9	14·3	13·7	13·1	12·3
5–9	14·3	14·3	13·6	13·0	14·0	13·7	13·1	12·6
10–14	16·7	14·2	14·2	13·5	16·2	13·9	13·6	13·1
15–19	15·9	16·5	14·1	14·1	15·7	16·0	13·8	13·5
20–24	16·4	15·6	16·3	13·9	17·7	15·5	15·8	13·6
25–29	16·6	16·2	15·4	16·0	19·0	17·4	15·3	15·6
30–34	15·9	16·3	15·9	15·1	18·0	18·7	17·2	15·1
35–39	13·9	15·5	16·0	15·6	17·0	17·7	18·4	16·9
40–44	13·0	13·5	15·1	15·6	16·3	16·6	17·3	18·0
45–49	12·1	12·5	13·0	14·5	14·7	15·8	16·1	16·7
50–54	11·2	11·4	11·9	12·3	13·7	14·0	15·1	15·4
55–59	10·4	10·3	10·5	11·0	12·9	12·9	13·2	14·2
60–64	8·9	9·3	9·2	9·4	10·5	11·9	11·9	12·2
65–69	6·4	7·4	7·7	7·7	8·2	9·3	10·6	10·5
70–74	4·3	4·8	5·5	5·7	6·2	6·7	7·6	8·7
75–79	2·2	2·7	3·0	3·4	3·9	4·4	4·8	5·4
80–84	1·0	1·1	1·3	1·4	1·8	2·1	2·4	2·6
85–89	0·3	0·3	0·3	0·4	0·6	0·6	0·7	0·8
90–94	—	—	—	0·1	0·1	0·1	0·1	0·1
Total	194·5	196·4	196·7	195·5	220·6	221·1	220·0	217·2

TABLE XI

Estimated total future population of Bristol (in 000's)

	Males	Females	Total		Males	Females	Total
1937	194·5	220·6	415·1	1972	178·8	192·1	370·9
1942	196·4	221·1	417·5	1977	172·5	183·6	356·1
1947	196·7	220·0	416·7	1982	165·4	174·7	340·1
1952	195·5	217·2	412·7	1987	157·8	165·4	323·2
1957	192·9	212·7	405·6	1992	150·0	156·2	306·2
1962	189·0	206·8	395·8	1997	142·1	147·3	289·4
1967	184·3	199·8	384·1	2002	134·5	138·9	273·4

In Table XII we give the quinquennial percentage change in the population. The estimated future population is also shown as a percentage of the 1937 population.

From these tables it is seen that the male population will go on increasing slightly until about 1947, and will then begin decreasing at an increasing rate until the end of the century, when the rate of decrease shows signs of becoming stable. In the case of females the decrease will set in earlier, but again the rate shows signs of stability at the end of the present century.* The decline of the

* It should be emphasized that these percentage rates of decrease are only rough approximations. The calculation of the stable population and the corresponding true rates of decrease are left to the next section.

TABLE XII

Quinquennial percentage changes in the population

% Change			1937 Population = 100		
Period	Males	Females	Year	Males	Females
1937–42	+1·0	+0·2	1937	100·0	100·0
1942–47	+0·2	−0·5	1942	101·0	100·2
1947–52	−0·6	−1·3	1947	101·1	99·7
1952–57	−1·3	−2·1	1952	100·5	98·5
1957–62	−2·0	−2·8	1957	99·2	96·4
1962–67	−2·5	−3·4	1962	97·2	93·7
1967–72	−3·0	−3·9	1967	94·8	90·6
1972–77	−3·5	−4·4	1972	91·9	87·1
1977–82	−4·1	−4·8	1977	88·7	83·2
1982–87	−4·6	−5·3	1982	85·0	79·2
1987–92	−4·9	−5·6	1987	81·1	75·0
1992–97	−5·3	−5·7	1992	77·1	70·8
1997–2002	−5·3	−5·7	1997	73·1	66·8
			2002	69·2	63·0

female population begins earlier than that of the male because of the different age distributions of the two sexes. Whilst in 1937 48·8 per cent. of the male population were under 30, the corresponding proportion among females was only 43·9 per cent. The mortality under 30 being fairly low, the decline begins to show itself earlier among females, a larger proportion of whom is subject to heavier mortality experience.

In Table XIII the estimated population of schoolchildren is

TABLE XIII

Estimated number of schoolchildren in Bristol (aged 5 to 14) and percentage changes

Year	In (000's)		% Decrease		1937 Population = 100	
	Males	Females	Males	Females	Males	Females
1937	31·0	30·2	8·1	8·6	100·0	100·0
1942	28·5	27·6	2·5	3·3	91·9	91·4
1947	27·8	26·7	4·7	3·7	89·7	88·4
1952	26·5	25·7	5·3	5·4	85·5	85·1
1957	25·1	24·3	6·4	6·6	81·0	80·5
1962	23·5	22·7	6·4	6·2	75·8	75·2
1967	22·0	21·3	5·9	5·6	71·0	70·5
1972	20·7	20·1	4·8	5·5	66·8	66·6
1977	19·7	19·0	5·1	5·3	63·5	62·9
1982	18·7	18·0	5·3	4·4	60·3	59·6
1987	17·7	17·2	5·6	6·4	57·1	57·0
1992	16·7	16·1	5·6	6·4	53·9	53·3
1997	15·6	15·2	5·1	5·9	50·3	50·3
2002	14·8	14·3	—	—	47·7	47·4

shown together with its percentage decreases. The decline in numbers among them is much more rapid than among the population as a whole, because the effects of the low fertility rate are quickest in making themselves felt among the youngest section of the population.

Table XIV shows the changes in the age distribution of the population. We have divided it into four age groups, one of which comprises the population of working age (aged 15 to 64) and into three classes of dependents aged 0 to 4, 5 to 14, and 65 and over respectively. It is seen that the proportion of the working population does not show any marked decrease, and more especially the male working population remains proportionately almost constant. Thus the fear that the burden of maintaining a declining population will fall on a smaller proportion of the population than is the case to-day is seen to be unfounded.

TABLE XIV

Per cent. age distribution of the estimated future population

Year	Males				Females			
	0–4	5–14	15–64	65–	0–4	5–14	15–64	65–
1937	7·8	15·9	69·1	7·3	6·5	13·7	70·5	9·4
1947	7·0	14·1	69·9	9·1	6·0	12·1	70·1	11·9
1957	6·2	13·0	70·8	10·0	5·4	11·4	69·3	13·9
1967	5·8	11·9	71·2	11·0	5·1	10·7	68·1	16·1
1977	5·6	11·4	69·7	13·3	5·0	10·4	65·7	19·0
1987	5·4	11·2	68·7	14·8	4·9	10·4	64·9	19·9
1997	5·4	11·0	68·2	15·5	4·9	10·3	65·1	19·7

But, whilst the proportion of the population of working age will remain constant, the position of the different groups in the non-working population is almost wholly reversed. To-day people aged 65 and over constitute less than 10 per cent. of the population and the number of dependent children is almost double that of the aged. In 60 years' time, however, the number of dependent children will fall short of the number of aged people. This will, of course, have important effects on social life, and especially on family life. The trend should also be taken into account when the building of schools and health clinics is considered. In Table XIII we showed that the decline in the number of schoolchildren will, in the absence of any net inward migration, set in almost immediately. Expenditure on schools or infant clinics should therefore aim at improving quality rather than quantity.

In this section we have used the indicative mood throughout,

but the use of the subjunctive would be much more appropriate. The calculations in this section cannot show what will happen to the future population of Bristol, they do show what is implied in the present level of fertility and mortality as projected into the future. It might be argued that these assumptions are arbitrary and that they do not allow for migration. Let us deal with the second objection first. Mr. Shannon, in a yet unpublished study on "Migration and the Bristol Area," has shown that the growth of Bristol in the past has been due mainly to natural increase and that inward and outward migration have been almost equal. Moreover, it is difficult to see where the migrants of the future are to come from, as Dr. Charles has shown * that the experience of the country as a whole will be similar to that of Bristol. And the one conclusion that emerges most clearly from our discussion is that quite a considerable amount of net inward migration will be necessary in order even to maintain the population of Bristol at its present level.† As regards the first objection, it is undoubtedly true. It is almost certain that there will be changes both in fertility and mortality. But any attempt to estimate the extent, and indeed even the direction, of these changes must be mere guesswork.

We are therefore left with the conclusion that a decline in the population of Bristol is inevitable. It may be larger or smaller than our figures indicate, but a decline there will be. Again, this will have important effects on the life of Bristol, and the expectation of a declining population should be taken into account when new long-term plans are put into effect. Housing and sewage schemes, the provision of parks and open spaces, hospital and maternity facilities, urban transport—all these must take the population factor into account. It would need too much space to go into detail here, a full discussion of the economic consequences of a declining population in the national sphere, to which further reference should be made, has recently been published.‡

The picture will be rounded off in section 4, which will deal with the stable population and with its corresponding real rate of decrease. Taken in conjunction with the figures of this section, it should give

* *The Effect of Present Trends in Fertility and Mortality upon the Future Population of England and Wales and upon its Age Composition.* By Enid Charles. London and Cambridge Economic Service, Special Memorandum No. 40. London, August 1935. Dr. Charles's is only one of many similar studies, all pointing towards a substantial decline of the population of this country in the near future.

† It may be argued that the influx of evacuees into Bristol may lead to some inward migration, as some firms may stay in Bristol after the end of the war. This appears improbable and no reasoned estimate of migration could be made at the present stage, even if the argument were true.

‡ W. B. Reddaway, *The Economics of a declining Population*, London, 1939.

a fairly comprehensive picture of the meaning of present fertility and mortality rates.

4. *The stable population*

In the preceding section we have demonstrated the effects that a continuation of present fertility and mortality rates will have on the Bristol population. These calculations are very laborious, and to carry them farther than we have done would serve no useful purpose. At the same time they do not tell us much about the ultimate fate of the Bristol population.

In order to obtain that knowledge we must make use of the methods first devised by Bortkiewicz and Lotka.* They have shown that if a population is subject to constant fertility and mortality it will ultimately settle down to a stable rate of increase or decrease and to a stable age distribution. Both the stable rate of decrease and the stable age distribution may be computed from a knowledge of the net fertility schedule. The method and formulae we have used are given in Appendix 2.

We find that if present fertility and mortality trends continue unchecked, and if there is no net inward migration, the female population of Bristol will ultimately settle down to a stable rate of decrease of 0·01148 or 1·15 per cent. per annum. We have shown in Table XII that the female population as computed by our arithmetical method decreases by 5·7 per cent. over the two five-year periods 1992–97 and 1997–2002. This corresponds approximately to the yearly stable rate of decrease computed above. Small oscillations about the stable rate of decrease may take place after 2002, but they will continuously diminish in magnitude. The average age of mothers at confinement will be 29·32 years.

We have also computed the age distribution of the stable female population. It is given in five-yearly age groups in Appendix 2. For our purposes it will be sufficient to give a summary table, comparable with Table XIV.

It is easily seen that the age distribution of the estimated 1997 population corresponds nearly to that of the stable population.

What is true of the female population will also hold of the males if the sex ratio at birth is assumed to be constant. The five-yearly decreases of the males show signs of becoming stable at the end of the century, but their stable rate of decrease will probably be slightly lower than that of the females.

* For the most recent statements of the theory see E. C. Rhodes, "Population Mathematics," ch. V–VI, *Journ. Roy. Stat. Soc.*, Part 2, 1940, and Lotka, *Analyse démographique avec application particulière à l'espèce humaine*, pp. 64–71.

TABLE XV

Per cent. age distribution of the stable female population compared with that of the estimated 1997 *population*

Age	Stable population	1997 population
0–4	4·8	4·9
5–14	10·2	10·3
15–64	64·8	65·1
65–	20·2	19·7

To sum up, the effect of an unchecked continuation of present fertility and mortality rates means that in the next 65 years the population of Bristol will, in the absence of net inward migration, decrease from 415,000 to about 273,000, a decrease of about 34 per cent., and will thereafter continue to diminish by about 5½ per cent. of itself every 5 years. This rate of decrease roughly corresponds to one third every 30 years.

5. *Mother's age and parity order*

Before beginning to analyse differential fertility in Bristol we shall have to devote some attention to the relation of the age of the mother to the parity order of the child. The information as to parity order may be obtained from the Birth Register, and theoretically parity should, of course, include any product of conceptions; that is to say, stillbirths and abortions should be counted. When we investigated the Health Visitors' Records * which gave the previous pregnancy history of the mother it became clear, however, that the number of admitted abortions was so low that we were forced to the conclusion that there had been some concealment. This factor, could not, of course, be taken into account in the tables. The tables in Appendix 3 give us the distribution of live births by age of mother and parity order for each of the years 1932 and 1937. (We have excluded stillbirths because no information regarding them could be gathered from the follow-up study of Health Visitors' Records described below.) The number of children born appears to decline sharply after the mother has reached the age of about 32 or 33, but in neither of the tables is there a sharply defined peak at which the maximum number of births takes place. We have calculated mean parity ages for the mothers, and in Table XVI we show them for the two years studied together with the ratio of their difference to its standard error.

* See Section 6 below.

TABLE XVI

Average age of mothers for various parities

Para	Live births, Bristol, 1932 and 1937		
	1932	1937	Diff./S.E. diff.
1	26·27	26·41	1·08
2	28·84	29·07	1·28
3	31·03	31·04	0·04
4	33·22	32·85	1·12
5	34·83	33·98	2·07
6	35·50	35·79	0·62
7	37·63	37·27	0·65
8	38·09	37·16	1·37
9	38·10	38·78	0·89
10	38·88	40·43	1·91
11–	41·08	41·62	0·98
All	29·87	29·31	5·09

Regarding a difference that exceeds twice its standard error as significant, there is only one such difference between individual mean parity ages. The average age for all mothers is, however, significantly lower in 1937 than in 1932. This must be due to the changing weights given to individual mean parity ages, or, in other words, to a change in the proportion of children of low parity order. There is also an irregularity in the smooth progression of average ages at parity 8. In 1932 the mean age at eighth confinement is lower by only 0·01 of a year from the mean age at ninth confinement, and in 1937 it is actually lower than the mean age at seventh confinement. These irregularities must be ascribed to chance fluctuations.

In Table XVII we show the proportion of children of various parity orders born in 1932 and 1937, and compare it to two similar distributions, one relating to Stockport in 1937 and the other to Glasgow in 1913.*

On testing the 1932 and 1937 Bristol distribution for significant difference by the χ^2 test,† we find a χ^2 of 90·26, which for 10 degrees of freedom gives us a P of less than 1 per cent. We thus conclude that the difference between the two distributions cannot be due to random fluctuations, and an inspection of Table XVII will show that there has been a real tendency for the proportion of children of high parity to fall in 1937, as compared with 1932.

* Reid and Mackintosh, "The Incidence of Anæmia in Pregnancy," *The Lancet*, vol. 232, 1937, p. 43. Pearson and others, "On the Correlation of Fertility with Social Value, Section III, Glasgow Data," by H. J. Laski, *Eugenics Laboratory Memoirs*, 1913, No. 18, p. 47.

† The formula used was that given by Fisher in *Statistical Methods for Research Workers* (7th Edition), p. 91.

TABLE XVII

Proportion of children of various parity orders

Para	Bristol, 1932	Bristol, 1937	Stockport, 1937	Glasgow, 1913
1	37·47	43·13	39·71	14·14
2	24·64	25·32	23·92	16·46
3	12·97	12·59	13·36	14·09
4	7·96	7·28	8·03	11·97
5	5·68	4·19	5·87	11·90
6	3·97	2·36	3·34	9·07
7	2·51	1·75	2·08	7·33
8	1·82	0·98	1·08	5·60
9	1·08	0·68	0·99	4·02
10	0·85	0·68	0·63	2·22
11–	1·05	1·04	0·99	3·20
All	100·00	100·00	100·00	100·00

In the Stockport sample the authors examined a series of 1,108 pregnant women. There was no selection as to social circumstances or as to parity, the patients being those who attended Stockport Corporation ante-natal clinics. The Stockport results agree fairly well with the Bristol results for the same year. On testing the two distributions by the χ^2 test, we obtain a χ^2 of 15·84, which for 10 degrees of freedom corresponds to a P of about 10 per cent. The two distributions are thus not significantly different. From these two samples it might be concluded that the percentage of first-born children in a 1937 urban population was in the neighbourhood of 40 per cent., and that roughly three-quarters of the children born were either first, second or third children.

These results provide a striking contrast to the pre-war situation in Glasgow. The data consisted of "reports on upwards of 8,000 babies born in Glasgow most kindly provided by Dr. Chalmers, the Medical Officer of Health for the City. This material . . . is not drawn from a special class." The tables really give the size of the family, which is not quite the same as parity order, but, even so, the discrepancies are striking. It appears from the context of the Glasgow study that the data related to the working class, but if these figures were compared to corresponding ones for Bristol unskilled labourers in 1937, there would still be a large discrepancy. There appears to have been a decline in family size even amongst the lowest occupational groups.

A similar study was made by Kennedy in Edinburgh in 1933.* He collected records of 10,000 cases in the gynæcological wards of

* Kennedy, "The Menarche and Menstrual Type," *Journ. Obst. & Gyn. Brit. Empire*, 1933, p. 792.

the Edinburgh Royal Infirmary, and from his data Pearl * computed the percentages shown in the following table.

TABLE XVIII

First-born children per cent. of all live-born children

Mother's age	Bristol, 1932	Bristol, 1937	Edinburgh, 1933
Under 25	69·0	70·5	63·8
25–34	35·7	40·2	28·8
35–44	8·9	11·8	18·8
Over 45	—	7·7	9·9

Pearl was surprised at the high percentages obtained by Kennedy, and thought that this was due to the fact that Kennedy used gynæcologically defective material, among which he expected to find a preponderance of primiparae. The Bristol figures suggest that but for the age groups 35 and over Kennedy's figures underestimate rather than over-estimate the percentage of primiparae in the different age groups. Women over 35 who experience their first pregnancy are probably more apt to go to hospital for their delivery, so that the higher percentage in those age groups may be explained by that fact.

In Table XIX we show children of various parities as a percentage of all live-born children.

TABLE XIX

Children of various parities per cent. of all live-born children, Bristol.

Mother's age	1932			Para	1937			
	1	1–3	4–8	Over 9	1	1–3	4–8	Over 9
15–19	93·84	100·00	—	—	91·07	100·00	—	—
20–24	65·74	98·75	1·26	—	67·90	98·36	1·63	—
25–29	45·34	89·75	10·19	0·06	48·52	90·50	9·49	—
30–34	23·04	67·07	31·48	1·45	29·08	75·05	24·24	0·70
35–39	11·00	41·53	50·18	8·27	13·90	52·19	40·43	7·38
40–44	3·26	20·52	56·68	22·80	5·65	28·64	47·18	24·19
45–49	—	11·11	61·11	27·79	7·69	12·81	41·03	46·15

We may disregard the percentages in the first and last age groups as being subject to large fluctuations. In almost all the other age groups we observe an increase in the proportion of first, second and third births. There have been substantial decreases in the proportion of children of parity order four to eight, but the

* Pearl, *The Natural History of Population.* Oxford, 1939, p. 133.

percentage of children of very high parity order has held its own. This suggests that there is a number of persons who are liable to have large families and who do not seem to be influenced much by external conditions. This problem will be dealt with in greater detail in the section devoted to differential fertility.

In Table XX mean parities for various age groups have been computed. The concept of mean parity has no well-defined meaning when taken by itself, but it might be useful for purposes of comparison, especially for comparison between different occupational groups. Between 1932 and 1937 there has been a decrease in mean parities in all age groups except the first and the last, which are known to be subject to large fluctuations.

TABLE XX

Mean parities at various ages, Bristol, 1932 *and* 1937

Age of mother	1932	1937
15–19	1·07	1·10
20–24	1·44	1·42
25–29	1·95	1·88
30–34	3·04	2·65
35–39	4·49	4·00
40–44	6·13	5·81
45–49	6·78	7·51
All	2·72	2·40

In this case the increase in the last age group may be explained, as there was a larger number of births to women aged 45 to 49 in 1937 than in 1932. As these births tend to be of a high parity order, the average for this group has been raised.

TABLE XXI

Standard deviations of mother's ages about mean parity ages

Para	1932	1937	Difference / St. error of diff.
1	4·60	4·69	1·00
2	4·76	4·78	0·17
3	4·95	4·99	0·22
4	4·91	4·83	0·35
5	4·61	4·96	1·21
6	3·99	4·49	1·52
7	4·17	4·25	0·21
8	3·56	4·43	1·81
9	3·42	3·94	0·96
10	3·51	3·84	0·40
11–	2·98	3·06	0·20
All	5·98	5·83	1·88

In Table XXI we give the dispersion of mother's ages about the mean parity ages, as measured by the standard deviation. If $\frac{\sigma}{\sqrt{2n}}$ is taken as the standard error of the standard deviation, none of the dispersions has changed significantly between 1932 and 1937. These dispersions will be valuable for comparison with those found in the different occupational groups.

6. *Differential fertility—description of the material*

So far we have been concerned with the fertility of the Bristol population as a whole, and we have utilized the material available in the Bristol Births Notifications Register. We shall now attempt to investigate differences in fertility among the different occupational groups in Bristol. In order to do this we have had to use information collected by the Health Visitors employed by the Maternity and Infant Welfare Department of the Corporation of Bristol.

The Health Visitors visit every child born in the city at least once, and in the great majority of cases they continue regular visits until the child reaches the age of 5 and passes from their care into that of the School Medical Service. In a number of cases the families of the children are classified as O.V.S., or "over visiting standard," and no further visits are then made. As a rule O.V.S. families are middle-class families. The Health Visitors have no compulsory powers of entry, and in some cases they are refused admission to the houses. But it says much both for the tact and the skill of the Health Visitors that the proportion of refusals is very small and appears to be diminishing.

The Health Visitor keeps a record card for each family she visits, and proceeds to complete this card during the course of her visits. From these cards we have extracted certain social and medical data. For our purposes the most important one is the occupation of the father. It is these data that we shall be using in the rest of this paper. Unfortunately some mothers appear to resent giving this information, and as it is the primary duty of the Health Visitor to look after the child's welfare, and not to collect statistics, she may not press for information, for fear of being refused entry the next time she calls. Some of the record cards were therefore incomplete. Information is especially scanty where the occupation of the father is concerned, and we have this information in about 50 per cent. of the cases only.

Apart from the incompleteness of the records there are a number of cases where the entry in the birth register cannot be matched

with a corresponding entry in the files of the Maternity and Infant Welfare Department. Apart from the O.V.S. families which we have already mentioned, no information exists about stillborn children, children who have died before the investigation was made,* or children whose parents have moved outside the city's boundaries. Moreover, the department occasionally loses track of one family moving from one part of the city to another. In investigating the records we are not, therefore, dealing with the population as a whole, but with a sample of live-born children, born in Bristol in 1937 and surviving into 1939.

Is this sample likely to be a random sample? Unfortunately we can make no tests to answer that question. But certain considerations must be taken into account. In the first case the Health Visitor is likely to be more inquisitive if the standard of living of the family is low, for it is in these cases that the social environment of the child becomes important from the point of view of its medical progress. And, in the second place, there is a tendency for people to be more reticent about their private affairs the higher they stand on the social ladder. We would therefore expect to have more information about families at the lower end of the social scale, and this slight downward bias must be borne in mind when the results of the sample are interpreted.

We have used the occupational code of the Bristol Social Survey, reprinted in Appendix 4. This code is almost identical with that of the Merseyside Social Survey. But we did not have the same information that the Bristol Social Survey investigators had when they came to code their occupations. In particular we had no information whatever about wages, and some small differences in coding are inevitable.

The Bristol Social Survey Code has 10 occupational classes. This is too large a number for the purposes of this paper, and we have condensed them into five as follows:

0123 = middle class and clerical.
45 = minor commercial, shopkeepers and shop assistants
67 = workmen in supervisory positions and skilled artisans.
8 = semi-skilled workers.
9 = unskilled labourers.

In Table XXII we show the percentages in each of the last four occupational groups in our sample and in the Social Survey. We have compared the last four groups only, as the Social Survey excluded all non-manual workers earning more than £250 per annum.

* That is to say children who died under the age of 2 years. This is a big gap, especially as no information is available about infantile and neo-natal deaths beyond the purely biological information given in the Birth Register.

TABLE XXII

Proportions in different occupational groups

Occupational group	Social survey		Population sample	
	Number	Per cent.	Number	Per cent.
45	294	8·77	344	13·37
67	1,051	31·35	796	30·94
8	1,180	35·20	609	23·67
9	827	24·67	824	32·02
All	3,352	100·00	2,573	100·00

It is evident from this table that we have more information about Class 9 (unskilled workers) than about the other classes. This bears out the *a priori* considerations advanced above.

7. *Mean parity ages in different occupational groups*

We may now turn to an examination of differences in fertility between different occupational groups. For this purpose we have

TABLE XXIII

Mean parity ages in different occupational groups

Occupational group	Para			
	1	2	3	All
0123	27·95	30·50	31·91	29·64
	0·40	0·42	0·89	0·30
45	27·06	29·64	31·73	29·34
	0·37	0·43	0·70	0·28
67	26·19	28·80	30·94	28·51
	0·23	0·33	0·45	0·19
8	25·98	28·72	31·11	29·15
	0·30	0·35	0·58	0·24
9	24·56	27·60	29·63	30·10
	0·30	0·32	0·47	0·23
All	26·41	29·07	31·04	29·31
	0·09	0·12	0·18	0·08

constructed tables showing the number of births to mothers whose husbands are known to be in certain occupational groups. These births have been tabulated by the age of the mother and by parity order. Mean Parity Ages have been computed in all cases where the number of births of a given parity order exceeds 25. If the number fell short of 25 the sampling error would become so large that no reliance could be placed on comparisons. The relevant

information is given in Table XXIII. The figure below each mean parity age gives its standard error.

If we concentrate our attention on mean parity ages at parity 1 we see that there is a pronounced fall in the mean age as we go down on the social scale. If there were no interference with fertility by birth control or similar means the age at first delivery would serve as an index of the age at marriage. As we know that birth control is in fact being practised, and as no statistics for differential age at marriage are available, we cannot analyse the relative importance of these two factors. We shall have to be content with establishing the fall in the age at first delivery that occurs in the lower occupational groups.

In Table XXIV the ratio of the difference in age at first delivery in different occupational groups to its standard error is tabulated. This table exhibits some interesting results. The most striking of them is that the differences between group 9 and all other occupational groups are significant. There appears to exist a real difference between the average age at first delivery of labourers' wives and that of all other women. We have mentioned above that our sample may be biased in a downward direction—that is to say,

TABLE XXIV

Ratio of differences in mean parity ages at parity 1 to their standard error

Occupational group	45	67	8	9
0123	1·65	3·83	3·94	6·78
45	—	1·98	2·25	5·21
67		—	0·55	4·29
8			—	3·38

there may be a tendency for labourers who are in the lower section of group 9 to be selected. But if we accept the measure of dispersion given by the standard deviation of the distribution of the mothers' ages as correct, we find that the difference between group 8 and group 9 would have to be reduced by 0·58 or by 41 per cent. of itself in order to become statistically insignificant. The difference seems undoubtedly to be a real one.

At the other end of the scale the average age at first delivery in the highest group (0123) is significantly different from the corresponding average ages of all, but the next highest group (45). The age at first delivery in group 45 differs significantly from the corresponding ages in groups 8 and 9, and the difference between it and group 67 is almost significant. One might conclude that there

is a significant difference in the average age at first delivery between the middle and shopkeeping classes on one hand and the manual workers on the other.

The spread of the births around the mean age, as shown by the standard deviation of the distribution of mothers' ages, is given in Table XXV. At parity 1 it does not seem to differ much between different occupational groups.

TABLE XXV

Standard deviation of mothers' ages at given parities in different occupational groups

Occupational group	Para			
	1	2	3	All
0123	4·65	3·70	4·65	4·77
45	4·54	4·51	4·86	5·19
67	4·50	4·86	4·47	5·39
8	4·69	4·61	5·22	5·96
9	4·32	4·33	5·04	6·58

As regards second births, the same trend holds. If we construct a table similar to Table XXIV for second births, it is seen that the differences between group 9 and all other groups are again significant. The differences between the highest group and all manual workers' groups are significant, but there appears to be no significant difference between the shopkeeping class and the upper working-class groups. The low value of the standard deviation in group 0123 should in my opinion be ascribed to chance.

TABLE XXVI

Ratio of difference of mean parity ages at parity 2 to its standard error

Occupational group	45	67	8	9
0123	1·43	3·21	3·24	5·47
45	—	1·56	1·67	3·78
67		—	0·17	2·61
8			—	2·38

A similar analysis may be made for parity order 3, although the standard errors tend to become larger there. The results are shown in Table XXVII.

This table is even more striking than the previous ones. In spite of the much larger size of the standard error, group 9 still stands quite distinct from all the other groups. (The difference

between group 8 and group 9 (just fails to be significant, but the chance of a difference of $1{\cdot}97\sigma$ or larger is 0·04884.) The distinction between group 9 and the other groups is emphasized by the fact that none of the other differences are significant.

TABLE XXVII

Ratio of difference of mean parity ages at parity 3 to its standard error

Occupational group	45	67	8	9
0123	0·16	0·97	0·75	2·26
45	—	0·95	0·68	2·50
67		—	0·23	2·02
8			—	1·97

If the average age of all mothers in different occupational classes be compared, it is seen that it tends to fall as we go down from group 0123 to group 67, but that it rises again in groups 8 and 9. This is due to different weights being given to the elements making up the average, the proportion of children of high parity order who tend to be born to mothers of higher age groups being larger in groups 8 and 9. A comparison of means would therefore be useless, the percentages in various parity groups will have to be investigated. This will be done in the next section. But it might be profitable before proceeding to this investigation to compare the standard deviations of mothers' ages in different occupational groups. A glance at Table XXV shows that they tend to become larger in the lower occupational groups. Taking the standard error of the standard deviation to be $\frac{\sigma}{\sqrt{2n}}$, we can construct a table similar to Tables XXIV, XXVI and XXVII in which the ratio of the differences of the standard deviations to their standard errors are shown. This is done in Table XXVIII.

TABLE XXVIII

Ratio of the differences of the standard deviations in different occupational groups to their standard error

Occupational group	45	67	8	9
0123	1·45	2·48	4·41	6·96
45	—	0·83	2·96	5·35
67		—	2·71	5·67
8			—	2·70

Again group 9 stands apart from all others, and group 0123 is significantly different from all other groups except group 45. It should be noted, however, that in this case there is a distinction between group 8 and higher social classes. The standard deviations are related to the range of the distributions, and they might be taken as a very rough index of the length of the actual as distinct from the potential child-bearing period. Taking the standard deviation of group 0123 as 100, we find that the standard deviations in groups 45, 67, 8 and 9 become 109, 113, 125 and 138 respectively. The spread of mothers' ages in group 9 is higher by about 40 per cent. than that in group 0123. This is an indication that the actual child-bearing period of an unskilled labourer's wife is materially higher than that of a woman of the middle classes.

From this section the following conclusions emerge: The age of the mother at first, second and third delivery decreases with occupational status of the husband, and the average ages in the lowest occupational group (group 9) especially, differ significantly from those of all the other groups. The average age of all mothers decreases with a decrease in the husband's occupational status, but rises again in groups 8 and 9 mainly because of the greater incidence of births of high parity order in these groups. The standard deviation of the age distribution of the mothers increases with a decrease in occupational status, signifying a longer active child-bearing period among women whose husbands are in a low occupational group.

8. *Parity patterns in different occupational groups*

The different parity patterns in the occupational groups are illustrated by Table XXIX, showing the proportion of children of

TABLE XXIX

Proportion of children of various parity orders

Para	0123	45	67	8	9
1	52·11	43·31	47·24	40·07	25·00
2	29·12	32·85	26·51	27·91	22·33
3	10·73	13·95	12·31	13·46	13·83
4	4·21	4·65	7·91	9·52	9·71
5	1·53	2·62	2·64	3·61	9·47
6	1·15	0·87	1·26	1·15	5·83
7	0·38	0·87	0·75	1·81	4·25
8	0·77	0·29	0·75	0·33	1·94
9	—	—	—	0·66	2·43
10	—	—	0·38	0·82	2·18
11–	—	0·58	0·25	0·66	3·03
All	100·00	100·00	100·00	100·00	100·00

various parity orders in the different occupational groups of the population.

Again group 9 exhibits a completely different pattern from all other groups. The proportion of first-born children, which is about the same in groups 0123, 45 and 67, drops in group 8 to 40 per cent. and in group 9 to 25 per cent. of all children. Group 9 differs even from group 8, and the proportion of children of high parity order is much larger in that group than in any other group. In order to see whether any of these differences could have arisen from sampling fluctuations, we have applied the χ^2 test to each pair of distributions. The values of χ^2 are given in Table XXX. For 10 degrees of freedom a χ^2 of 21·161 corresponds to a P of 0·02 and a χ^2 of 23·209 to a P of 0·01. Thus group 9 again differs from all other groups, and only one other difference (that between group 0123 and group 8) is significant. The size of the χ^2 obtained from a comparison of group 9 with other groups is of quite a different order of magnitude from all the other χ^2 s in the table.

TABLE XXX

Values of χ^2

Occupational group	45	67	8	9
0123	8·05	8·79	25·40	114·87
45	—	11·63	16·38	109·04
67		—	18·78	187·38
8			—	101·07

A similar result is obtained when mean parities for various age groups are considered. These are shown in Table XXXI.

TABLE XXXI

Mean parities in different occupational groups

Mother's age	0123	45	67	8	9	All
15–19	1·00	1·00	1·04	1·20	1·04	1·10
20–24	1·15	1·43	1·37	1·47	1·67	1·42
25–29	1·62	1·69	1·83	1·98	2·47	1·88
30–34	2·09	2·23	2·48	2·53	4·15	2·65
35–39	2·85	3·21	3·57	3·64	5·58	4·00
40–44	2·20	4·78	4·86	5·62	7·39	5·81
45–49	—	1·00	3·50	8·60	9·64	7·51
All	1·83	2·02	2·07	2·37	3·55	2·40

Disregarding the figures in the first and last age groups, where the number of births is so small as to render the averages meaning-

less, we see that for every age group the mean parity increases as we descend the social scale. This is partly due to the differences in the age at marriage in different occupational groups and partly to different degree of the use of contraceptives. In this connection the low value for the age group 20–24 in group 0123 is especially noteworthy. The values in group 9 are again of a different degree of magnitude from all the other values given in the table.

9. *Conclusion*

We have shown that the fertility habits of persons in occupational group 9 differ significantly from those of the rest of the population in Bristol when any one of three criteria is applied. The ages of the mothers at their first three confinements are lower on the average than the corresponding ages in all other groups, the dispersion of the distribution of mothers' ages is higher, and the parity pattern of the class as a whole is different from that of other classes, there being a much higher proportion of children of high parity order.

The most direct way of estimating differences in fertility would have been the calculation of net reproduction rates for different occupational groups. This was impossible in our case, as the female population in different occupational groups could not be estimated with any degree of accuracy. It was for this reason that the fertility habits of different groups had to be compared by what may be termed secondary characteristics. But all our comparisons point to the conclusion that there is a certain group with fertility habits distinct from the rest of the population and that that group consists mainly of people in occupational group 9.

The differences in parity patterns are specially significant. Mean parity may be taken as an index of family size, and mean parity is undoubtedly greater in group 9 than in other groups. The grading of group 9 as the lowest occupational group does not imply any judgment as to their social value, but it is true that unemployment and poverty are more prevalent amongst them than amongst the rest of the population. In 1937 29·7 per cent. of families whose man-head was in group 9 were below the poverty line of the Bristol Social Survey,* as against 17·0 per cent. of group 8 families and less than 10 per cent. of families in other occupational groups. Thus the part of the population that is most fertile has to bring up its children in the worst environmental conditions. The Social Survey figures show † that 51·3 per cent. of the families having

* H. Tout, *The Standard of Living in Bristol*, p. 49.
† *Ibid.*, p. 39.

more than four children under 14 years of age have an income below their standard needs, and that some 20 per cent. of all children under 14 in Bristol are below the poverty line. Our figures suggest that these alarmingly high proportions are due mainly to the large number of children in the families of unskilled labourers.* A large amount of the distress caused could undoubtedly be relieved by a system of family allowances, and in a recent article † based on Social Survey data it has been shown that the cost of family allowances that would remove a great deal of the poverty would be small in relation to the total wage bill.

There remains one question. Does the method by which our sample has been selected affect our results? We have mentioned that the Health Visitor will press for information more insistently when conditions in the visited family are poor. This tendency is reflected by the representation of group 9 in our sample, which is proportionately higher than the strength of that group in the general population. But there is also a large amount of distress among families of group 8, and the tendency to select people from the lower strata of that group would be as operative as in the case of group 9. Yet the differences between group 8 and group 9 are in most cases significant, and very large. It would seem that an increase in the size of our sample would not appreciably affect the differences that we have established.

I am indebted to the National Institute of Economic and Social Research, which has supplied the funds by which this research has been made possible, to the Medical Officer of Health for Bristol and his staff for allowing me access to his records and for help in carrying out the investigation, and to the University of Bristol for providing me with facilities for work and with clerical assistance.

APPENDIX 1

The calculation of the future population

To compute the future population in five-yearly age groups, we obtain from the life-table the function ${}_5L_x = 2\frac{1}{2}(l_x + l_{x+5})$, which gives us approximately the numbers x to $x + 5$ in the stationary population. Writing ${}_5p_x = {}_5L_{x+5}/{}_5L_x$, we obtain the reduction factor by which each age group must be multiplied in order to give us the population in the age group $x + 5$ to $x + 10$, 5 years hence.

* Similar results were obtained in the *Social Survey of Merseyside*, vide vol. 3, ch. 21.

† H. Tout, "A Statistical Note on Family Allowances," *Econ. Journ.*, March 1940.

The number of female births is obtained by applying the specific fertility rates given in Tables III and VI to the number of women in the relevant age groups. The relevant number of women is estimated by averaging the number of women in the age group at two five-yearly intervals. Thus, in order to obtain the births that are to give us the population aged 0 to 4 in 1942, we would estimate a female population by averaging the numbers in the different age groups in 1937 and 1942. The resulting number of births must be multiplied by $_5L_0/500{,}000$ in order to give us the population aged 0 to 4 in 1942.*

Male births are computed by assuming the sex ratio at birth to remain at the 1937 level, when 51·45 per cent. of all live births were males. The number of females born is thus multiplied by $\frac{51\cdot 45}{48\cdot 55} = 1\cdot 0597$ to give us the number of males born.

The formula $_5L_x = 2\frac{1}{2}(_x + l_{x+}l_5)$ is an approximate formula, and assumes that the rate of mortality remains approximately constant over a five-yearly period. This assumption is not fulfilled in the 0 to 4 age group, where mortality at the beginning of the period is very much heavier than during the latter part. An error is thus introduced into our calculations, but in view of their hypothetical nature, it was not thought worth while to use a more accurate method.

APPENDIX 2

The stable population

Denoting the yearly real rate of increase by r, and writing R_0 for the net reproduction rate and m_1, m_2, m_3, m_4 for successive semi-invariants of the net fertility schedule, we have:

$$- rm_1 + \frac{1}{2!}r^2m_2 - \frac{1}{3!}r^3m_3 + \frac{1}{4!}r^4m_4 = - \log_e R_0 \dagger$$

Having computed r from the net fertility schedule, we may compute the stable age distribution, by using the following formula:

$$c(a) = \frac{e^{-ra}l(a)}{\int_0^A e^{-ra}l(a)da}$$

* For a short account of the method see D. V. Glass, "European Population Movements in the Union of South Africa," *South African Journ. of Econs.*, vol. 7, No. 1, pp. 49–51.

† E. C. Rhodes, "Population Mathematics," *Journ. Roy. Stat. Soc.*, Pt. 2, 1940. A. Lotka, *Analyse démographique*, p. 69.

where $c(a)$ is the proportion of people aged a in the stable population, and where $l(a)$ is the l_x of the life table and A the upper limit of age of the population. Applying these formulae to the net fertility schedule we obtain:

$$r = -\ 0{\cdot}01148.$$

The stable age distribution is shown in the table below:

Age	%	Age	%	Age	%	Age	%
0–4	4·8	25–29	6·0	50–54	7·0	75–79	4·4
5–9	5·0	30–34	6·3	55–59	7·0	80–84	2·6
10–14	5·2	35–39	6·5	60–64	6·9	85–89	0·9
15–19	5·5	40–44	6·8	65–69	6·5	90–94	0·1
20–24	5·8	45–49	6·9	70–74	5·7		

APPENDIX 3

Age of mother and parity order of live-born babies, Bristol, 1932

Age of mother	1	2	3	4	5	6	7	8	9	10	11–	Un-kwn.	Total
15	2	—	—	—	—	—	—	—	—	—	—	—	2
16	3	—	—	—	—	—	—	—	—	—	—	—	3
17	19	1	—	—	—	—	—	—	—	—	—	—	20
18	35	2	1	—	—	—	—	—	—	—	—	1	39
19	78	5	—	—	—	—	—	—	—	—	—	1	84
20	105	25	2	—	—	—	—	—	—	—	—	2	134
21	130	45	7	—	—	—	—	—	—	—	—	4	186
22	154	41	12	1	—	—	—	—	—	—	—	9	220
23	148	84	26	3	—	—	—	—	—	—	—	9	270
24	196	91	32	9	1	—	—	—	—	—	—	12	341
25	182	119	39	16	5	—	—	—	—	—	—	16	377
26	195	100	43	21	4	—	1	—	—	1	—	21	386
27	167	102	52	22	7	3	—	—	—	—	—	14	367
28	131	104	35	21	16	6	1	—	—	—	—	21	335
29	104	110	59	24	14	10	4	—	—	—	—	22	347
30	92	89	58	38	26	11	3	—	1	—	1	14	333
31	78	72	50	27	22	18	5	1	—	—	—	16	289
32	60	87	57	27	26	22	7	6	2	—	—	20	314
33	43	50	38	29	25	12	7	8	3	3	—	12	230
34	30	46	32	36	23	20	8	7	7	2	—	12	223
35	31	44	26	30	16	15	10	13	4	6	2	11	208
36	16	23	30	32	26	19	10	8	5	2	3	9	183
37	14	20	27	18	13	15	14	7	13	1	2	7	151
38	13	19	25	16	21	15	12	9	3	2	3	9	147
39	15	15	18	17	22	26	10	12	4	9	8	7	163
40	3	8	15	9	13	7	11	8	5	7	9	7	102
41	3	7	5	12	12	8	8	4	3	5	3	4	74
42	1	8	4	8	10	4	11	6	4	1	10	5	72
43	2	2	2	5	5	4	11	6	3	4	6	—	50
44	1	1	1	5	2	1	—	4	2	1	7	1	26
45	—	—	2	1	3	—	1	2	—	1	2	1	13
46	—	—	—	1	—	—	—	—	1	—	1	—	3
47	—	—	—	—	1	—	—	—	—	—	—	1	2
48	—	—	—	—	—	1	—	—	—	—	—	—	1
49	—	—	—	—	—	—	1	—	—	—	—	1	2
Unknown	26	43	21	13	2	3	4	—	—	2	1	79	194
Total	2,077	1,366	719	441	315	220	139	101	60	47	58	348	5,891

Age of mother and parity order of live-born babies, Bristol, 1937

Age of mother	1	2	3	4	5	6	7	8	9	10	11-	Un-kwn.	Total
15	1	—	—	—	—	—	—	—	—	—	—	—	1
16	6	—	—	—	—	—	—	—	—	—	—	—	6
17	24	2	—	—	—	—	—	—	—	—	—	—	26
18	43	4	1	—	—	—	—	—	—	—	—	1	49
19	79	7	1	—	—	—	—	—	—	—	—	—	87
20	125	23	1	—	—	—	—	—	—	—	—	—	149
21	177	47	8	—	1	—	—	—	—	—	—	1	234
22	187	58	10	3	—	—	—	—	—	—	—	1	259
23	225	79	23	3	1	—	—	—	—	—	—	3	334
24	200	115	46	9	5	—	—	—	—	—	—	1	376
25	209	86	40	17	8	1	—	—	—	—	—	1	362
26	195	106	48	19	5	1	1	—	—	—	—	3	378
27	197	124	44	22	8	3	—	1	—	—	—	2	401
28	150	120	50	29	7	2	—	1	—	—	—	2	361
29	169	114	64	29	17	4	4	1	—	—	—	2	404
30	122	93	55	31	21	13	4	2	2	—	—	—	343
31	91	77	36	22	12	4	4	2	—	1	—	3	252
32	81	102	53	35	27	14	5	4	2	1	1	5	330
33	60	90	41	32	8	10	6	2	1	1	—	1	252
34	61	59	50	36	28	13	7	4	1	—	—	1	260
35	26	40	27	24	16	9	8	7	1	1	—	1	160
36	32	40	31	24	10	10	7	5	3	3	1	2	168
37	22	32	33	30	17	13	13	6	7	5	7	1	186
38	9	11	22	17	13	6	8	5	5	2	2	1	101
39	9	15	19	8	8	10	8	3	5	3	7	—	95
40	7	15	12	10	11	7	8	4	3	4	12	—	93
41	2	3	10	8	5	5	6	3	2	5	3	—	52
42	2	6	3	6	5	4	3	3	4	5	6	—	47
43	2	2	5	3	3	3	5	1	1	3	3	1	32
44	1	1	—	2	4	6	2	—	1	—	8	—	25
45	3	1	1	2	1	—	3	1	—	2	5	—	19
46	—	—	—	1	1	—	1	2	—	3	5	—	13
47	—	—	—	—	—	1	—	1	1	—	—	—	3
48	—	—	—	—	1	—	—	—	—	1	—	—	2
49	—	—	—	1	—	—	—	—	1	—	—	—	2
Unknown	24	20	8	6	4	—	—	—	—	—	1	5	68
Total	2,541	1,492	742	429	247	139	103	58	40	40	61	38	5,930

APPENDIX 4

Occupational code used

0. *Highest Professions, Administrative, Business.* All professions and all higher administrative posts in state or business. Merchant. Consulting Engineer, Analytical Chemist.

1. *All School Teachers*, including teachers of Art, Music and Domestic Science, Physical Training and Kindergarten.

2. *Lower Professional, Commercial, Technical and Managerial.* Chemists, Clerks (special training or experience), Commercial Travellers, Customs Officers, Engineers (good class street), Farmers, Market Gardeners, Licensed Victuallers, Master Mariners, Pilots (1st class), Trained Nurses (sick or children's), Police Inspectors, Relieving Officers, Sanitary and Building Inspectors, Shopkeepers (Proprietors and Supervisory Assistants), Trained Library Assistants, Reporters.

3. *Ordinary Clerks*, including G.P.O., Municipal, Insurance, Shipping, Telephone Operators, Draughtsmen, Cashiers, Secretaries.

4. *Insurance and other Agents.* Motor Salesmen, Furniture Salesmen, Typewriter Salesmen, Commercial Travellers (in a small way), School Attendance and Assistant Relieving Officers.

5. *Shop Assistants, Minor Commercial.* Small Shopkeepers, Dealers (in a small way), Fish, Fruit or Vegetable Roundsmen (on own account), Coal Merchant, Manager of Branch Shops, Trade Union Official, Assistant Manager, Canvasser.

6. *Manual Worker in Supervisory Position.* Foreman, General Contractor (own account, employing a few others), Haulage Contractor, Boatswain, Chief Steward, Dock Official, Park Superintendent, Police Sergeant, Pilot (2nd or 3rd class), Head Warehouseman, Railway and Bus Inspector.

7. *Skilled Manual Worker.* Plumber, Fitter, Boilermaker, Carpenter, Painter, Engine-driver, Chef, Policeman, Working Engineer, Bookbinder, Coppersmith, French Polisher, Mason, Overn-builder, Piano-tuner, Plasterer, Signalman, Sign-writer, Watchmaker, Wheelwright, Bricklayer, Tiler, Compositor, Lithographer, Press Minder, Upholsterer, Hairdresser, Passer, Checker, Printer, Colourman, Beater, Typecaster, Tile Slabber, Boot Repairer, Turner, Lens Surfacer, Colour Matcher, Sculptor, Brass-finisher, Rounder, Varnish-maker, Tailor, Cooper, Electric Welder, Riveter, Brush-maker, Iron Moulder, Stevedore, Yeast Separator, Confectioner, Steeplejack.

8. *Semi-skilled Manual Worker.* Barman, Carter, Domestic Servant, Factory Worker, Fireman, Gardener, Laundry Worker, Motor-driver, Park-keeper, Postman, Bus Conductor, Guard, Cinema attendant, Male Nurse, Furniture Remover, Meter Tester, Chimney Sweep, Storekeeper, Bus and Tram Driver, Bricksetter, Roundsman, Fryer, Demolition Labourer, Crane-driver, Waiter, Soldier, Foundry Assistant.

9. *Unskilled Manual Worker.* Docker, Seaman, Labourer, Hawker, Watchman, Stop and Go Man, Lamplighter, Pointsman, Setter, Porter, Loader, Fettler, Scavenger, Craftsmen's Helpers—*e.g.*, Builder's Labourer, Electrician's Mate, Cellarman, Rag-and-Bone Man.

Discussion on Mr. Grebenik's Paper

The following comments on the paper have been received in writing :—

Mr. D. Caradog Jones : In the limited ration of permitted space I will confine my comments on Mr. Grebenik's paper to congratulations on the able use he has made of his material and a caution to the less critical reader that it is not safe to draw conclusions concerning other urban areas on the basis of experience

confined to one or two districts. The need for this caution will be illustrated by figures taken from an unpublished study of English fertility statistics which contains some valuable material relating to Liverpool, the work of one of my recent students, Dr. K. T. Lim. These Liverpool figures, while revealing in certain respects the same general tendencies as those of Bristol, show a remarkable lag behind Bristol in point of time. The likeness and the difference between the figures are understandable. Bristol and Liverpool are both ports, the centres of large mixed populations; but—to mention only two points of dissimilarity which certainly affect fertility—Bristol has a far greater variety of industries apart from those connected with seafaring, especially factories employing many women and girls; the people of the north-western port is generally believed to be more virile and has a greater admixture of immigrant stocks with strong racial characteristics, in particular the Irish Roman Catholic element.

The time lag in the Liverpool figures, as indicated in the following table, is approximately ten years behind the figures for England and Wales :—

Specific fertility rates : all live births

Age of Mother	Bristol (1932) [1]	Liverpool (1931)	England and Wales (1921) [2]
15–19	8·7	14·6	15
20–24	62·6	89·7	101
25–29	104·2	151·5	152
30–34	84·1	118·6	135
35–39	53·6	109·0	96
40–44	22·4	43·0	42

[1] Derived from Mr. Grebenik's Appendix 3 data.
[2] *Registrar-General's Statistical Review*, 1932, Text, p. 136.

The gross reproduction rate for England and Wales given by Kuczynski for 1921 was 1·326; the corresponding rate given by Lim for Liverpool for 1931 was 1·310. Compare with these the figures for England and Wales and for Bristol in 1931, recorded in Table IV of the paper, namely 0·929 and 0·860.

The relative fertility at various ages is closely similar for Bristol and Liverpool, as will be seen by placing the following figures for Liverpool, 1931, alongside those for Bristol, 1932, in Table IX :—

Age of Mother ...	15–19	20–24	25–29	30–34	35–39	40–44	45–49
Per cent. of Births ...	3·23	19·61	30·19	21·47	18·14	6·74	0·62

But there is danger in the generalization which some may be tempted to draw from Bristol and Stockport experience in 1937 (encouraged by the second paragraph following Table XVII) concerning the proportion of children of various parity orders. Witness the contrast between the Bristol figures in 1932 and the following figures for Liverpool in 1931 :—

Para	1	2	3	4	5	6	7	8
Per cent. ...	24·3	19·5	17·8	11·5	7·6	5·2	5·3	8·8

One further comparison is of interest. Here I take figures from Table XXIX, which analyses the proportion of children of various parity orders in relation to the father's occupational grade, bearing in mind that the grading here adopted by Mr. Grebenik is almost identical with that of the Merseyside Social Survey which was used by Dr. Lim. To save space I have shortened the table and, as Dr. Lim combines the grades up to 5, I have taken a mean between the Bristol figures for grades 0123 and 45; the classification I have adopted compensates for such large differences as are found in these grades. L stands for Liverpool and B for Bristol in the following table :—

Proportion of births of various orders according to occupational grade

Para	Grades 0–5		Grades 6–7		Grade 8		Grade 9	
	L	B	L	B	L	B	L	B
1 or 2 ...	68·4	78·7	49·1	73·8	45·9	68·0	32·2	47·3
3 or 4 ...	22·2	16·8	32·7	20·2	33·5	23·0	28·4	23·5
5– ...	9·2	4·5	18·4	6·0	20·6	9·0	39·4	29·2

Although Liverpool, for every occupational grade, has a strikingly larger proportion of high-grade births, precisely the same trends are evident in both cities as we pass down the occupational scale. The lower the scale the greater the fertility; also, in Liverpool as in Bristol, the unskilled manual working class, occupational grade 9, stands out significantly from the other groups with an exceptionally high fertility incidence.

As to conditions in urban areas generally, Bristol is probably more typical than Liverpool, but the differences between them in fertility experience point to the importance of extending Mr. Grebenik's valuable researches to other parts of the country.

Dr. E. C. Rhodes : Mr. Grebenik has rendered a useful service to the community by his study of the population of Bristol. To the local authorities of Bristol the first part should be specially valuable. No planning for the welfare of the people can be made without due regard to the changes which are likely in the number and kind of the population. The old days of budgeting on the basis of a continual increasing number of consumers of various commodities have past. For nearly fifty years we have been warned of the approaching maximum of the population in this country. Some authorities put the date of this event in the immediate future; this year, or next year, or within five years. During the past decade we have been further warned of the consequences of the coming changes in the population, changes involving not merely the total number but the distribution between different ages. Some of the more important changes affect children under 15. Education authorities have to take notice of the decline in the number

of school children. Unfortunately the problem is not too simple, on account of the changing geographical distribution of population, and of the clearing of certain areas in large towns under slum clearance schemes. So it is quite likely that schools will still be built, partly to serve new areas of population, partly to replace older buildings.

The present study is particularly interesting because it deals with a particular large urban area. Now, it is conceivable that the total population of the country might begin to decline in the next few years while the populations of the big towns might continue to increase for some years owing to the continued attraction of urban centres for the population of the rural areas. If this kind of movement does take place within the next ten years, say, in Bristol, the authorities will be able to find its extent by comparing the population estimated by Mr. Grebenik with the recorded population in the future. One can foresee the difficulties which local authorities will face, with services of various kinds planned to efficiency when serving a population of say 400,000, when it is found that there is only a population of 350,000 to use them and pay for them. One can foresee the large towns making urban life more attractive still in order to induce the rest of the population to join them.

At the same time, it is difficult to foresee what will be the effect of the War on the populations of large towns. In a certain sense it seems futile to compute future populations based on known fertility and mortality records of peace time when a European War is raging, but it is still true that these forces will be operating when additional forces operate. The relative force of the peace-time factors and the war-time factors is unknown as yet. What the post-war period will bring is unknown. One can foresee that every effort will be made to make goods for export in exchange for food and raw materials. It is possible that the towns will continue to attract rural communities as before, though it is conceivable that there will be a lower standard of living. It is possible that there will be more and more communal services, so the large towns will have to be prepared to organize still more services for their populations.

The second part of Mr. Grebenik's paper dealing with differential fertility confirms some of the conclusions of previous workers in this field. It seems, in particular, that unskilled manual workers are still in a separate class in this regard, in spite of the spread of education and information which has been proceeding during the many years since differential fertility became a subject for investigation by scientists. Mr. Grebenik and the Bristol Medical Officer of Health are both to be congratulated, the first on the manner in which he has surmounted the difficulties of measuring the tendency to differential fertility, the second for arranging for the compilation of the records.

Dr. R. R. Kuczynski: The greatest merit of Mr. Grebenik's most valuable paper consists, I think, in showing how much information can be derived from local birth records containing such data as the mother's age and the parity order of the child, and it seems desirable

that similar studies of the past be made for other localities because the vital statistics based on the new Population Statistics Act will tell us very little about fertility prior to 1938. But preference certainly should be given to records, if there are such, which show also the duration of the marriage. Mr. Grebenik rightly suggests that the rise in the specific fertility rates of females under 25 years of age and in the number of first births between 1932 and 1937 was due more or less to an increase in the number of newly-married couples. But the figures he quotes in support of this view are, it seems to me, not convincing. He starts from the numbers of marriages registered in Bristol, assumes that (as in England and Wales in 1936) 44·5 per cent. of all brides marrying in Bristol were under 25 both in 1932 and in 1937 and that one-half of these brides had "their first child within a year or so of marriage," and reaches the conclusion that 740 first births to women under 25 in 1932 and 856 in 1937 were births to newly-married couples. If this were actually so, the number of first births to women under 25 not newly married would have been 130 in 1932 and 211 in 1937, which seems most unlikely. As a matter of fact, the changes in the number of newly-married couples living in Bristol do not necessarily correspond to the changes in the number of marriages registered in Bristol, because many young couples who were married in Bristol may live elsewhere, while many who were married elsewhere may live in Bristol. Moreover, with a large rise in the number of marriages, the proportion of brides under 25 may have increased. Finally, the assumption that one-half of the newly-married women had their first child within a year or so may be far off the mark. Every attempt to estimate the influence of nuptiality upon fertility must, I am afraid, fail without a knowledge of the duration of marriage at birth. The lack of this knowledge unfortunately also reduces the importance of Mr. Grebenik's conclusions regarding differential fertility. It is certainly interesting to learn that the proportion of first and second births was much smaller and the proportion of fifth and later births much larger among the wives of unskilled labourers than among other wives, but what we are particularly anxious to know is whether or not fertility of the wives of unskilled labourers who married in the 1930's is higher than that of the wives of the well-to-do.

The President: Though this is not a field in which I have any special knowledge, I can appreciate the importance of the new class of data that has fortunately become available at Bristol, and the skill and industry with which Mr. Grebenik has developed most interesting results. At an age when most of us were working nervously for our first degree, he has been able to produce an original and finished piece of research,—and the Society can confidently look forward to many valuable contributions from one of its newest recruits.

Mr. Grebenik, in reply: I should like to deal first with Dr. Kuczynski's remark on the necessity for investigating data relating

to fertility in its connection with the duration of marriage. Such data are available in Bristol, and I had planned to investigate them after the completion of the present study. Unfortunately the war prevented this investigation from being undertaken, but I hope to be able to carry it out and thus supplement my paper after the war.

The figures of Bristol marriages that I gave were meant merely as an illustration, and I am aware of their insufficiency for proving a direct connection between the increased number of marriages in 1936 and the rise in the specific fertility rate for ages 20 to 24 in 1937. I tried to indicate, however, that I regarded both these phenomena as being in a large measure due to economic revival, thus pointing to an indirect connection.

As regards the very interesting figures quoted by Mr. Caradog Jones for Liverpool, it is admitted that Liverpool is one of Britain's most fertile towns. Glass places it fifth in the scale of county boroughs in order of 1931 gross reproduction rates. Bristol and Stockport have a slightly lower fertility in 1931 than the average of all county boroughs, but they are probably more typical of urban conditions as a whole than Liverpool. I should therefore regard the Liverpool figures for the proportion of first births as unduly low. But the correct percentage of first births for the country as a whole can, as Mr. Jones says, only be obtained from further studies of different urban areas.

In conclusion, I should like to thank Professor Bowley for his very kind remarks.

THE REPORT OF THE ROYAL COMMISSION ON THE DISTRIBUTION OF THE INDUSTRIAL POPULATION

[A Discussion before the ROYAL STATISTICAL SOCIETY on April 16th, 1940, the PRESIDENT, PROFESSOR A. L. BOWLEY, C.B.E., in the Chair.]

PROFESSOR J. H. JONES, M.A.

I

THE Royal Commission on the Distribution of the Industrial Population was appointed on July 8th, 1937, and its Report was presented to Parliament in January of the present year. The Report consists of three parts: A Majority report signed by the Chairman and nine of the twelve other members, a note of reservation by three of the nine, and a minority report by the three remaining members of the Commission. But the Commission reached nine unanimous conclusions, called objectives. Differences of opinion only emerged when the Commission discussed the methods by which those objectives might be reached; they were concerned with means, not with ends: with policy and machinery, not with the purpose for which they were designed. In opening the present discussion I shall confine myself to the consideration of ends rather than of means. The former constitute the fundamental issue. Unless and until we are agreed about the purpose of action, there is no point in discussing the kind of action that should be taken.

I venture to stress this point. Before we can suggest whether anything should be done in any sphere we must know what is and what we want. The first step is to know the facts, in the full or scientific sense of the word. The second is to separate the inevitable from the controllable or avoidable. That which is inevitable is neither right nor wrong: whether we like it or not we have to accept it, like the rain of Manchester. It raises no moral issue, no question of human responsibility. But that which, like the fog of London, Leeds or Manchester, is not inevitable (or is presumed not to be) raises a moral issue. It challenges our judgment. If it coincides with that which we desire there is clearly no problem to be solved, but if it differs from that which we desire, it presents such a problem.

The Commission reached the unanimous conclusion, not only that a problem existed, but also that certain "objectives" should be sought. I venture to submit that such unanimity constituted a real achievement. The Commission was composed of people from many branches of industrial and professional life, representing different political outlooks and social interests. If anyone had predicted, at the beginning, that so diverse a group could have unanimously

agreed upon " objectives " representing a big advance upon previous public assumptions, he would have been accused of unwarranted optimism. Nor should it be forgotten that the Reports were completed several weeks before the outbreak of war, and long before its imminence could have influenced the Commission's judgment. The conclusions, though now strengthened by the war, were based upon long-term considerations in a world living in a period of uneasy peace.

The Commission was asked " to inquire into the causes which have influenced the present geographical distribution of the industrial population of Great Britain, and the probable direction of any change in that distribution in the future; to consider what social, economic or strategical disadvantages arise from the concentration of industries or of the industrial population in large towns or in particular areas of the country; and to report upon what remedial measures if any should be taken in the national interest." It will immediately be agreed that the terms of reference were extremely wide : it is, indeed, doubtful if any other Royal Commission in modern times was presented with such wide terms. The problem of deciding the scope of the investigation was by no means simple, if a Report was to be presented within a reasonable period. This point is of considerable importance, for it was bound to influence the character of the recommendations. Thus, for example, the Commission could not be expected to embark upon that kind of research which, in the recommendations of all three groups, was to be carried out by the authorities to be set up under the respective schemes.

Two distinct and separate, though not wholly independent, questions appeared to have been submitted to the Commission. The first was concerned with the regional distribution of industry and population: the adjective " geographical," in relation to Great Britain, seemed to indicate the intention of those who drafted the terms of reference, while the discussions in the House of Commons before the Commission was appointed helped to remove lingering doubt. The second question was concerned with the concentration of large numbers of people in relatively small areas. I shall refer to these as the question of location and the question of congestion.

It would be possible to conclude that the location of industry raised no issue of importance and created no problem provided that the problem of congestion could be solved. Given a desirable distribution of industry and population within small areas—that is to say, given well-planned towns, suburbs, and garden cities—it might be argued that the distribution of industry as between different parts of the country, such as the North-west and South-east, represented no more than a phase in the natural economic development of the country, and that any interference with such development would

retard human progress. On the other hand, it would be possible to conclude that the present regional distribution of industry and population would be undesirable even if all towns and villages had been planned in the most desirable way. Those who have examined the " objectives " to which reference has been made will have observed that the Commission unanimously agreed that the present regional distribution of industry was not satisfactory, and that it therefore constituted a problem to be solved. It was to be expected that congestion should be unanimously regarded as a problem.

II

We have seen that the Commission was presented with two tasks, which may now be examined separately. The first (with which I am mainly concerned) was concerned less with statistical facts than with causes. The examination of statistical facts was important, but only as a preliminary to the investigation of causes and of future trends. For the essential facts, relating to the present time, the Commission naturally turned to those Government departments that were likely to possess or to be able to secure information—namely, the departments of the Registrars General, the Board of Trade and the Ministry of Labour. None of these Departments, however, had been required in the past, and as part of their official duties, to investigate the causes of industrial change. The memorandum submitted by the Board of Trade contained a discussion of such causes, but it was a discussion necessarily based, in the main, upon the researches of private investigators.

I have no desire to elaborate truisms. The present industrial structure can only be understood in the light of the past. Information about the past is restricted to that provided by historians, among whom I venture to mention Professor Clapham and Mr. A. Redford. One of the minor problems facing the Commission was to decide how far into previous history it would be necessary or desirable to delve. Readers of the Report will have observed that a very brief analysis of a century of development was presented as a background to the fuller analysis of those changes that took place after the last war.

It requires no profound knowledge of economic history to realise that the industries of a community may be broadly divided into three groups. The location of some industries is determined by factors over which we have little or no control. Every community requires products and services—mainly services—that can only be provided on the spot. Although such industries and services give employment to a large proportion of the total industrial population, they raise

no question of location and call for no further comment. But neither they nor the community could exist without other industries the products of which are "exported" to other communities. The latter constitute the foundation of the industrial structure of the community, be it village, town or region, and are therefore called basic industries. The location of some basic industries is also determined by factors over which we have little or no control. We could not plead the inefficiency of the Clyde as an excuse for building warships in Birmingham, or of South Wales as an excuse for digging for coal in Hyde Park. We know at once, as we watch the navvies in the Park, that they are digging shelters. Local industries, and those basic industries the location of which is determined by nature, constitute the industries in the first group.

The second group consists of those that have grown in certain areas as the result of a combination of circumstances which may slowly change but which, for practical purposes, must be regarded as fixed. Natural factors combined with external economies of a specialized or technical character are so important that the location of such industries must be accepted (except for the most urgent strategic reasons) in any conceivable Government action. No one would be bold enough to suggest, for example, that the Government should remove the cotton industry from Lancashire, the woollen industry from Yorkshire or the steel industry from Lincolnshire or South Wales. The third group of industries consists of those that appear to grow up in places selected for no very obvious technical reason. They continue where they start, and under skilful management flourish wheresoever they may be. If they enjoy external economies, the latter are general rather than technical or specialized. Their location is clearly more under human control than that of industries falling into the first two groups; changes in location may be made with less economic loss—even, it may be, without economic loss.

The distinction that I have drawn between the three groups of industries is obviously very broad, its purpose being merely to distinguish between that which must be accepted as inevitable and that which may be nationally directed, if direction be regarded as desirable. Nor is it necessary, in a gathering of scientists, to emphasize the fact that if, as in coal-mining, the general location of the industry is determined by nature, the active part of the industry within the coalfields is determined by numerous factors the combined effects of which vary from time to time. In short, the regional distribution of industry is determined by comparative costs; the theory of comparative costs in international trade is no more than a theory of trade in general.

If I might venture upon a bold generalization I would submit that, during the nineteenth century, the relative growth, first of one region and then of another, was due to the relative growth of one or more of the basic industries falling into the first two groups—mainly coal-mining, steel production, engineering, shipbuilding and the textile industries. In the early stages of relative growth there was a time lag in the development of local industries. In due course the rate of growth, measured by output, of the specialized industries was reduced to the rate of growth of population in the country as a whole. But the changing technique of production tended to reduce the number of people required to produce a given output, with the result that the rate of growth of employment in the industry within the region fell below the rate of increase in the population. Allowing for the time lag to which I have referred, it is clear that at this stage additional basic industries became necessary if the region was to maintain its growing population; and these were attracted by a combination of natural factors and external economies of a specialized character. The phase of extreme specialization was followed by a phase in which basic industries became more numerous.

The relative growth of the London and Home Counties area after the last war differed from that of most other areas in the nineteenth century. It was due, in the main, to the growth of industries in the third group. But this case differs from those of the nineteenth century in two other important respects. In the first place, the rate of growth of the new specialized industries in the rapidly expanding areas (such as coal-mining in Durham) was usually higher than in the rest of the country. The post-war rate of growth of the expanding industries was not much if at all higher, on the whole, in the London and Home Counties area than in the rest of the country, considered either as a unit or by area. In the second place, nineteenth-century relative growth in one area was not usually associated with intense and persistent depression in other areas. But the post-war relative growth of the London and Home Counties area was associated with deep and persistent depression in the major basic industries of other areas. Thus the relative growth of the former was but the expansion of an area almost filled with expanding industries and relatively free from declining industries, and able to recruit the labour essential to relative growth by the fact that workers in other areas were being driven out by unemployment and the threat of poverty and destitution.

I venture to submit that these differences are material to the issue. The theory of comparative costs, which underlies the location of industry, is closely related to the theory of relative profitability, which determines the industries that are developed at any given time.

In the nineteenth century it dictated coal or shipbuilding or textiles at a given time, so that others remained on the horizon, awaiting their turn. Since the last war there has been no such competition for the available supply of resources of labour and capital. For the expanding industries the competition of other areas has been, relatively, not much, if any, less effective than that of London, in spite of adverse psychological factors. To have prevented the post-war trend of the industrial population it would have been necessary for other areas to offer far greater relative attraction than industries of the third group could be expected to make possible. Their location was largely influenced by external economies of a non-technical or general character. Among these was an adequate supply of labour. Such labour, however, was not already in the London area, where the rate of natural increase was no higher than elsewhere. It could only be provided by immigration, and the necessary scale of immigration was made easy by the depression of the depressed areas. Thus the cost, economic and social—in so far as these terms can be separated—of providing such labour was relatively heavy, although its incidence was not borne by the expanding industries.

III

The Commission unanimously reached the following conclusion: "The continued drift of the industrial population to London and the Home Counties constitutes a social, economic and strategical problem which demands immediate attention." It was also agreed (unanimously) that one of the "objectives of national action" should be the "encouragement of a reasonable balance of industrial development, as far as possible, throughout the country." These and other unanimous conclusions were influenced by "economic, social and strategic" considerations, so that the Commission did not commit itself to the view that its "objectives" could be achieved without some economic loss. Evidence was submitted, upon what I have called the question of congestion, by the Ministry of Health, the Registrars General, representative bodies in the medical profession and other expert bodies and individuals. Some of the evidence was statistical, some dealt with intangibles. It guided the Commission to other unanimous conclusions to which I cannot refer in so brief an introduction, and strengthened the unanimous view that "a Central Authority, national in scope and character, is required" to deal with the problems before the Commission. In so far as the Commission was concerned with problems associated with "town planning," the unanimous recommendation that "national action" should be taken and a "national authority" established will probably be

regarded as representing the greatest step in advance of present practice.

The Commission separated into three groups, who submitted three separate proposals for achieving the agreed ends or "objectives." But none of the schemes was based upon the assumption that the necessary reforms could be introduced without economic loss. It was assumed that the economic loss, if any, would be offset by other gains. The Commission was concerned with human welfare, which includes many things that cannot be reduced to economic statistics or a business balance sheet. Nevertheless it is important to discover, if it can be discovered, whether the agreed ends can be achieved without economic loss, or, if not, how much loss is inevitable. This field of enquiry has hitherto been completely neglected.

In its evidence the Board of Trade stated, in effect, that business men were rational beings and were generally successful. When, therefore, a firm built an establishment in or near London or any other large city, it was reasonable to conclude that it was a favourable site, if not the best that the country could provide. The Board's statement was severely criticized by some who had failed to observe the limits of interpretation indicated by the official witnesses. Assuming the site to be the best from the firm's point of view, it may still be uneconomic if it destroys some of the advantages previously enjoyed by those already in the neighbourhood. The process of decentralization which is already so evident in large cities is itself an indication that as the city grows the relative costs of production at the old sites tend to rise. Moreover, the cost of transfer is itself so heavy that the disadvantages of the old sites are endured for a long period before actual transfer takes place. It would be generally agreed that what appears to be an industrial development of an advantageous character from the point of view of the newcomers may prove an economic loss from the point of view of the community as a whole.

Statistics of national income are of very little value in estimating the true economic value of large-scale industrial development in a given area. Everything is included in the national income because income is measured by the value of the product and has nothing to do with the purpose for which the product is supplied. A redistribution of industry may change the character of the national income, and a change in its character or constituents destroys the value of statistical comparisons in the present field of enquiry, even assuming that the value of money has not been changed by changes in money costs. One of the most striking examples of the danger of using statistics of this type is to be found in conclusions that have been drawn about the relative productivity of London and the Home Counties and the

contribution of the area to the total national income. We have been told by publicists that the Census of Production shows the productivity per head in the London area to be greater than in such places as Lancashire and the North-east, and that therefore any so-called reform that would prejudice further growth of the London area would be injurious to the growth of the national income. But the Census of Production merely shows that the London industries were prosperous and that the basic industries of other areas were depressed.

I submit these illustrations in a short paragraph merely to show that the statistics already available are of very little value in any investigation into the relative advantages of different parts of the country, and that far closer investigation than has yet been undertaken is necessary to discover whether the country would suffer from any economic loss if industry were guided in the direction suggested by the Commission. All that we can say is that the success already achieved, in other parts of the country, by those industries associated with the growth of London and the Home Counties after the last war affords presumptive evidence that industry could be redirected without injury to the individuals primarily concerned and with considerable economic advantage to the community as a whole. Strap-hanging, an inevitable accompaniment of underground transport in London, constitutes part of the measured national income!

Discussion on the Report of the Royal Commission on the Distribution of the Industrial Population

Opened by Professor J. H. Jones, M.A.

Sir Montague Barlow (Chairman of the Royal Commission) said that he would have preferred to be called upon to take part in the discussion at a somewhat later stage, when possibly points of criticism or controversy had developed. First he wanted to congratulate Professor Jones on the outlook adopted in his paper: he had taken the useful course of emphasizing not so much the points of disagreement beloved of controversialists as the points of agreement, which were of immense importance if anything effective was to emerge. It might be right or it might be wrong, but it was the habit of statesmen when harassed by the difficulties that faced them every day, particularly in a national crisis like the present, if there was disagreement in a report of this kind, to fasten upon the disagreement rather than upon the agreement, probably as an excuse for inaction. Therefore he was glad to find that Professor Jones had emphasized two important conclusions of the work of the Commission as a whole.

The first of these was the fact that thirteen gentlemen and ladies of varying outlook and experience, enclosed in a room for the

best part of two years, did arrive unanimously at nine agreed objectives of policy. That in itself was a fact of primary importance. Secondly, Professor Jones had emphasized the agreement of the whole Commission on the necessity of not proceeding piecemeal or by localities in this problem so far as planning was concerned, but of securing a national policy. At the moment there were some 225 planning authorities in London and the Home Counties alone, and it was obviously impossible with such multiplicity of authorities that central planning could proceed on any well-co-ordinated lines.

Professor Jones in his address had analysed with care some of the causes of the growth of London and the Home Counties: it was necessary to throw the net as wide as to include not only the county of London, but also the seven adjacent counties, because the growth in some of the outlying towns—Luton, for instance—had been greater even than in the outer fringe of London—*i.e.*, in what was usually known as Greater London. The magnetic attraction of the London centre drew industry and labour into a concentration with a radius of twenty or thirty miles, not merely a radius of ten or fifteen.

It was seldom realized how great that growth had been. In the 1801 census the area in question—which, by the way, was under 7 per cent. of the total area of Great Britain—had something under 2 million population; in 1937 the population was nearly 12 million. During the sixteen years since 1921 the population of the area had risen by nearly 2 million, or 18 per cent., while the population of Great Britain as a whole had risen by $3\frac{1}{4}$ million, or $7\frac{1}{2}$ per cent. In other words, of the total increase of population in the sixteen years, more than half had taken place in London and the Home Counties.

This was a very remarkable phenomenon. Further, it must be borne in mind that under conditions of modern civilization the big cities had a magnetic attraction; this had been a curious feature during the last hundred years. It was often said that this "draw" to the big town was the result of the cinema, and undoubtedly recreation had some influence. But many other factors were involved—the possibility of advancement in life, better educational opportunities, more elaborate hospital treatment, and so on. This phenomenon occurred not only in this country, where there were seven great concentrations—a word which he preferred to "conurbations"—of over a million, but elsewhere; and in the world to-day there were some fifty or more of these concentrations of over a million, whereas before 1800 there appeared to have been none.

This magnetic attraction was working double time in south-east England. The British problem was immensely complicated as compared with the American; in that great country the Federal and State Governments were dealing with much the same situation as that which faced us in Great Britain, and excellent research work was being carried out by a powerful committee set up by President Roosevelt. A volume called *Our Cities* was recently issued by that Committee and covered much the same ground as the Report of our Royal Commission.

In the case of Great Britain there were causes for the "pull" of London besides the merely economic one. London was the great metropolitan centre, the focus of entrepot trade, the seat of Government, and so on, and all these factors drew people towards it. If this concentration were not checked or directed on other lines by some superior force, it seemed likely to proceed until one-third or one-half of the population of these islands became concentrated in their south-eastern corner. And this was what constituted one of the great differences between the American problem and ours: it had to be borne in mind that the area of this country was very small and densely populated. England and Wales was the most densely populated national area in the world. People often thought of Belgium as more crowded, but while the population of Belgium was 702 to the square mile, that of England and Wales was 762, and of the United States only 36. It was necessary to examine with the greatest care the proper disposal of this valuable national asset in the shape of the land on which we lived, and to see that the claims of population, of industry, and of agriculture were all given their proper weight.

It might be asked whether it was not a good thing to let people crowd into the big cities, leaving a major portion of the land free for cultivation; it had been said that big cities were the price we paid for the retention of some agricultural land. The growth of London as recommended by the regional planning committee would extend into Essex for industrial purposes over some of the finest market-gardening land in the country, and it was very doubtful if this was making the best use of the national resources. Instructed public opinion would, he believed, support the view that if it could be done without serious economic loss and without grave inconvenience, it would be desirable to secure some better distribution of population, with the view at least of preventing the continued accumulation of population in the south-eastern area.

Professor Jones had left at large the question of economic loss which might be involved if the extension of population in London and the Home Counties were controlled, or possibly still more, if an attempt were made, as some of those interested in town planning were advocating, to break up the great London concentration altogether. He was not one of those who thought it was either desirable or possible to break up London into towns of 50,000 people, but the Royal Commission made, in fact, no such suggestion. The Commission did, however, suggest a control of further extension and the question was, supposing such control was imposed on this south-eastern area, was it going to involve economic loss? Professor Jones had rightly said that all three portions of the Royal Commission admitted that some economic loss might be entailed. When Mary MacArthur visited the chainmakers of Cradley Heath she found women working there under miserable conditions, naked to the waist, and earning a wretched wage. On purely economic grounds, at any rate from the point of view of the employers, some justification could doubtless have been urged for that state of things, but it was very doubtful whether, after the Trade Board Acts were

passed, and chain-making under such wretched conditions had been put a stop to, there was any considerable economic loss even to the employers concerned. Equally he was doubtful whether, with reasonable administration, any real economic loss would be involved to employers in the area by the measure of control proposed by the Commission. Certain leading industrialists themselves took the line that some control was desirable. There was also the experience of the Commissioners in the Special Areas, who had been enabled in recent years to direct to a very considerable extent new industries into the depressed areas or their neighbourhood. The action of the Commissioners was largely experimental in character so far, but it seemed there was considerable ground for thinking that Government direction of location could operate, certainly with regard to what were called the light industries, without economic loss being involved. It would always be borne in mind, that if an industry could make out a special case on evidence that it could succeed only within the London area, then permission should be given whereby that industry under certain conditions could start in London. In conclusion, to an audience such as the Royal Statistical Society, the economic issue would bulk largely, but it was of course obvious that the human aspect and the general welfare and development of the nation during the next fifty years must be the care of statesmen as much as economic considerations.

Sir Gwilym Gibbon said that he had a grievance against this paper. He had been hoping to have some friendly fun with his friend Professor Jones over the dogmatism of the reservation to the Royal Commission's report, but he found that the dogmatic Saul had now become the Paul of grace. He agreed entirely with the main conclusion of the paper that more research was urgently required. He had been preaching it for years, especially to planners, and what they did was to appeal to the Lord, and the Lord in this case was the State, and the State delivered unto them a Royal Commission. As to whether the "baby" was worth the prayer, he dared not venture to express an opinion in the presence of the chairman of the Commission.

By research he did not mean just superficial investigation, he meant genuine research, and in this connection he might mention the part of Professor Jones's paper in which he said: "It was assumed that the economic loss, if any, would be offset by other gains. The Commission was concerned with human welfare, which includes many things that cannot be reduced to economic statistics or a business balance sheet. Nevertheless, it is important to discover, if it can be discovered, whether the agreed ends can be achieved without economic loss, or, if not, how much loss is inevitable." Surely, not only important but essential. Before it could be decided whether a national policy was going to result in national good, he would have thought that one wanted at least some pretty good guesses, a fair estimate of what the economic loss, or gain, was going to be. He agreed, of course, that there were other considerations of human welfare, but one could not judge of the

balance sheet without some clear idea of what was to be put down on both sides.

One fault which he found with the paper—not exactly a fault, because he knew that it had to be brief—and also with the report of the Royal Commission and the reservation to it, was that not sufficient account was given to the more recondite factors which concerned the location of industry. For instance, one of the biggest factories in its line in this country was originally started in the back room of a dwelling-house, and he had been assured by the man who started it that it grew because he was able to put his hand in the district on exactly the type of workmen and the type of materials which were essential for its growth. That was only one illustration, but considerations of this kind applied almost all over the country, except in the case of those industries which were determined closely by geographical conditions. He presumed that Professor Jones intended to include that class of consideration among the special or technical factors to which he referred in his paper. It was a far more important factor in the welfare of the country than was generally realized, as also, and even more so, was the necessity of doing everything to encourage personal initiative and enterprise. That was how the British industrial system had grown, and that more than anything else was the source of the prosperity that had come to this country. Too much control would damp down these qualities.

It was all very well to say "Set up a board," but did anyone in that room think that, if there were a board for regulating in some detail the location of industry, it would always decide strictly on the merits of national welfare? They knew perfectly well—they had had examples enough in recent years from the special areas themselves—that other considerations would come into operation, including some which might be good in themselves but which did not necessarily have regard to national welfare as a whole. Some foolish things had been done in the special areas already, and there was a danger that the pressure of interests of various kinds—this far more than any individual pressure—might influence the decision of a board, as against the only issue which should matter—namely, the national welfare.

Professor Jones said in one place that no one would think of moving the wool industry from South Yorkshire. That might be, but it was fairly sure that, if a controlling board with wide powers were set up, the inertia and pull of local interests would hinder industries from being moved from unsuitable to suitable areas. Any Government Board was likely in this matter to lag behind the necessary changes. The *tempo* of change had increased and was increasing with every decade, and it was important that action should not lag behind more than could possibly be avoided.

As one who had had a good deal to do with planning, he wished to emphasize another point which was not brought out very clearly in any of the reports except that of the minority. The location of industry could not be determined to the best advantage except as part of a general plan. He must also say something in particular

about Greater London. Of course, London was a "mess." He knew of no big town which was not a mess. But there was nothing magnetic, magical, or mystical about the growth of Greater London. The growth in the main was due primarily to the fact—there were, of course, other considerations, including the enormous growth of Government departments in recent years—that the manufacture of consumption-goods now played a much bigger part in the national economy, and for these there was in London a tremendous market on the doorstep. That was the main cause, there was nothing mysterious about it, and it was an inevitable economic process. He himself was country-bred, and was still a countryman in spirit, but he refused to be intimidated by fear of the growth of big cities, by the "metrophobia" (if he might coin the word) from which many of his friends suffered.

The real fault in London was much less its growth (though he would like to see it curtailed) than its lamentable lack of planning, which was a disgrace to us all. In his own little way he had tried to get some body which would look at the problem as a whole. It was grievous that in this country we could get, at the best, a few thousand pounds from the various bodies to investigate this problem, whereas from a private source New York got £200,000. It was planning that was most needed in London and the area round it, and he would add, to remove a misapprehension which generally prevailed, not simply the planning of the outer area, whether that part which had already been developed or that which might be developed in the future, but also the replanning of the inner core of London, which was even more important from the general standpoint. All that the Royal Commission had done in that respect had been to "pass the buck," and he did not blame them for that, because the underlying problem was that of compensation.

If he might state in a few words the conclusions which he would wish to draw from the whole business they would be these. More central guidance and help in planning was needed. It was generally not advantageous or desirable that there should be state national planning in detail, except in some matters which could be seen and dealt with properly only from the national standpoint. In most matters the big problems of planning should be settled and applied for regions adequate in size for this purpose, with due regard to local needs. And he would urge that we should stop talking so much nonsense about the location of industry and try to find out what did in fact govern location and what were the factors which did really count on the profit and loss side of the national balance sheet in this matter. There was urgent need for more thorough research.

MR. ROY GLENDAY said that he thought the work of the Royal Commission should be welcomed as the first official step taken in this country to face up seriously to the problem of the distribution of population. Those who had hoped for a scientific study of the problem were bound, of course, to feel disappointed. But this should not to be put to the blame of the Royal Commission, which had carried out a very valuable piece of pioneer work.

Neither its personnel nor its terms of reference really gave ground for expecting a scientific report. Indeed, its terms of reference were in effect self-contradictory. The Commission was appointed to enquire "what social, economic or strategical disadvantages arise from the concentration of industries or of the industrial population in large towns or in particular areas of the country." Strategical considerations meant war conditions. Now, the organization of our industries and population for war involved a deliberate and direct reversal of the tendencies which the Commission had found operating under conditions of peace, and to which they might be said to have objected. Population concentrations were being evacuated from the large towns; such occupations as building, public utilities, distributive trades and urban services which had expanded by nearly 70 per cent. over the past fifteen years were losing men; while coal-mining, shipbuilding and repairing and engineering, which during the same period had decreased by some 25 per cent., were urgently in need of more labour, with the result that everything possible had to be done to induce workers from the former set of occupations to migrate to them. The tragedy was that after the war, if peace planning was going to be different from war planning, a good deal of this redistribution would have to be undone. His second criticism of the terms of reference was that, though wide, they were not wide enough. They involved halting, as it were, in midstream in the consideration of what, after all, was the real problem that was troubling everyone to-day—the industrial future of this country, especially from the employment aspect. Mr. Glenday felt that it had been this wider consideration which had really weighed with the members of the Royal Commission in arriving at their recommendations. The underlying objective was to provide employment and a worth-while existence for the unemployed and infuse new life into the distressed areas where so many of them were still located. Unfortunatély the problem was not one that was confined to this country, but world-wide. It was inherent in the modern system of economic development to which the world had pinned its faith for the past hundred and fifty years or so.

Mr. Glenday wished to congratulate Professor Jones on what must have been a very courageous act of renunciation in throwing over his belief in the traditional theory of international trade. If only other economists would follow him and face realities, then economic theory might at long last take a great step forward. But he could not help wondering whether Professor Jones realized that when he committed himself to an out-and-out policy of industrial diversification as a solution for the unemployment *within* this country, he was at the same time justifying the policies of national self-sufficiency which were being advocated as a solution for the international unemployment problem *outside* this country. Mr. Glenday said that there were a number of questions of detail which he would have liked to raise had time permitted, such as, to enquire, for example, what scientific justification there was for including the Home Counties with London? He would, however, confine himself

to a general observation regarding the suitability of the traditional method of economic analysis for dealing with the problem of urban development.

When a town and its planning were under consideration he felt that if one was to understand what was happening one must deal with the history of the town as a "whole" and consider changes which took place in the town structure, also as a "whole." To break up the pattern into its constituents was to lose the substance for the shadow. To attempt, for example, to use the doctrine of "comparative cost" to relate conditions before and after such revolutionary developments as the introduction of the motor-car and cheap electric supply, which had caused a sudden and *discontinuous* change in industrial mobility and flexibility, was to misconceive the situation. He agreed with the last speaker that there was no mystery in the recent growth of large towns. The explanation was that the direction of industrial development had changed, and also that the motor-car which had assisted in that change had created new urban occupations. Before the war of 1914–18, export trade had been the major preoccupation, since that war the centre of industrial development had shifted to domestic consumption and the problem of raising the standard of living of the poorer sections of the community. Naturally the industries and services required for this latter purpose had developed in the areas where the consumers were located. It was no good providing motor-coach services, chain stores, catering facilities and the like in the distressed areas of the North of England for the use of people living in, say, London. This did not mean that he personally was not wholeheartedly in favour of measures, however drastic, being taken to restore prosperity to the distressed areas. Even the recommendations of the minority reports were too tame for him in this respect.

But the question which everyone appeared to shirk—including the Royal Commission—was: What are the new industries which can restore life to the distressed areas? Professor Jones's classification of industries did little more than obscure the issue, and the report of the Commission was equally vague. Mr. Glenday said that he had examined in a general way the recent development of London and the new estates which had been set up in the distressed areas and had found no answer to the question, provided re-armament industries were excluded from the solution.

The same question existed in the international field. The traditional overseas agricultural and mining areas were rapidly industrializing, while the older industrial countries were finding it increasingly difficult to replace fast enough by new industries those which were losing markets as a result. Mr. Glenday's own personal conclusion was that this and many other countries must be prepared to face revolutionary changes in their traditional ways of living before the problem could be solved.

Sir Duncan Wilson said that he had some hesitation in speaking because all that he knew of the subject had been derived from a

recent perusal of the Royal Commission's report, which he thought full of interest and wise deductions. But he knew from past experience that the Society tolerated, and even appeared to welcome as a kind of light relief, the comments of people who knew practically nothing about the subject under discussion. In his past official life he had been concerned mainly with conditions of work in factories, and he had been rather surprised at being unable to discover any reference to this in the report. There was a section dealing with health, in which a description was given of bad housing conditions, sanitation, and so forth, in the early part of the last century, but no mention was made of the conditions under which many men, women and children passed almost half their lifetimes, and which might have had as great an effect upon them as the other factors with which the Commission had dealt. It might be said that this sphere was irrelevant to the inquiry and was already governed by factory legislation, and that the conditions in a factory would be much the same wherever it was located. But he ventured to think that there were certain aspects of factory life and amenity which had a direct bearing on the questions at issue. He would give two examples.

The Royal Commission in its Report had pointed out that in the expansion of London the factories which had sprung up were generally well planned, well managed, and comfortable to work in. They suffered, however, in his view, from certain disadvantages arising from their location. In the first place, they drew their workers from such various and distant parts of London that the employer was completely out of touch with them, excepting during their working hours. He could have no interest in their leisure. This was in marked contrast to the conditions in some industrial towns which were much more homogeneously industrialized.

The second point was that these factories were as a rule small. They could not offer the same amenities to their workers as the large ones. By amenities he meant recreation, facilities for meals, medical supervision, and so on. The obvious remedy was for a number of factories to pool together in order to offer such amenities in common; and yet, apart from some of the Trading Estates, mentioned in the Report, no attempt—or at least no successful attempt—in this direction appeared to have been made. Again, he remembered that some years ago in order to relieve traffic congestion in a certain part of London, the suggestion was made that the factories concerned should "stagger" their hours of work and meal-times, but without any effect. The managements of neighbouring factories in some parts of London seemed in fact to be as shy of collaboration as neighbouring householders in a London square.

He had ventured to mention these matters because he thought that the conditions of work inside the factory would be an important subject to take into account in any future action that might be decided upon.

MR. G. L. SCHWARTZ said that he wished to deal with an economic point that arose out of the discussion. He thought that

they could leave on one side the strategic aspect, because if they had to distribute the population after this war from strategical considerations, sensible people would clear out altogether.

The point he wished to discuss was the economic argument that people should be prevented from moving for their own advantage because of a possible net social loss that might arise from that movement. The favourite example of that was the argument that houses, schools, and so forth might be left derelict in distressed areas and have to be duplicated in other places, and that the people who moved had not to take that into consideration. That was a very specious argument and, he thought, wholly incorrect.

If he might take a personal example, the Chairman of that meeting had been "guilty" of a good deal of movement in his time. To the speaker's own knowledge he had inhabited at least four different towns, and every time he had moved it might be that he had left the local sewers, roads, and so forth to their fate, and had also increased the congestion in the place to which he had gone. Was it suggested that he should have compensated the people in the town which he had left, or that he should apologize for the congestion which he created in the town to which he went? Could it be argued that a movement which was of an advantageous character from the point of view of himself and the people who induced him to migrate could possibly have been an economic loss from the point of view of the community as a whole? Was there any case for preventing Professor Bowley from moving from one town to another in this way? Was there any reason why he should pay compensation for supposed loss that might have arisen in the place he had vacated? To give another personal example, the doctor had just cut him (the speaker) off from alcohol, and he felt very sick about it, but he would feel more sick still if he had to compensate the local brewer for the loss of his custom. It might just as well be argued that the migration of millions of people from Europe to the United States was a net social loss to the world because sewers and roads had to be duplicated in the United States, whereas they were abandoned in Europe. He saw no validity at all in these arguments.

Of course, people always objected to the congestion caused by others. One heard continually that London was too crowded, but nobody was prepared on that account to clear out of it himself. They always objected to the other man getting into the railway carriage; they might one day tell him that his presence there represented a net economic or social loss and see what happened.

Professor Jones concluded his paper with the remark "Strap-hanging, an inevitable accompaniment of underground transport in London, constituted part of the measured national income." That was a clever way of putting it, but a distorted way. People came to London after weighing up the balance of advantages and dis-advantages. One of the advantages of being in a big city was the enjoyment of its ordinary amenities. One of the disadvantages was that one might have to straphang on the way into the centre. One of the advantages of being in London was that one could enjoy the

meetings of the Royal Statistical Society. One of the disadvantages was that one might get a headache in listening to the discussions. But if one was perfectly free to decide whether to go or not, and did go, one could say that there had been a net addition to the national income, and there was no need to point to the headache and say that that must inevitably result in a national loss.

MR. DUDLEY WALTON said that there were in this country seven aggregates of population of the same general quality, social and economical, as the aggregation of London and the Home Counties, and it seemed to him that the problems and disadvantages in respect of London were similar to those in the other six. Each of those other six aggregations had grown up historically under known conditions. London was the centre of government and also the centre of distribution of the national income. If one took south-east England as an area, it was the district in which the bulk of the national revenue was expended.

A development since the last war had been the discovery by manufacturers that it was more advantageous to produce commodities in the centre of a purchasing district than at a place where they could be more easily and economically manufactured. Factories had been built all around London, where commodities had a market, and from which they could be more easily distributed than from, say, the coalfields. Those two considerations seemed to him to account for a great deal of the congestion of south-east England. There were, of course, various illusions when one applied statistics to this kind of problem. If one drew a circle of eighty miles radius from Leeds it would include 50 per cent. of the entire population of England, and one could take other parts of the map and draw circles which would cover 50 per cent. of the population. The arguments derived from congestion could be applied to a great many different centres.

DR. EDELBERG said that he could not pretend to have studied the question of location from a practical point of view, but he was rather struck by the absence from this paper—and the same thing applied to the report of the Royal Commission—of very much attempt to present the economic side of the problem analytically and to analyse the balance of forces tending to a concentration and of the opposite, or centrifugal, forces which tended to produce the other effect.

In this connection, when the question was considered of what might conceivably be done to direct the location of industries and population, it was important to remember that the relevant thing was not the present position of each industry or population unit, but how it was likely to develop in the future. He thought that on considering the problem analytically it was obvious that the relevant thing was the trend. Was there an upward trend, and was it safe to extrapolate it? It seemed to have been assumed in the paper that there was an upward trend, and further that it could be extrapolated safely.

He thought it possible that in the post-war period there was an upward trend of concentration around London, which need not continue with the same force as previously, and that that trend was caused by the decline of the export industries, which depended on factors of location, such as coal mines and so forth. But as this kind of factor was not going to operate in future there was no need to fear quite the same amount or speed of concentration as in the past, and it was possible that the problem of the alleged dangers of concentration around London was exaggerated by the Commissioners because it was conditioned in their minds, perhaps unconsciously, by the experience of the last post-war period. It was well to remember that there were forces working in the opposite direction as well as forces in the direction of concentration, and it was the balance of the two that mattered. They should be able to picture in their minds the probable course of development if the process was left to itself, and only then could they see what they should do about it if any action were considered necessary. The danger was in exaggerating the upward trend because of the peculiar post-war experience, and it was also well to bear in mind that there were forces working in the opposite direction.

Professor Jones said that he did not intend to attempt to reply in detail, particularly to Sir Gwilym Gibbon, Mr. Schwartz, and Dr. Edelberg. He would merely desire to put this general point. He was quite aware of the apparent dilemma to which Mr. Glenday referred, and, like the rest of the Royal Commission, he was aware of the special issues raised by Mr. Schwartz. He himself deprecated the underlying assumption of a good deal of present-day economic discussion that " whatever is, is right " and that if one could extrapolate—to use Dr. Edelberg's term—then one would have to allow for balancing forces. He would merely say as a general reply to those points of theory that a very detailed statistical analysis was carried on behind the scenes. Not only was there a trend towards London of a character that, rightly or wrongly, was deemed undesirable by the Commission as a whole, but also, as far as the Commission could discern—they were not economic astrologists, and that was what extrapolation meant—there was a real danger that the trend would not only continue but even accelerate. The acceleration was envisaged as of such extent that having regard to the social consequences of such a trend—he used the word " social " in the broadest possible sense—it would be positively dangerous to the community as a whole not to try to prevent it. If there were balancing forces, these would come out. If the balancing forces were inherent in the economic structure no authority which was set up would interfere with them, presuming that they were in the right direction, as balancing forces ought naturally to be.

He would add one point in reply to what Mr. Schwartz had said. He was not sure whether Mr. Schwartz was criticizing anything he himself had said or anything said by people other than those who appeared as witnesses before the Commission or by the members of the Commission themselves. He did not think he himself had ever

suggested that it was a bad thing for—should he say?—an aged community, speaking in terms of houses and roads and sewers, to disappear. There was nothing in what he had said that afternoon, nothing in what he was aware of having signed, that implied such a view. He knew it was true that some people stated that one of the costs to the community of migration was the loss of the public services in the declining area. He himself thought it would be a great social gain if they destroyed the public services in some of the Welsh towns and mining districts and rebuilt them either there or in some other place. He desired to remove the misapprehension from the mind of Mr. Schwartz, because nothing that he had said was intended to convey what Mr. Schwartz had inferred. There was one point in which Mr. Schwartz had misread what he had written. The question he had posed was not whether it was good for the man who was actually moving, but whether it was good for the other people whom he joined. The balancing of individual and social welfare was not the balancing of the individual interest and the wider interest of the same individual, but a balancing of the individual interest of the man who went to the large town and the effect of his going upon the people who already lived in the town. He thought he had tried to bring that out in the introduction to the discussion in which he had said that people suffered a very long time before they took active steps even to restore the economic loss they had sustained through the concentration of industry.

He had wanted to put these points right at the beginning, and he might perhaps be allowed to refer to Mr. Glenday's comments in a written reply.

In a written communication PROFESSOR JONES continued :—

I wish merely to refer to one comment made by Mr. Glenday. In supporting the view that greater regional diversification of industry was advantageous I was keenly aware of the apparent dilemma to which he referred, and I dealt with it, individually, in the memorandum published as an Appendix to the Report. In view of the fact that an adequate reply would necessitate a long and highly theoretical discussion, I must rest content with the statement that I made no conscious "act of renunciation in throwing over" my "belief in the traditional theory of international trade." In relation to the practical application of the so-called doctrine of comparative costs, I would venture to refer to the discussion in the Appendix, where, in effect, I try to show that comparative costs are materially affected by external economies of a special and general character. Past experience suggests that in a highly specialized area external economies of a general character tend to be below the average. The result is that new industries fight shy of the area, so that the people surplus to the main industry migrate to other areas. Moreover, under diminishing returns the main industry may be pushed farther than would be the case if the evidence of general economies of a high level were to affect comparative costs. This extreme specialization may represent a failure to make the fullest use of resources, while diversity of the type I indicated

bears no resemblance to economic nationalism. There remains a further practical issue. If, irrespective of its merits, protection is, in fact, adopted by a Government, it is impossible for that Government to ignore its regional effects. The argument for regional subsidies to reduce injury in the areas previously favoured by free trade is of the same type as the argument for national protection. Finally, it is nowhere stated that diversification is a cure for unemployment.

As a result of the ballot taken during the meeting, the 48 candidates named below were elected Fellows of the Society :—

Neil Geoffrey Calvert, A.I.A.; Alfred Charles Edward Caplen; Harry Ryde Barnes, B.Sc.; George Albert Bauer, B.A.; Mrs. Chameli Bose; William Reed Bowles, F.C.W.A.; A.C.I.S.: Annie Henrietta Chevis; Leonard Dodworth Coe; A. Noel Curphey; Alfred Rudolph D'Abreo; Goronwy Hopcyn Daniel, D.Phil.; Stanley Harold George Edmonds; Gordon Quested Evison, B.Com.; Wilfred John Foster; Harold Thomas Edmonds Gambrell; Durgadas Ganguli, B.Sc.; Walter John Goshawke, A.I.A.; David Henry Gustavus Hamilton-Russell; John Hyslop Hamilton; Sydney Charles Harris, B.Com.; Harry Henry, B.Sc.; Geoffrey Heywood, F.F.A.; Arthur Hazel Lionel Johnson; Ahmad Khan, Ph.D.; Wilfred Ernest King; John Dennis Kuipers, B.A.; Maxwell Lander; Kaethe K. Liepmann, Dr.Phil.; William Henry Nathan Loveday; George Donald Alastair MacDougall; Vicente Mills; Henry Peat; V. N. Poornapregna, M.A.; Norman William Ross, F.I.A.; I. Rubin; Martin Rudd; Professor M. Schollenberg-Orloff; Jesse Scott; Dhrubjyoti Sengupta, M.A.; David Montagu de Silva, M.R.C.S., L.R.C.P., D.T.M., D.P.H.; Thomas Hilary Sinclair; Jaggit Singh, M.A.; Erika Anitz Spitzer, M.Sc. (Prague); Clarence Reginald Thompson; John Edward Wall.

Corporate Representatives.

Joan Ching, *representing* Messrs. Buckmaster and Moore.

James Lythgoe, *representing* the Corporation of Manchester.

John William Neubert, *representing* the International Sugar Council.

Wholesale Prices in 1939

By the Editor of "The Statist"

(*The Statist's* Index Numbers in continuation of Mr. A. Sauerbeck's figures)

The Sauerbeck-*Statist* annual index numbers of wholesale prices are set out in the following table. Annual averages are shown for every year since 1846 (*i.e.* from the beginning of Mr. Sauerbeck's calculations) and Jevons's figures for the years 1810 and 1818, adjusted to Sauerbeck's standard, are also included. These all-commodities index numbers embrace forty-five commodities and are calculated, with few exceptions, from the average of fifty-two weekly quotations for each commodity, the averages for the standard period 1867–77 being taken as 100. Up to the end of 1912 the compilation of the statistics was undertaken by Mr. Augustus Sauerbeck and since that date by *The Statist*.

Table I

The Statist's *Annual Index Numbers* (*in continuation of Sauerbeck's figures*)
(1867–77 = 100)

Year.	Average No.	Year.	Average No.	Year.	Average No.	Year.	Average No.	Year.	Average No.
1939	95	1919	206	1900	75	1881	85	1862	101
'38	91	'18	192	1899	68	'80	88	'61	98
'37	102	'17	175	'98	64	'79	83	'60	99
'36	89	'16	136	'97	62	'78	87	1859	94
'35	84	'15	108	'96	61	'77	94	'58	91
'34	82	'14	85	'95	62	'76	95	'57	105
'33	79	'13	85	'94	63	'75	96	'56	101
'32	80	'12	85	'93	68	'74	102	'55	101
'31	83	'11	80	'92	68	'73	111	'54	102
'30	97	'10	78	'91	72	'72	109	'53	95
'29	115	1909	74	'90	72	'71	100	'52	78
'28	120	'08	73	1889	72	'70	96	'51	75
'27	122	'07	80	'88	70	1869	98	'50	77
'26	126	'06	77	'87	68	'68	99	1849	74
'25	136	'05	72	'86	69	'67	100	'48	78
'24	139	'04	70	'85	72	'66	102	'47	95
'23	129	'03	69	'84	76	'65	101	'46	89
'22	131	'02	69	'83	82	'64	105	'18	159*
'21	155	'01	70	'82	84	'63	103	'10	171*
'20	251								

* Jevons's numbers adjusted.

For the year 1939 the annual index number is 95, which represents a rise of 4·3 per cent. upon the 1938 figure of 91 and a fall of nearly 7 per cent. from the 1937 level, expressed by the index number 102. This reversal of the downward trend in the two previous years arose from the price conditions prevailing in the last four months of the year, the average for the eight-month period January–August, at 89·6, having shown a continuation of that trend. With the outbreak of war, the rise in prices anticipated, in the event of war, in our review of 1938 prices, immediately manifested itself, the September index-number rising slightly more than 10 per cent. over that for August. For the three remaining months of the year, the rise proceeded at a slower rate, the indices for October, November and December showing increases over those for preceding months of 6, 6 and 7 per cent. respectively, bringing the December index to 120·1, the highest for that month since 1927.

TABLE II

THE STATIST'S *Annual Index Numbers—ten-year averages* (1867–77)

1838–1847 = 93	1900–1909 = 73	1916–1925 = 165
'48– '57 = 89	'01– '10 = 73	'17– '26 = 164
'58– '67 = 99	'02– '11 = 74	'18– '27 = 159
'68– '77 = 100	'03– '12 = 76	'19– '28 = 152
'78– '87 = 79	'04– '13 = 77	'20– '29 = 142
'88– '97 = 67	'05– '14 = 79	'21– '30 = 127
'90– '99 = 66	'06– '15 = 82	'22– '31 = 120
'91–1900 = 66	'07– '16 = 88	'23– '32 = 115
'92– '01 = 66	'08– '17 = 98	'24– '33 = 110
'93– '02 = 66	'09– '18 = 110	'25– '34 = 104
'94– '03 = 66	'10– '19 = 123	'26– '35 = 99
'95– '04 = 67	'11– '20 = 146	'27– '36 = 95
'96– '05 = 68	'12– '21 = 148	'28– '37 = 93
'97– '06 = 70	'13– '22 = 153	'29– '38 = 90
'98– '07 = 71	'14– '23 = 157	'30– '39 = 88
'99– '08 = 72	'15– '24 = 162	

The annual increase of 4·3 per cent., noted above, was spread unevenly over the principal categories of commodities entering into the general index. For all foods, indeed, the 1939 index was 1 per cent. lower than in the previous year, a fall of 8·6 per cent. in the annual level of vegetable food outweighing the rises which affected animal foodstuffs and the plantation foods, amounting to 3·6 and 9·3 per cent. respectively. For all materials, on the other hand, the yearly index number showed a rise of 7·3 per cent., resulting mainly from the 24 per cent. increase in textile prices, which overshadowed the relatively slight increases in the cases of minerals—the price level of which rose by less than 1 per cent.—and of sundry materials, for which the index rose by 3·5 per cent. over the 1938 level.

These comparisons do not, however, provide sufficient clue to the price history of the past year, which is more clearly indicated by the table given below, relating to quarterly movements of prices. The movements reflected in that table are more easily appreciated from the following table, which gives, for each quarter of 1939, the percentage evaluation of the principal group indices on the basis of the corresponding period in the preceding year.

Quarterly Movements—1939 *Index Numbers as Percentages of those of* 1938

Year	Quarters	Vegetable Food (Corn, etc.)	Animal Food (Meat, etc.)	Sugar, Coffee, and Tea	Total Food	Minerals	Textiles	Sundry Materials	Total Materials	Grand Total	Silver
1939	I	73·7	95·3	101	86·7	99·6	102	91	96·8	92·8	95·7
	II	74·8	99·0	111	90	102	116	98·6	104	98·4	99·6
	III	83·6	109	112	99·2	100	123	103·5	107	104	91
	IV	135	123	133	129	104	166	125	127	128	102

Here is plainly to be seen the unevenness of price developments between the main groups of commodities, particularly in regard to the third quarter, which included the first month of war. For this quarter the total food index still remained below that for the same three months of 1938 while total materials had risen by 7 per cent., a notable increase in respect of textiles—which had already made itself felt in the second quarter—being damped down by the equally notable consistency of mineral prices, which remained at the same level as for 1938. By the end of the fourth quarter a different picture is presented. The "total food" index has risen slightly more, in proportion to October-December 1938, than "total materials," vegetable foods, which had remained the least active of all the principal groups, having jumped from 84 per cent. of 1938 level in the third quarter to 135 per cent. in the fourth. Equally noticeable are the soaring tendency of textile prices and the continued quietness of minerals.

While it is too early to attempt to elucidate the effects on prices of the various commodity controls imposed by the Government from the outbreak of war, it is necessary to refer to the fact of those controls and to the uncertainties created by them in the field of prices. During the last quarter of 1939 the quoted prices of certain commodities became metaphysical concepts rather than definite market indicators and no student who approaches this problem in the future should overlook the fact that some of the prices he will

be handling have no reference whatever to anything approaching market conditions. For obvious reasons the c.i.f. price of American wheat disappeared after August 1939 and from September onwards statistical irregularity has been caused by the use of the f.o.b Montreal price in its place. Tea and sugar prices have also been sources of difficulty, and it will be noted that in the detailed tables below relating to the average annual prices of individual commodities, the quotation for common Congou tea applies to the first nine months of the year only.

In view of the circumstances in which this brief review is prepared, it may be permissible to look beyond the period to which it properly relates and to remark that the upward movement observed in the last four months of 1939 has since continued to a point at which the general index for May 1940, stands at over 41 per cent. higher than in August 1939. This continuance has not been an uninterrupted one. In no one of the first five months of 1940 has the proportional rise in the general index been so great as it was in any one of the war months of 1939. In February it amounted to less than 0·5 per cent., while in the following month a drop of 1 per cent. on the February index was recorded, a movement shared by all principal groups save minerals. By May the greatest increase shown by the group indices, compared with a year previously, was that for textiles, amounting to 76 per cent., followed by vegetable food, with a 57 per cent. rise on the year. The "all foodstuffs" index rose in the same period by 36 per cent. and total materials by 44 per cent.

Monthly Fluctuations of the Index Numbers of* 45 *Commodities*, 1867–77 = 100

	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Year
1888	70·9	70·6	69·9	69·8	68·1	67·4	69·0	70·1	71·9	72·4	72·7	73·2	70
1896	61·4	61·4	60·7	60·3	60·1	59·3	59·2	59·7	61·2	62·6	62·6	62·0	61
'97	62·0	61·9	61·9	61·5	61·2	61·3	61·7	63·2	63·4	62·7	62·4	62·4	62
'98	62·8	63·4	63·0	65·5	66·4	64·7	64·3	64·0	63·9	63·6	63·9	63·8	64
1899	65·4	65·8	65·6	66·1	66·6	66·9	67·9	68·3	70·0	71·5	71·6	72·3	68
1900	74·0	75·1	75·7	75·6	75·5	75·7	76·2	76·0	75·5	74·7	73·9	73·4	75
'01	72·2	71·7	71·0	70·6	70·5	69·8	69·5	69·8	69·6	69·6	69·0	68·4	70
'02	68·8	68·9	69·2	69·7	70·9	70·4	70·0	69·5	69·3	68·8	68·6	69·1	69
'03	69·5	70·2	70·4	69·4	69·6	69·5	69·5	70·0	69·1	69·0	69·0	70·0	69
1904	70·4	70·8	70·8	70·5	69·9	69·4	69·9	70·4	70·7	71·0	71·2	70·9	70
'05	71·2	71·4	71·8	72·0	71·7	72·0	72·5	72·3	72·4	73·2	74·2	74·9	72
'06	75·2	75·0	75·7	76·5	77·0	76·9	76·4	76·7	77·5	78·5	78·6	79·7	77
'07	80·0	80·7	80·0	80·7	82·4	82·0	81·1	79·4	79·1	78·8	76·7	76·2	80
'08	76·0	74·5	74·1	73·8	73·6	72·9	73·1	72·2	72·5	72·2	72·2	72·3	73
1909	72·0	71·9	72·4	74·3	75·4	75·1	75·2	74·9	74·7	75·2	75·5	76·3	74
'10	77·1	78·1	79·1	78·5	78·2	76·9	78·1	78·2	77·6	77·2	77·8	77·9	78
'11	78·5	78·6	78·9	80·0	80·3	80·0	78·9	79·5	80·3	80·7	80·6	80·9	80
'12	81·8	82·9	84·4	85·0	85·3	85·5	86·5	85·9	86·7	85·8	85·3	86·4	85
'13	86·4	86·1	86·7	86·2	85·7	84·1	84·2	85·0	85·7	84·5	83·3	83·8	85
1914	83·5	83·8	82·8	82·3	82·3	81·2	82·4	87·9	89·3	89·8	88·8	91·6	85
'15	96·4	100·9	103·7	105·9	107·2	106·4	106·4	107·0	107·8	110·0	113·1	118·4	108
'16	123·6	127·0	130·4	134·2	135·4	131·0	130·5	134·5	134·4	141·5	150·8	154·3	136
'17	159·3	164·0	169·0	173·0	175·0	180·4	176·9	175·7	176·4	180·6	182·9	185·1	175
'18	186·2	187·3	188·0	189·8	191·1	192·3	192·9	195·9	197·1	197·8	195·3	196·0	192
1919	192·1	187·5	184·7	184·6	194·6	199·4	206·4	212·7	214·8	224·3	231·0	235·2	206
'20	245·3	260·4	261·8	266·1	260·0	255·7	254·6	253·5	248·7	239·9	223·8	207·2	251
'21	197·2	183·0	177·2	169·8	162·2	155·8	158·2	154·3	149·4	138·4	136·7	133·6	155
'22	132·5	132·2	133·3	134·8	135·5	135·6	134·0	129·6	127·9	130·1	130·6	129·1	131
'23	130·2	131·9	132·7	134·0	132·2	127·9	124·8	125·0	127·8	127·7	132·4	133·2	129
1924	137·2	138·8	137·0	136·8	136·4	136·3	138·4	138·0	141·3	146·1	145·5	147·7	139
'25	144·8	143·1	140·1	137·5	135·7	131·2	134·3	134·3	132·7	130·2	132·9	130·4	136
'26	129·3	127·9	126·1	125·5	125·7	124·9	126·0	127·0	128·0	131·0	130·8	123·9	126
'27	123·1	124·1	123·6	123·3	123·8	123·1	122·0	122·8	121·5	120·6	121·5	121·4	122
'28	120·9	121·1	123·6	125·6	126·2	122·6	120·3	118·0	116·8	116·8	117·9	117·9	120
1929	117·0	120·1	120·5	116·5	113·0	113·1	115·2	113·9	112·6	111·1	108·3	108·8	115
'30	106·6	104·8	103·0	101·5	98·8	95·8	94·4	92·2	90·8	90·4	88·6	86·9	97
'31	85·7	85·5	85·5	84·4	82·2	82·6	80·2	79·1	80·7	82·3	83·0	85·4	83
'32	84·7	86·7	84·1	82·5	80·2	77·0	78·9	80·7	80·4	77·8	77·9	77·7	80
'33	77·8	77·0	77·0	78·5	80·9	81·3	81·7	81·2	80·7	80·5	79·3	80·0	79
1934	82·5	82·5	82·2	81·0	81·1	80·7	82·4	83·4	82·1	81·1	81·0	82·8	82
'35	83·6	83·4	82·9	84·1	85·2	83·7	84·3	84·1	85·1	85·8	86·3	86·7	84
'36	87·1	87·1	86·7	86·2	85·6	84·8	87·1	89·0	90·4	91·7	94·5	98·9	89
'37	99·6	102·1	107·3	104·7	106·2	104·7	105·9	104·4	103·3	100·8	96·7	97·3	102
'38	96·5	96·4	94·2	93·5	91·4	91·4	91·1	88·6	88·6	88·8	87·4	89·1	91
'39	88·7	88·6	89·0	90·5	90·6	90·6	88·7	90·4	99·7	105·8	112·2	120·1	95
'40	124·1	124·5	123·4	126·0	128·0								

* The average of the twelve monthly figures of each year does not necessarily coincide with the annual figures, as the latter are calculated mostly from the average of 52 weekly quotations, while the former are based on end-of-the-month prices.

Summary of Index Numbers. Groups of Articles, 1867–77 = 100

	Vegetable Food (Corn, etc.)	Animal Food (Meat, etc.)	Sugar, Coffee, and Tea	Total Food	Minerals	Textiles	Sundry Materials	Total Materials	Grand Total	Silver *	Wheat Harvest †	Average Price of Consols ‡	Average Bank of England Rate ‡
												£	Percent.
1873......	106	109	106	107	141	103	106	114	**111**	97·4	80	$92\frac{1}{2}$	4·750
1896......	53	73	59	62	63	54	63	60	**61**	50·5	112	$110\frac{3}{4}$	2·483
1911......	70	90	61	75	93	76	81	83	**80**	40·4	110	$79\frac{5}{16}$	3·467
1914......	75	100	58	81	99	81	87	88	**85**	41·6	109	$72\frac{3}{16}$	4·038
'15......	108	126	70	170	126	92	109	108	**108**	38·9	106	$65\frac{1}{2}$	5·000
'16......	133	152	86	130	158	129	136	140	**136**	50·4	97	$58\frac{1}{80}$	5·470
1917......	177	192	113	169	172	192	174	179	**175**	65·8	102	$54\frac{3}{5}$	5·15
'18......	168	207	130	174	192	222	202	206	**192**	76·4	111	$56\frac{7}{8}$	5·0
'19......	179	213	147	185	220	228	219	222	**206**	85·3	98	$54\frac{1}{12}$	5·166
'20......	227	263	198	234	295	262	244	264	**251**	76·1	96	$47\frac{1}{60}$	6·71
'21......	143	218	83	158	181	140	145	153	**155**	48·1	118	$47\frac{23}{30}$	6·092
1922......	107	184	82	130	142	134	124	132	**131**	51·6	105	$56\frac{15}{32}$	3·692
'23......	98	162	101	122	155	140	117	134	**129**	49·4	105	$57\frac{63}{64}$	3·496
'24......	119	158	105	130	158	170	120	146	**139**	50·7	107	$56\frac{57}{64}$	4·0
'25......	118	162	89	128	154	165	119	143	**136**	52·5	114	$56\frac{3}{8}$	4·575
'26......	108	150	88	119	154	133	114	131	**126**	47·1	99	$54\frac{29}{30}$	5·0
1927......	108	138	83	114	141	131	118	129	**122**	42·8	109	$54\frac{39}{48}$	4·650
'28......	107	142	78	114	123	136	117	124	**120**	44·0	109	$55\frac{13}{16}$	4·5
'29......	99	146	72	110	126	122	111	119	**115**	40·2	114	$54\frac{5}{16}$	5·508
'30......	77	142	54	96	112	84	97	97	**97**	29·0	99	56	3·4
'31......	68	119	50	83	100	63	85	82	**83**	20·4	99	$55\frac{3}{16}$	3·975
1932......	72	105	50	79	99	64	81	81	**80**	19·5	105	$66\frac{25}{32}$	3·017
'33......	60	106	47	74	107	67	80	83	**79**	18·7	114	$73\frac{23}{32}$	2·0
'34......	63	108	50	77	109	72	80	85	**82**	20·0	120	$80\frac{21}{32}$	2·0
'35......	66	107	42	76	112	80	83	90	**84**	26·4	112	$86\frac{1}{2}$	2·0
'36......	76	109	41	81	118	83	88 §	94	**89**	18·5	100	$85\frac{11}{64}$	2·0
'37......	93	117	49	93	142	93	101	110	**102**	18·4	99	$76\frac{9}{32}$	2·0
'38......	81	111	43	84	136	75	87	96	**91**	17·6	122	$74\frac{5}{32}$	2·0
'39......	74	115	47	83	137	93	90	103	**95**	17·1	112	$66\frac{21}{64}$	2·5
Average 1904–13	68	91	53	73	95	74	76	81	**77**	44·1	106	$82\frac{25}{32}$	3·733
1890–99	61	80	63	68	71	56	66	64	**66**	55·8	103	$103\frac{9}{16}$	2·958
'78–87	79	95	76	84	73	71	81	76	**79**	82·1	97	$99\frac{9}{16}$	3·264
1818–27	109	90	151	111	128	105	106	112	**111**	98·0	—	—	3·692

* Silver (see note on p. 353), parity of 1 gold to $15\frac{1}{2}$ silver = 100.
† Wheat harvest in U.K. to 1895: 29 bushels = 100; from 1896: 30 bushels = 100.
‡ Average price of Consols and the average Bank of England rate of discount are actual figures, not index-numbers; Consols 3% to 1888, $2\frac{3}{4}$% from 1889, $2\frac{1}{2}$% from April, 1903.

THE STATIST'S *Index Numbers—monthly averages by groups* (1867–77 = 100)

	Vegetable Food	Animal Food	Sugar, Tea, and Coffee	Foodstuffs	Minerals	Textiles	Sundry Materials	Total Materials	All Commodities
1937									
Jan. ...	91·3	108·1	46·2	**88·0**	134·5	95·9	100·3	**108·1**	**99·6**
Feb. ...	91·1	111·5	46·7	**89·3**	144·8	96·1	101·6	**111·5**	**102·1**
March ...	97·0	118·4	48·3	**94·6**	153·5	100·2	104·9	**116·5**	**107·3**
April ...	96·3	120·1	48·5	**95·0**	138·5	100·0	103·7	**111·8**	**104·7**
May ...	95·6	121·8	50·2	**95·7**	146·1	100·9	102·7	**113·8**	**106·2**
June ...	90·9	121·2	50·2	**93·5**	145·9	98·9	102·2	**112·9**	**104·7**
July ...	92·9	123·3	50·5	**95·2**	149·4	97·6	102·7	**113·7**	**105·9**
Aug. ...	92·4	121·7	49·5	**94·2**	148·0	95·1	100·9	**111·8**	**104·4**
Sept. ...	94·5	119·2	50·5	**94·3**	145·8	91·0	100·7	**109·9**	**103·3**
Oct. ...	96·4	117·6	49·0	**94·2**	138·6	85·4	99·3	**105·6**	**100·8**
Nov. ...	94·4	114·0	47·1	**91·7**	135·5	78·7	93·7	**100·3**	**96·7**
Dec. ...	95·7	117·2	45·6	**93·1**	134·7	79·5	93·8	**100·4**	**97·3**
1938									
Jan. ...	95·0	115·2	44·2	**91·7**	135·0	79·4	92·8	**100·0**	**96·5**
Feb. ...	92·0	117·1	42·7	**90·9**	136·2	78·6	93·7	**100·5**	**96·4**
Mar. ...	89·9	116·9	42·3	**89·8**	133·3	75·5	90·6	**97·4**	**94·2**
April ...	89·1	116·9	42·4	**89·5**	132·3	75·3	89·1	**96·5**	**93·5**
May ...	89·1	114·3	42·3	**88·5**	129·6	72·3	85·9	**93·5**	**91·4**
June ...	89·5	111·1	42·0	**87·5**	134·2	72·8	84·6	**94·3**	**91·4**
July ...	83·5	109·6	42·0	**84·4**	136·0	75·1	85·7	**96·0**	**91·1**
Aug. ...	76·2	106·9	42·9	**80·5**	134·6	73·3	84·6	**94·5**	**88·6**
Sept. ...	74·3	105·7	43·7	**79·4**	137·1	73·4	84·8	**95·4**	**88·6**
Oct. ...	70·7	104·1	43·0	**77·2**	141·1	74·4	85·9	**97·2**	**88·8**
Nov. ...	67·0	103·6	43·0	**75·4**	140·6	73·1	84·6	**96·1**	**87·4**
Dec. ...	68·7	109·6	44·0	**78·6**	140·0	75·4	84·8	**96·8**	**89·1**
1939									
Jan. ...	68·9	110·9	43·3	**79·0**	134·8	78·0	84·0	**95·9**	**88·7**
Feb. ...	67·7	110·0	43·1	**78·2**	133·8	80·1	83·9	**96·2**	**88·6**
Mar. ...	67·3	111·7	44·1	**78·8**	133·9	80·7	84·2	**96·5**	**89·0**
April ...	66·7	113·2	47·0	**79·7**	135·1	85·4	84·7	**98·5**	**90·5**
May ...	66·3	112·6	46·6	**79·2**	134·3	85·5	86·2	**99·0**	**90·6**
June ...	67·0	113·0	47·0	**79·8**	133·9	86·3	84·9	**98·5**	**90·6**
July ...	61·6	110·6	43·4	**76·0**	134·3	84·5	84·6	**97·9**	**88·7**
Aug. ...	62·2	112·8	44·4	**77·1**	136·2	87·0	86·6	**100·2**	**90·4**
Sept. ...	71·9	127·6	56·5	**89·1**	136·9	101·7	92·8	**107·4**	**99·7**
Oct. ...	85·6	130·3	56·0	**95·8**	138·1	108·0	100·9	**113·1**	**105·8**
Nov. ...	92·3	130·3	58·5	**99·2**	143·4	124·1	106·4	**121·8**	**112·2**
Dec. ...	101·7	130·3	57·6	**103·0**	158·7	138·4	111·8	**132·6**	**120·1**
1940									
Jan. ...	106·2	142·7	57·3	**109·4**	158·1	142·9	114·1	**134·8**	**124·1**
Feb. ...	103·7	140·5	57·5	**107·5**	161·0	140·8	118·6	**136·8**	**124·5**
March ...	102·6	140·5	57·3	**107·0**	161·2	139·9	115·5	**135·3**	**123·4**
April ...	102·7	140·5	57·6	**107·0**	163·4	145·6	121·1	**140·0**	**126·0**
May ...	104·2	140·5	56·9	**107·6**	166·6	150·7	122·0	**142·9**	**128·0**

Quarterly Movements of Prices *

Summary of Index Numbers, 1867–77 = 100

Years	Quarters	Vegetable Food (Corn, etc.)	Animal Food (Meat, etc.)	Sugar Coffee, and Tea	Total Food	Minerals	Textiles	Sundry Materials	Total Materials	Grand Total	Silver †
1928	I	108·9	143·8	80·3	115·7	123·6	136·7	120·6	126·3	**121·9**	43·2
	II	118·0	152·0	81·1	122·8	122·9	140·6	117·9	126·3	**124·8**	44·7
	III	101·1	142·0	77·9	111·2	121·0	135·3	116·7	123·6	**118·4**	44·3
	IV	101·9	138·1	76·3	109·8	126·0	131·5	115·3	123·5	**117·5**	43·7
'29	I	102·9	142·7	75·4	111·8	130·2	130·7	116·6	124·6	**119·2**	42·8
	II	92·8	148·3	73·2	109·1	125·0	121·2	111·1	117·9	**114·2**	41·1
	III	99·9	143·4	71·7	110·0	126·9	115·6	111·1	116·7	**113·9**	39·6
	IV	91·3	145·2	64·2	105·4	122·9	107·9	108·7	112·3	**109·4**	37·3
'30	I	80·8	152·1	58·3	102·3	121·1	96·4	104·9	106·7	**104·8**	33·0
	II	76·7	142·4	56·5	96·5	110·8	92·4	99·5	100·4	**98·7**	29·8
	III	77·4	132·1	48·6	91·5	109·0	77·3	94·6	94·6	**92·5**	26·8
	IV	71·9	130·0	51·7	89·0	105·4	68·9	91·6	88·3	**88·6**	26·5
'31	I	69·0	127·3	48·6	86·2	103·4	62·7	89·8	85·1	**85·6**	21·8
	II	69·5	123·3	48·8	85·0	98·6	61·4	85·6	81·7	**80·1**	21·3
	III	70·0	117·4	47·0	81·4	98·5	58·6	81·6	79·0	**83·0**	21·9
	IV	75·7	107·9	53·7	82·9	102·2	66·9	85·4	84·1	**83·6**	21·5
'32	I	80·7	109·2	52·2	83·2	101·1	67·5	87·8	85·2	**85·2**	21·0
	II	77·6	107·6	50·2	82·9	95·1	59·4	79·6	77·3	**79·9**	19·4
	III	68·2	105·3	49·6	77·9	100·6	65·9	80·7	81·5	**80·0**	19·4
	IV	64·2	98·2	48·4	73·4	101·2	64·5	80·2	81·0	**77·8**	18·4
'33	I	60·2	106·0	47·3	74·4	99·7	62·0	79·1	79·4	**77·3**	18·2
	II	59·1	108·4	47·4	74·8	109·7	68·7	79·8	84·2	**80·2**	20·0
	III	62·0	105·8	47·9	75·2	111·0	71·0	80·0	85·6	**81·2**	18·5
	IV	58·5	106·4	47·4	73·8	110·9	67·7	79·7	84·4	**79·9**	18·5
'34	I	59·4	110·1	53·0	76·7	111·7	73·6	79·9	86·5	**82·4**	19·1
	II	58·7	110·3	52·2	76·1	108·4	70·8	79·1	84·4	**80·9**	18·6
	III	71·0	109·5	48·0	80·4	108·5	70·7	78·7	84·3	**82·6**	19·7
	IV	66·7	107·3	44·8	77·0	109·7	70·9	79·6	85·0	**81·6**	22·1
'35	I	64·4	111·2	41·3	76·8	108·9	77·4	82·6	88·0	**83·3**	23·0
	II	67·1	107·9	42·3	76·9	112·3	79·4	82·8	89·8	**84·3**	29·3
	III	68·8	106·6	40·6	76·8	113·8	80·1	82·3	90·1	**84·5**	27·3
	IV	70·7	104·1	41·8	76·9	116·7	83·6	84·9	93·0	**86·3**	25·7
'36	I	72·1	104·6	41·2	77·6	116·1	84·6	86·4	93·8	**87·0**	18·2
	II	71·2	107·1	39·7	77·8	114·7	79·4	84·8	91·2	**85·5**	18·6
	III	75·3	112·6	39·1	81·5	116·7	80·5	90·0	94·2	**88·8**	18·6
	IV	85·8	110·7	42·6	85·9	129·6	88·2	93·8	101·7	**95·0**	18·9
'37	I	93·1	112·7	47·1	90·6	144·3	97·4	102·3	112·0	**103·0**	18·6
	II	94·3	121·0	49·6	94·7	143·5	99·9	102·9	112·8	**105·2**	18·7
	III	93·3	121·4	50·2	94·6	147·7	94·6	101·4	111·8	**104·5**	18·4
	IV	95·5	116·3	47·2	93·0	136·3	81·2	95·6	102·1	**98·3**	18·0
'38	I	92·3	116·4	43·1	90·8	134·8	77·8	92·4	99·3	**95·7**	18·5
	II	89·2	114·1	42·2	88·5	132·0	73·5	86·5	94·8	**92·1**	17·4
	III	78·0	107·4	42·9	81·4	135·9	73·9	85·0	95·3	**89·4**	17·3
	IV	68·8	105·8	43·3	77·1	140·6	74·3	85·1	96·7	**88·4**	17·4
'39	I	68·0	110·9	43·5	78·7	134·2	79·6	84·0	96·2	**88·8**	17·7
	II	66·7	112·9	46·9	79·6	134·4	85·7	85·3	98·7	**90·6**	17·3
	III	65·2	117·0	48·1	80·7	135·8	91·1	88·0	101·8	**92·9**	15·7
	IV	93·2	130·3	57·4	99·3	146·7	123·5	106·4	122.5	**112·7**	17·8

* The averages of the four quarterly figures to each year do not necessarily coincide with the annual averages, as the latter are based as far as possible on average weekly prices. See also the *Journal*, 1893, p. 221; 1895, p. 144; 1901, p. 90; and 1909, p. 70.

† Silver, parity of 1 gold to 15½ silver = 100.

Construction of the Tabular Statements

The following table illustrates the method of construction of the index numbers. The index numbers here given are based on the average prices for the eleven years 1867–77. Take, for instance, the *Gazette* price of English wheat :—

		s.	d.	
Average,	1867–77 ...	54	6	= 100, average point.
,,	1914 ...	35	0	= 64, or 36 per cent. *below* the average point.
,,	1930 ...	80	7	= 148, ,, 48 ,, *above* ,, ,,
,,	1936 ...	53	3	= 98, ,, 2 ,, *below* ,, ,,

The individual index numbers, therefore, represent simple percentages of the average point.

The articles are grouped in six categories :—

	Index Nos.	1867–77 Total Numbers	Example for 1939	
			Total Numbers	Average
1. Vegetable food, corn, etc. (wheat flour, barley, oats, maize, potatoes, and rice)	8	800	589	74
2. Animal food (beef, mutton, pork, bacon, and butter)	7	700	808	115
3. Sugar, coffee, and tea	4	400	187	47
1–3. *Food*	19	1,900	1,584	83
4. Minerals (iron, copper, tin, lead, and coal)	7	700	958	137
5. Textiles (cotton, flax, hemp, jute, wool, and silk)	8	800	741	93
6. Sundry materials (hides, leather, tallow, oils, soda, nitrate, indigo, and timber)	11	1,100	991	90
4–6. *Materials*	26	2,600	2,690	103
General Average	45	4,500	4,274	95

The general average is drawn from all forty-five descriptions, which are treated as of equal value, and is the simple arithmetic mean as shown above.

Index of Silver Prices

The base of the index numbers given below is 60·84*d.* per standard oz. = 100, this being a parity of 1 fine oz. of gold to 15½ standard ozs. of silver.*

	Price per oz. standard	Index number		Price per oz. standard	Index number
	d.			*d.*	
Average 1873 ...	59¼	=97·4	*Lowest* Nov., 1902	21 11/16	=35·6
,, '90–99...	34	=55·8	End Dec., 1906 ...	32 5/16	=53·1
,, 1917–26...	40½	=66·6	,, Dec., '08 ...	23 3/16	=38·1
,, 1893 ...	35⅝	=58·6	,, Dec., '12 ...	29	=47·7
,, '96 ...	30¾	=50·5	,, Dec., '13 ...	26 9/16	=43·7
,, 1909 ...	23 11/16	=38·9	,, June, '14 ...	26	=42·7
,, '14 ...	25 5/16	=41·6	,, Dec., '14 ...	22 11/16	=37·3
,, '15 ...	23 11/16	=38·9	,, Dec., '15 ...	26¼	=43·1
,, '16 ...	31 5/16	=50·4	,, Dec., '16 ...	36½	=58·7
,, '17 ...	40⅞	=65·8	,, Dec., '17 ...	43½	=70·0
,, '18 ...	47½	=76·4	,, Dec., '18 ...	48 7/16	=77·9
,, '19 ...	57	=85·3	,, Dec., '19 ...	77¼	=98·3
,, '20 ...	61 9/16	=76·1	,, Dec., '20 ...	40⅞	=49·2
,, '21 ...	36⅞	=48·1	,, Dec., '21 ...	34⅝	=49·3
,, '22 ...	34 7/16	=51·6	,, Dec., '22 ...	31 9/16	=49·6
,, '23 ...	31 15/16	=49·4	,, Dec., '23 ...	33 7/16	=49·0
,, '24 ...	34	=50·7	,, Dec., '24 ...	31 13/16	=50·4
,, '25 ...	32⅛	=52·5	,, Dec., '25 ...	31 11/16	=52·1
,, '26 ...	28 11/16	=47·1	,, Dec., '26 ...	25	=41·1
,, '27 ...	26 1/32	=42·8	,, Dec., '27 ...	26½	=43·6
,, '28 ...	26¾	=44·0	,, Dec., '28 ...	26⅜	=43·3
,, '29 ...	24 7/16	=40·2	,, Dec., '29 ...	21 7/16	=35·2
,, '30 ...	17 11/16	=29·0	,, Dec., '30 ...	14 7/16	=23·7
,, '31 ...	14⅗	=20·4	,, Dec., '31 ...	20 7/16	=21·6
,, '32 ...	17⅘	=19·5	,, Dec., '32 ...	16½	=17·2
,, '33 ...	18⅛	=18·7	,, Dec., '33 ...	19 1/16	=19·5
,, '34 ...	21 7/32	=20·0	,, Dec., '34 ...	24⅝	=22·6
,, '35 ...	29	=26·4	,, Dec., '35 ...	22½	=20·6
,, '36 ...	20 1/16	=18·5	,, Dec., '36 ...	21 5/16	=19·4
,, '37 ...	20 1/16	=18·4	,, Dec., '37 ...	19 1/16	=17·7
,, '38 ...	19 17/32	=17·6	,, Dec., '38 ...	20 1/16	=17·3
,, '39 ...	20 13/32	=17·1	,, Dec., '39 ...	22½	=17·3

* All the index numbers in the table from 1916 to 1925 inclusive and from 1931 to date are calculated on the basis of the gold prices of silver instead of the sterling prices, though the latter are the price quotations given in the table. In arriving at the index numbers for these dates the prices of gold are taken as follows.
fine oz., derived from
1919 the average price
with the U.S. dollar, the average New York exchange in that year being $4·429. For the other dates the index numbers are based on the quotations in the London market for exportable gold. The quotation at the end of 1919 was 109*s.* 8¼*d.* per fine oz. At the end of 1920, 1921, 1922, 1923 and 1924 the quotations per fine oz. were 116*s.* 1*d.*, 98*s.* 0*d.*, 88*s.* 11*d.*, 95*s.* 4*d.*, and 88*s.* 2*d.* respectively and the average quotations in these years were 112*s.* 11½*d.*, 107*s.* 0½*d.*, 93*s.* 4*d.*, 90*s.* 3*d.*, and 93*s.* 8¼*d.* respectively, while the average price in 1925 was 85*s.* 5½*d.* The prices at the end of 1931, 1932, 1933, 1934, 1935, 1936, 1937, 1938 and 1939 were 121*s.* 11*d.*, 123*s.* 9*d.*, 126*s.* 6*d.*, 141*s.* 0*d.*, 141*s.* 2*d.*, 141*s.* 7*d.*, 139*s.* 6*d.*, 149*s.* 7½*d.* and 168*s.* respectively, and the average prices in these years were 92*s.* 6¼*d.*, 118*s* 0¾*d.*, 124*s.* 10½*d.*, 137*s.* 7¾*d.*, 142*s.* 1¼*d.*, 140*s.* 3½*d.*, 140*s.* 8¾*d.*, 142*s.* 6½*d.* and 154*s.* 4*d.* respectively.

World's Production of Silver (in millions of ounces)

	United States	Mexico	Canada	Australia	Other Countries	Total
1903... ...	54·3	70·5	3·1	9·7	30·1	167·7
'04... ...	57·7	60·8	3·7	14·5	27·5	164·2
'05... ...	56·1	65·0	5·9	15·0	30·3	172·3
'06... ...	56·5	55·2	8·5	14·2	30·6	165·0
'07... ...	56·5	61·0	12·8	19·0	34·8	184·2
'08... ...	52·4	73·6	22·1	17·2	37·8	203·1
'09... ...	54·7	73·9	27·5	16·3	39·7	212·1
'10... ...	57·1	71·4	32·9	21·5	38·8	221·7
'11... ...	60·4	79·0	32·7	16·6	37·5	226·2
'12... ...	63·8	74·6	31·6	18·1	36·2	224·3
'13... ...	66·8	70·7	31·5	3·5	51·4	223·9
'14... ...	72·4	27·5	28·4	3·6	36·5	168·4
'15... ...	74·9	39·5	28·4	4·1	37·3	184·2
'16... ...	74·4	38·2	25·4	4·2	26·6	168·8
'17... ...	71·7	35·0	22·2	10·0	35·3	174·2
'18... ...	67·8	62·5	21·2	10·0	35·9	197·4
'19... ...	56·7	62·7	15·7	7·4	32·0	174·5
'20... ...	55·5	66·8	12·6	7·5	33·0	175·4
'21... ...	53·1	64·5	13·1	4·9	35·7	171·3
'22... ...	56·2	81·1	18·6	11·3	46·3	213·5
'23... ...	73·3	90·9	17·8	13·3	50·7	246·0
'24... ...	65·3	91·5	19·7	10·8	52·2	239·5
'25... ...	66·1	92·9	20·2	11·1	54·8	245·1
'26... ...	62·7	98·3	22·4	11·2	59·0	253·6
'27... ...	60·4	104·6	22·7	9·0	57·3	254·0
'28... ...	58·4	108·5	21·9	9·0	59·5	257·3
'29... ...	61·2	108·7	23·1	9·0	59·7	261·7
'30... ...	51·0	105·0	26·0	8·9	57·1	248·0
'31... ...	31·0	86·0	21·0	7·6	50·4	196·0
'32... ...	24·0	69·0	18·0	6·5	47·5	165·0
'33... ...	22·8	68·1	15·2	11·0	52·0	169·1
'34... ...	32·5	74·1	16·4	10·8	56·6	190·4
'35... ...	45·6	75·6	16·6	11·4	71·5	220·7
'36... ...	63·4	77·5	18·3	12·7	81·8	253·7
'37... ...	71·3	84·7	22·7	14·3	80·9	273·9
'38... ...	61·7	81·0	22·2	103·0		267·9
'39* ...	63·8	81·0	24·7	103·3		272·8

* Provisional. (Estimate by Messrs. Samuel Montagu & Co.)

Gold.—The table below shows the world's annual gold production since 1851. Prior to 1911 the estimates are those of the Bureau of the U.S. Mint and other authorities. The estimates since 1926 are those of the Union Corporation, Limited. The value is taken throughout at £4·25 per fine oz. The figures show clearly the increase in output caused by the departure of the large producing countries from the gold standard (especially the departure of South Africa at the end of 1932).

World gold production increased by some 5·8 per cent. in 1939, the eleventh successive year of expanding output since the slight

downward movement recorded for 1928. During that period the average annual increase in production, according to the following estimates, has amounted to 6·6 per cent. by value.

(000's omitted)

Year	Value of output £	Year	Value of output £
1851	17,200	1896	41,559
'52	26,550	'97	48,509
'53	31,090	'98	58,949
'54	25,490	'99	63,027
'55	27,015	1900	52,312
'56	29,520	'01	53,630
'57	26,655	'02	60,975
'58	24,930	'03	67,337
'59	24,970	'04	71,380
'60	23,850	'05	78,143
'61	22,760	'06	82,707
'62	21,550	'07	84,857
'63	21,390	'08	90,995
'64	22,600	'09	93,302
'65	24,040	'10	93,544
'66	24,220	'11	94,930
'67	22,805	'12	95,783
'68	21,945	'13	97,481
'69	21,245	'14	92,709
'70	21,370	'15	97,114
'71	25,400	'16	92,597
'72	24,200	'17	87,236
'73	23,600	'18	78,605
'74	22,950	'19	73,078
'75	22,700	'20	68,522
'76	22,540	'21	67,848
'77	23,830	'22	66,723
'78	22,020	'23	77,888
'79	21,400	'24	81,807
'80	22,130	'25	82,267
'81	21,150	'26	82,211
'82	20,500	'27	82,582
'83	20,640	'28	82,400
'84	20,830	'29	84,500
'85	21,250	'30	88,500
'86	21,430	'31	95,100
'87	21,735	'32	103,400
'88	22,644	'33	107,700
'89	25,375	'34	116,000
'90	24,421	'35	125,700
'91	26,846	'36	140,900
'92	30,134	'37	147,900
'93	32,363	'38	157,300*
'94	37,229	'39 (provisional)	166,400
'95	40,843		

* Amended figure.

Average Prices of Commodities *

No. of Article	0	1	2	3	4	5	6	7	8	1–8	9	10
		Wheat		Flour	Barley	Oats	Maize §	Potatoes *	Rice	Vegetable Food	Beef ‡	
Year	Silver †	English *Gazette*	American	Town Made white (now "G.R.")	English *Gazette*	English *Gazette*	American Mixed	Good English	Rangoon Cargoes to Arrive	Total	Prime	Middling
	d. per oz.	*s.* and *d.* per qr.	*s.* and *d.* per qr.	*s.* per sack (280 lbs.)	*s.* and *d.* per qr.	*s.* and *d.* per qr.	*s.* per qr.	*s.* per ton	*s.* and *d.* per cwt.		*d.* per 8 lbs.	*d.* per 8 lbs.
1873 ...	59 1/4	58·8	63	51	40·5	25·5	30	160	9·6	—	65	56
1919 ...	57	72·10	74·10	46 3/4	75·8	52·3	78 1/5	198 1/2	25·10	—	108	108
'20 ...	61 9/16	80·7	92·4	66	90	57·4	90 2/3	242 1/2	41·10	—	125	125
'21 ...	36 7/8	72·9	73·9	64 1/2	54·4	34·5	38 1/5	198	18·5	—	115	109
'22 ...	34 7/16	47·10	52·11	45 3/4	40·1	29·1	31 1/4	130	14·10	—	88 5/8	82
'23 ...	31 15/16	42·2	47·3	39 1/2	33·8	26·8	36	101	14·10	—	79 1/4	74
'24 ...	34	49·3	53·9	43 5/6	46·9	27·2	39 2/3	186	16·9	—	82 1/8	76
'25 ...	32 1/8	52·2	62·4	50 1/8	42·0	27·2	38 9/16	154	16·0	—	80	73
'26 ...	28 11/16	53·3	58·9	49 1/4	36·11	25·1	29 13/16	127	16·3	—	74	67
'27 ...	26 3/32	49·3	58·3	44 9/10	42·0	25·4	30 9/16	136	15·11	—	70	62
'28 ...	26 3/4	44·8	50·10	40 1/6	39·0	29·0	38 15/32	133	15·0	—	74	66
'29 ...	24 7/16	42·2	51·3	38 1/2	35·5	24·7	36 1/4	111	14·3	—	71	66
'30 ...	17 11/16	34·3	36·10	33 1/12	28·3	17·2	23	93	13·0	—	73	68
'31 ...	14 3/5	24·0	25·1	22 7/8	28·0	17·8	15 9/16	146	9·8	—	67	61
'32 ...	17 27/32	25·0	27·5	24 7/32	27·1	19·3	18 13/16	152	9·8	—	65	59
'33 ...	18 1/8	22·10	25·7	23 29/32	28·7	15·10	17 1/16	86	7·9	—	61	52
'34 ...	21 7/32	20·2	28·0	23 1/8	30·11	17·5	19 19/32	97	7·8	—	58	52
'35 ...	29	22·2	31·1	25 3/4	28·7	18·9	17 1/4	107	8·10	—	54	49
'36 ...	20 1/16	30·9	35·1	31 17/32	29·5	17·8	19 1/2	146	9·0	—	54	50
'37 ...	20 1/16	40·0	49·7	40 1/8	39·0	23·11	26 9/16	136	10·5	—	61	57
'38 ...	19 17/32	28·11	39·3	30 31/32	36·4	21·2	28 1/16	111	10·7	—	62	58
'39 ...	20 13/32	21·5	30·1	22 9/32	31·7	19·3	26 1/2	117	11·1	—	61	58
Average 1904–13	26 3/4	31 1/2	36	30	25 1/2	18 1/2	24 1/2	78	7 3/4	—	51	44
1890–99	34	28 1/2	31 1/2	27 1/2	25 1/2	17 1/2	19 1/2	72	6 1/4	—	47	37
'78–87	50	40	43 1/2	34 1/2	31 1/2	21	25	102	8	—	55 1/2	46
'67–77	58 1/2	54 1/2	56	46	39	26	32 1/2	117	10	—	59	50
	Index Numbers (or Percentages) of Prices, the Average of 1867–77 being 100											
1873 ...	97·4	108	113	104	104	98	92	137	95	851	110	112
1919 ...	85·3	134	134	102	194	201	242	170	258	1,435	183	216
'20 ...	76·1	148	165	143	231	221	279	207	418	1,812	212	250
'21 ...	48·1	133	132	140	139	132	118	169	184	1,147	195	220
'22 ...	51·6	88	95	100	103	112	96	111	148	853	150	164
'23 ...	49·4	77	84	86	86	103	111	86	148	781	134	149
'24 ...	50·7	90	96	95	120	105	122	159	167	954	139	152
'25 ...	52·5	96	111	109	108	105	119	132	160	940	136	147
'26 ...	47·1	98	105	107	95	96	92	109	163	865	125	134
'27 ...	42·8	90	104	98	108	97	95	116	159	867	119	124
'28 ...	44·0	82	91	87	100	112	118	114	150	854	125	133
'29 ...	40·2	77	91	84	91	95	112	95	143	788	120	132
'30 ...	29·0	63	66	72	72	66	71	79	130	619	124	136
'31 ...	20·4	44	45	50	71	68	48	125	93	544	114	122
'32 ...	19·5	46	49	53	69	74	58	130	93	572	110	118
'33 ...	18·7	42	46	52	73	61	53	74	78	479	103	104
'34 ...	20·0	37	50	50	79	67	60	83	77	503	98	104
'35 ...	26·4	41	56	56	73	72	53	91	88	530	92	98
'36 ...	18·5	56	63	69	75	68	60	125	90	606	92	100
'37 ...	18·4	73	89	88	100	92	82	116	105	745	103	114
'38 ...	17·6	53	70	67	93	81	86	95	106	651	105	116
'39 ...	17·1	39	54	48	81	74	82	100	111	589	103	116

* The annual prices are the average monthly or weekly quotations, except potatoes, which are the average weekl quotations during the eight months January to April and September to December.
† Not included in the general average. ‡ Meat (9—18), by the carcase, in the London Central Meat Market
§ La Plata from 1924.

Average Prices of Commodities—Contd.

No. of Article	11	12	13	14	15	9–15	16A	16B	17	18A*	18B*	18
	Mutton		Pork	Bacon	Butter		Sugar			Coffee		
Year	Prime	Middling	Large and Small, average	Waterford	Friesland, Fine to Finest	Animal Food Total	British West Indian Refining ‖	Beet, German, 88 p. c., f.o.b.	Java, Floating Cargoes §	Ceylon Plantation, Low Middling †	Rio, Good	Mean of 18A and 18B
	d. per 8 lbs.	*d.* per 8 lbs.	*d.* per 8 lbs.	*s.* per cwt.	*s.* per cwt.		*s.* per cwt.	*s.* per cwt.	*s.* per cwt.	*s.* per cwt.	*s.* per cwt.	
873 ...	71	63	54	81	123	—	22 1/2	25	28	100	86	—
919 ...	114	114	128	190 3/5	252	—	38 2/5	34 1/3 ‡	43 1/2	145 3/4	114 7/8	—
'20 ...	144 2/3	144 2/3	168 1/4	239 2/3	301	—	58	65 1/2 ‡	74 3/4	148	111 1/2	—
'21 ...	130 7/8	125 1/4	121 4/5	179	250	—	19 1/2	18 1/2 ‡	22	120 3/4	63	—
'22 ...	125	121 2/5	101	145 1/3	202 1/2	—	15	14 1/8	15 5/16	120 2/3	74 1/3	—
'23 ...	114 3/8	107 3/4	89	113 3/4	186	—	25 1/2	23 1/3	24 2/3	117 1/2	55	—
'24 ...	111 1/2	103 1/2	70	106	211	—	23 3/8	20 1/4	21 1/2	152 5/8	85 1/3	—
'25 ...	106 3/8	98 5/8	84 1/2	128 1/2	206 7/8	—	16 1/4	11 5/6	12 1/3	153 13/16	98 5/16	—
'26 ...	89	80 1/2	98 5/8	130	173	—	16 7/12	11 2/3	12 5/8	154 2/3	89 1/16	—
'27 ...	86	79 3/5	85	102 1/4	178	—	16 7/16	12 1/2	13 9/16	143 1/2	71 1/2	—
'28 ...	92 1/4	87	77	101 3/4	185 7/8	—	13 1/2	10 11/16	11 1/2	143 9/32	81 11/32	—
'29 ...	89 1/2	83	91	116 13/32	180 1/8	—	11 7/32	8 5/16	8 7/8	141 1/8	74 27/32	—
'30 ...	92	86	89	105 5/8	146 13/16	—	8 5/8	5 27/32	6 13/32	106 23/32	42 11/32	—
'31 ...	79	73	65	83 3/8	130	—	7 13/16	5 11/16	6 7/16	101 1/4	33 15/16	—
'32 ...	63	55	54	77	126 3/8	—	7 1/2	5 5/16	5 19/32	105 3/32	54 5/16	—
'33 ...	69	63	60	81 1/2	105 9/16	—	7 5/32	4 23/32	5 1/4	86 25/32	42 1/8	—
'34 ...	74	70	65	90 9/32	79 3/8	—	6 5/32	4 5/32	4 11/16	87 7/32	42 13/16	—
'35 ...	75	70	62	89	92 5/8	—	6 3/16	3 21/32	4 3/4	67 9/16	29 21/32	—
'36 ...	73	68	65	93 27/32	98 21/32	—	6 13/32	3 19/32	4 3/4	58 1/2	30 21/32	—
'37 ...	78	74	68	94	108 3/32	—	7 31/32	5 1/4	6 1/2	75 3/4	36 3/4	—
'38 ...	62	56	69	97 3/16	114 9/16	—	7 1/32	4 5/16	5 15/32	75	19 3/4	—
'39 ...	68	64	70	97 3/4	122	—	9 3/32	6 1/32	7 7/32	73 3/4	22 1/2	—
verage 904–13	58 1/4	51 3/4	47 5/8	67	113	—	10 1/8	10 7/8	12	75 1/2	43 3/4	—
890–99	54 1/2	41 1/2	42 3/4	59	100	—	11 1/2	11 1/2	13 3/4	98	62	—
'78–87	64 1/2	53	49	71	116	—	17	18	21 1/2	78	52	—
'67–77	63	55	52	74	125	—	23	24	28 1/2	87	64	—
	Index Numbers (or Percentages) of Prices, the Average of 1867–77 being 100											
873 ...	113	114	104	109	98	760	101		98	115*	134*	125
919 ...	181	207	246	258	202	1,493	155		153	167	180	174
'20 ...	230	263	324	324	241	1,844	263		262	170	174	172
'21 ...	208	228	234	242	200	1,527	81		77	140	98	119
'22 ...	199	221	194	196	162	1,286	62		54	140	116	128
'23 ...	182	196	171	154	149	1,135	104		87	135	86	111
'24 ...	177	188	135	143	169	1,103	93		75	175	133	154
'25 ...	169	180	162	174	165	1,133	60		43	176	154	165
'26 ...	141	146	190	176	138	1,050	60		44	178	139	159
'27 ...	136	145	163	138	142	967	62		47	165	112	139
'28 ...	146	158	148	137	149	996	51		40	165	127	146
'29 ...	142	151	175	157	144	1,021	42		31	162	117	140
'30 ...	146	155	171	143	117	992	31		22	123	66	95
'31 ...	125	133	125	113	104	836	29		23	120	53	87
'32 ...	100	100	104	104	101	737	27		20	121	85	103
'33 ...	110	114	115	110	84	740	25		18	100	66	83
'34 ...	117	127	125	122	64	757	22		16	100	67	84
'35 ...	119	127	119	120	74	749	21		17	78	46	62
'36 ...	116	124	131	127	79	763	21		17	67	48	58
'37 ...	124	135	125	127	86	820	28		23	87	57	72
'38 ...	98	102	133	131	92	777	24		19	86	31	59
'39 ...	108	116	135	132	98	808	32		26	85	35	60

* Index numbers not included in general average. ‡ Comparative values.
† E. India good middling from 1908. § Raw Centrifugals, 96% Pol., from 1924. ‖ White Javas C.I.F., from 1924.

Average Prices of Commodities—Contd.

No. of Article.	19A*	19C*	19B*	19	16–19	1–19	20A	20B	21	22	—	23
	Tea				Sugar, Coffee, and Tea	Food	Iron			Copper		Tin
Year	Congou, Common	Indian, Good Medium	Average Import Price	Mean of 19A and 19B	Total	Total	Scottish Pig	Cleveland (Middlesbrough) Pig	Bars, Common	Standard	English Tough Cake	Straits
	d. per lb.	d. per lb.	d. per lb.				s. and d. per ton	s. and d. per ton	£ per ton	£ per ton	£ per ton	£ per ton
1873 ...	12	—	16·67	—	—	—	117·3	—	12 1/2	84	92	132
1919 ...	13 1/2	15	15·5	—	—	—	143·1	137·1	19 1/5	92	99 1/2	257
'20 ...	11 1/4‡	9 2/5	14·97	—	—	—	214·11	208·11	28 1/3	97 1/5	112 3/4	302
'21 ...	4 3/8	7	12·4	—	—	—	168·6	137·4	19 1/8	69 1/3	72 3/5	171
'22 ...	8 5/8	13 2/3	14·9	—	—	—	99·10	90·7	11 1/5	63 1/3	66 1/3	162
'23 ...	11	17 1/16	17·58	—	—	—	108·0	108·9	11 7/8	65 9/16	69 15/16	206
'24 ...	9 1/4	17 3/4	19·0	—	—	—	96·8	88·2	12 1/5	63 3/16	67 7/16	251
'25 ...	7 7/8	14 15/16	18·34	—	—	—	83·4	72·8	11 5/8	61 13/16	65 1/2	267
'26 ...	7 13/16	16 15/16	18·82	—	—	—	87·2	87·6	11 1/2	58 1/32	63 5/32	297 [illegible]
'27 ...	6 3/4	14 3/8	18·58	—	—	—	80·5	73·0	11 1/4	55 3/4	60 9/16	303 7/8
'28 ...	6 5/16	12 1/4	16·84	—	—	—	69·9	65·9	9 15/16	63 3/4	66 3/16	229 [illegible]
'29 ...	6 1/16	11 7/32	16·11	—	—	—	74·0	70·3	9 3/4	75 9/16	78 13/32	207 [illegible]
'30 ...	5 7/32	9 1/4	15·12	—	—	—	76·0	67·0	9 31/32	54 3/4	58 17/32	144 2/3
'31 ...	4 13/16	6 1/16	13·29	—	—	—	71·0	58·6	10 3/32	38 1/8	39 5/16	121 1/8
'32 ...	4 9/32	5 29/32	10·75	—	—	—	68·2	58·6	10	31 21/32	33 13/16	140
'33 ...	6 21/32	8 23/32	11·87	—	—	—	66	62·3	9 11/16	32 21/32	34 13/16	202 1/2
'34 ...	8 21/32	12	13·20	—	—	—	69·6	66·11	9 17/32	30 9/32	32 23/32	232 1/4
'35 ...	6 23/32	10 11/32	13·06	—	—	—	70·6	67·10	9 5/8	32 15/32	34 5/16	230 1/3
'36 ...	6 1/4	11 5/8	13·19	—	—	—	78·6	73·2	10 3/16	37 13/32	41 29/32	207 2/3
'37 ...	6 9/16	13 11/32	14·58	—	—	—	104·6	94·4	12 5/16	54 21/32	59 1/2	246 3/4
'38 ...	6 15/32	11 3/4	14·04	—	—	—	118·0	109	13 1/4	41 21/32	45 1/8	193 3/8
'39 ...	6 1/4§	11 5/32	14·18	—	—	—	104·3	100·7	12 3/8	44 1/4	—	232 1/3
Average 1904–13	7 7/8	7 1/8	8 1/8	—	—	—	57 3/4	51 1/2	6 7/8	67 3/4	72	164 7/8
1890–99	4 5/8	7 1/2	9 3/4	—	—	—	47	41 1/2	5 1/2	50	53	81
'78–87	6 3/4	—	12 3/4	—	—	—	46	38	5 1/2	55	60	89
'67–77	11 1/4	—	17 1/4	—	—	—	69	60	8 1/4	75	81	105

Index Numbers (or Percentages) of Prices, the Average of 1867–77 being 100

Year	19A*	19C*	19B*	19	16–19	1–19	20A	20B	21	22	—	23
1873 ...	*107	—	*97	102	426	2,037	170	—	152	112	—	126
1919 ...	120	—	90	105	587	3,515	217		234	123	—	245
'20 ...	100	—	88	94	791	4,447	329		343	130	—	288
'21 ...	39	—	72	55	332	3,006	237		232	92	—	163
'22 ...	77	—	86	82	326	2,465	148		136	84	—	154
'23 ...	98	—	102	100	402	2,318	168		144	88	—	196
'24 ...	82	—	110	96	418	2,475	143		152	84	—	239
'25 ...	70	—	106	88	356	2,429	121		144	82	—	254
'26 ...	69	—	109	89	352	2,267	135		139	77	—	283
'27 ...	60	—	108	84	332	2,166	119		136	74	—	289
'28 ...	56	—	98	77	314	2,164	105		120	85	—	219
'29 ...	54	—	93	74	287	2,096	112		118	101	—	198
'30 ...	46	—	88	67	215	1,826	111		121	73	—	138
'31 ...	42	—	78	60	199	1,579	100		123	52	—	115
'32 ...	38	—	62	50	200	1,509	98		121	43	—	131
'33 ...	58	—	68	63	189	1,408	99		117	44	—	193
'34 ...	77	—	77	77	199	1,459	106		116	40	—	221
'35 ...	60	—	76	68	168	1,447	107		117	43	—	219
'36 ...	56	—	76	66	162	1,531	118		123	50	—	198
'37 ...	58	—	85	72	195	1,760	154		149	73	—	235
'38 ...	58	—	81	70	172	1,600	176		161	56	—	185
'39 ...	56	—	82	69	187	1,584	159		150	59	—	221

* Index numbers not included in the general average. † Approximate. ‡ Nominal. § First 9 months only.

Average Prices of Commodities—Contd.

No. of Article	24	25A	25B	26	20–26	27	28	29A	29B	30A	30B	31
	Lead	Coal			Minerals	Cotton		Flax		Hemp		Jute
Year	English Pig	Wallsend Hetton in London §	Newcastle Steam	Average Export Price	Total	Middling American	Fair Dhollerah ‖	Petrograd ¶	Russian Average Import Price	Manila Fair Roping	Petrograd Clean (a)	Good Medium ††
	£ per ton	s. per ton	s. per ton	s. per ton		d. per lb.	d. per lb.	£ per ton	£ per ton	£ per ton.	£ per ton	£ per ton
1873 ...	23 1/2	32	—	20·90	—	9	6 3/16	47 1/2	44	43	36	18
1919 ...	29 3/8	45·3	45 3/8	46·2	—	19·65	14 3/4	120‡	174 3/8	58 3/4	147 3/8	50 1/8
'20 ...	40	32	51 1/8	79·8	—	23·14	13 2/3	120‡	345 1/8	65 3/5	145 1/3	44 4/5
'21 ...	24 1/5	32 1/4	29	34·83	—	9·4	5 13/16	112 1/2	118 1/3	40 1/4	145‡	27 5/8
'22 ...	25 1/5	34 2/5	24 5/8	24·16	—	12·10	8	95	84 7/8	33 1/5	57‡	30 3/4
'23 ...	28 13/32	32 13/32	28	25·13	—	15·25	10	83 3/8	84 1/2	33 1/4	57	26
'24 ...	35 13/16	27 1/2	22 3/32	23·38	—	16·26	11·03	120	104 1/8	44	81	31 3/4
'25 ...	37 7/16	29 5/8	16 15/16	20·08	—	12·64	11·01	92 1/5	120 3/4	46 1/8	89 1/4	49 9/16
'26 ...	32 1/8	**30 7/24	**16 5/24	18·59	—	9·40	7·75	65	72 15/16	43	74 5/8	43 7/8
'27 ...	25 9/16	23 1/16	14 13/16	17·80	—	9·54	8·27	95 13/16	74 1/12	43 3/8	66 5/16	32 3/4
'28 ...	22 5/16	21 7/16	13 21/32	15·67	—	10·92	8·66	98 1/2	91 15/16	37 11/16	63 13/32	33 13/16
'29 ...	24 1/32	23 3/8	15 27/32	16·13	—	10·26	7·73	76 7/16	71 7/32	37 7/32	61	32
'30 ...	19 1/2	24 25/32	14 7/32	16·64	—	7·49	5·12	53 1/8	60 7/32	26 1/2	48 7/32	20
'31 ...	14 9/32	24 5/8	13 3/4	15·98	—	5·90	4·60	36	35 9/16	18 3/4	27 1/2	15 15/16
'32 ...	13 5/16	23 13/32	13 5/8	16·27	—	5·24	4·85	45 7/32	42 5/32	18 23/32	36	16 3/32
'33 ...	13 1/4	22 5/8	13 25/32	16·08	—	5·54	4·53	51 13/32	48 15/32	15 7/8	37	14 7/8
'34 ...	12 13/32	20 1/4	14 3/32	16·08	—	6·70	4·80	60 11/16	50 29/32	14 29/32	42 5/16	14 3/32
'35 ...	16	20 9/32	14 13/32	16·30	—	6·71	5·42	79 3/8	72 5/16	19 11/32	43 1/4	16 15/16
'36 ...	19 15/32	23 3/32	15 29/32	16·98	—	6·71	5·12	63 25/32	60 15/16	28 11/32	42 27/32	17 13/32
'37 ...	24 29/32	24 1/2	20 29/32	19·05	—	6·21	4·80	78 13/32	70 3/32	34 5/16	38 3/8	19 11/16
'38 ...	17 1/16	25 11/16	18 3/4	21·32	—	4·93	3·67	66 29/32	63 1/2	21 25/32	38 5/16	17 21/32
'39 ...	17 3/8	25 11/32	—	21·12	—	5·95	4·41	90 21/32	72 9/16	22 5/16	48 5/16	26 19/32
Average 1904–13	15 5/8	18 1/8	11 7/8	11 7/8	—	6 1/2	5	32 1/2	36 7/8	30 3/8	31 3/8	18 7/8
1890–99	12	17 1/2	10 3/8	10 3/8	—	4 1/4	3	27	27	26 1/2	25	12 1/2
'78–87	14	16 3/4	8 5/8	9	—	6	4 1/4	33	34	35 1/2	26 1/2	15
'67–77	20 1/2	22	12 1/2	12 1/2	—	9	6 3/4	46	48	43	35	19

Index Numbers (or Percentages) of Prices, the Average of 1867–77 being 100

Year	24	25A	25B	26	20–26	27	28	29A	29B	30A	30B	31
1873 ...	117	145	—	167	989	100	92	97		101		95
1919 ...	143	206	—	370	1,538	218	219	313		264		264
'20 ...	195	145	—	638	2,068	257	203	495		270		236
'21 ...	118	147	—	279	1,268	104	86	246		237		145
'22 ...	123	156	—	193	994	134	118	191		116		162
'23 ...	139	147	—	201	1,083	169	148	179		116		137
'24 ...	175	125	—	187	1,105	181	163	239		160		167
'25 ...	183	135	—	161	1,080	140	163	227		174		261
'26 ...	157	138	—	149	1,078	104	115	147		151		231
'27 ...	125	105	—	142	990	106	123	181		141		172
'28 ...	109	97	—	125	860	121	128	203		130		178
'29 ...	117	106	—	129	881	114	114	157		126		168
'30 ...	95	113	—	133	784	83	76	121		96		105
'31 ...	71	112	—	127	700	66	68	76		58		84
'32 ...	65	106	—	130	694	58	72	93		70		85
'33 ...	65	103	—	129	750	62	67	106		68		78
'34 ...	61	92	—	129	765	74	71	119		73		74
'35 ...	78	92	—	130	786	74	80	161		80		89
'36 ...	95	105	—	136	825	74	76	133		91		93
'37 ...	121	111	—	152	995	69	71	158		93		104
'38 ...	83	117	—	171	949	55	54	139		77		93
'39 ...	85	115	—	169	958	66	65	174		91		140

* Approximate prices. † Approximate. ‡ Nominal. § Best Yorkshire house after 1916.
‖ Now No. 1 Oomra, Fine. ¶ Livonian Z.K. from 1921. ** Average price January–April, 1926.
†† Lightnings from 1931. (a) Russian Sirets Group 1, Sort 1 from 1931–33; Jugo-Slav Peasant from 1934.

Average Prices of Commodities—Contd.

No. of Article	32A	32B	33	34	27–34	35A	35B	35C	36A	36B	37
	Wool			Silk		Hides			Leather		Tallow
Year	Merino, Port Phillip, Average Fleece	Merino, Adelaide, Average Greasy	English, Lincoln Half Hogs	Tsatlee †	Textiles Total	River Plate, Dry	River Plate, Salted	Average Import Price	Dressing Hides	Average Import Price	Town
	d.* per lb.	d. per lb.	d. per lb.	s. per lb.		d. per lb.	d. per lb.	d. per lb.	d. per lb.	d. per lb.	s. per cwt.
1873 ...	25	11 3/4	24 1/2	21 3/4	—	11	8 1/4	—	18 1/4	—	44
1919 ...	67	32 1/8	22 5/8	26	—	22 1/2	19 3/8	17·1	36 1/2	40 5/8	87 [illegible]
'20 ...	79 7/8	32	22	38 3/5	—	20 1/2	18 1/4	20·1	43 2/5	71 1/4	75
'21 ...	31 7/8	11 5/8	8 3/5	26 9/16	—	9 11/16	8 7/8	9·58	25 1/8	46 1/4	36 1/4
'22 ...	39	17 1/8	9 3/4	28 3/4	—	9 1/8	8 7/8	8·06	24 1/3	36	34 5/8
'23 ...	43 11/16	20 1/3	12	24 1/4	—	9 15/32	8 3/16	8·23	23 7/16	31 7/16	36 3/8
'24 ...	53 7/16	25 1/2	18 7/8	23 7/16	—	10 3/64	8 5/8	8·63	22 13/16	33 7/8	42 3/8
'25 ...	41 1/16	17 3/8	17 1/6	18 1/8	—	11 7/16	8 13/16	9·87	23	33	42 1/4
'26 ...	36 1/2	16 7/16	15	15 13/16	—	10 5/16	8	9·32	21 1/5	35 11/16	38 1/6
'27 ...	38 1/16	17 7/16	15 11/32	15 7/16	—	12 5/16	10 7/16	9·85	22 15/16	36 15/16	33 7/8
'28 ...	37	17 5/16	17 29/32	14	—	15 7/16	11 29/32	12·09	23 13/16	37 7/16	36 [illegible]
'29 ...	35 3/8	13 3/32	16 1/16	13 7/8	—	10 9/16	8 1/4	10·80	19 1/4	38 1/4	36 [illegible]
'30 ...	18 9/32	8 13/32	10 3/4	10 15/16	—	6 25/32	6 1/2	7·80	18 1/2	33 1/2	28 2/5
'31 ...	14·7	7·1	8 1/2	8 15/16	—	5 1/2	5 9/16	6·12	17 3/4	32 1/2	19 1/3
'32 ...	15·0	7·2	5 25/32	8 5/32	—	4 9/16	4 3/4	5·47	17 3/32	28 7/8	21 [illegible]
'33 ...	19·9	9·3	5 29/32	6 3/4	—	5 1/4	4 11/16	5·65	17 11/32	26 29/32	19 [illegible]
'34 ...	21 1/4	10·4	7	5 7/16	—	4 13/16	4 7/8	5·71	17 1/2	25 5/16	17 3/4
'35 ...	20·1	9·5	7 11/32	5 13/32	—	5 5/32	5 1/8	5·51	17 1/2	25 13/16	24 5/8
'36 ...	24·7	12·2	10 5/32	5 23/32	—	6 1/4	6	6·47	17 1/2	27 11/32	23 [illegible]
'37 ...	26·9	12·7	16·9	8 7/16	—	8 21/32	7 9/16	8·62	18 9/32	28 5/16	23 [illegible]
'38 ...	18·6	8·9	11·9	7 31/32	—	6 21/32	5 9/16	6·35	14 5/16	24 1/32	17 [illegible]
'39 ...	17·9	9·0	12·2	13 3/32	—	6 3/4	6 1/8	6·39	18 5/32	23 1/16	16 [illegible]
Average 1904–13	17 3/8	9	10 7/8	11 3/4	—	9 5/8	7 1/2	6 7/8	16	17	31 1/2
1890–99	13 1/2	6 1/2	10	11 1/2	—	6 1/4	5 1/2	5	13 1/4	13 3/4	25
'78–87	18 1/2	8 3/8	11 3/4	15	—	8 5/8	6 3/4	6 1/2	15	17	35 1/2
'67–77	21 1/4	9 7/8	19 3/4	23	—	9	7	6 7/8	16	18 3/4	45
	Index Numbers (or Percentages) of Prices, the Average of 1867–77 being 100										
1873 ...	118	—	124	95	822	120	—	—	114	—	97
1919 ...	315		114	113	1,820	258			222		195
'20 ...	359		111	168	2,099	257			330		167
'21 ...	140		44	115	1,117	123			205		81
'22 ...	180		49	125	1,075	114			174		77
'23 ...	206		61	105	1,121	113			158		81
'24 ...	254		96	102	1,362	119			163		94
'25 ...	188		87	79	1,319	132			161		94
'26 ...	170		76	69	1,063	121			164		85
'27 ...	177		78	67	1,045	142			172		75
'28 ...	174		91	61	1,086	172			176		82
'29 ...	156		81	60	976	129			166		81
'30 ...	86		54	48	669	92			150		64
'31 ...	70		43	39	504	77			146		43
'32 ...	71		29	35	513	66			132		47
'33 ...	94		30	29	534	68			127		44
'34 ...	102		35	24	572	67			123		39
'35 ...	96		37	24	641	69			125		55
'36 ...	119		53	25	646	82			129		52
'37 ...	127		86	37	745	109			134		52
'38 ...	88		60	35	601	81			110		39
'39 ...	86		62	57	741	84			119		36

* Port Philip fleece washed nominal since 1895, exactly in proportion with the value of clean wool.
† Common New Style from 1921 to 1936. China, Extra "A" from 1937.

Average Prices of Commodities—Contd.

No. of Article	38	39	40A	40B	41	42	43	44	45A	45B	35—45	20—45	1—45
	Oil			Seeds	Petroleum *	Soda	Nitrate of Soda	Indigo	Timber		Sundry Materials	Materials	Grand Total
Year	Palm	Olive	Linseed	Linseed	Refined	Crystals		Bengal, Good Consuming	Hewn, Average Import Price.	Sawn or Split, Average Import Price	Total	Total	
	£ per ton	£ per ton	£ per ton	s. per qr.	d. per gall.	s. per ton	s. per cwt.	s. per lb.	s. per load	s. per load			
1873 ...	38	43	32	62	15 1/4	100	15 1/2	6 3/4	65	62	—	—	—
1919 ...	69 7/16	200†	92 3/4	139 1/4	17 3/8	118 1/8	24 7/8	9 1/8	137 1/2	232 1/2	—	—	—
'20 ...	69 3/4	200†	88 1/5	157	25 1/3	150 3/5	24 3/4	14 1/2	119 4/5	261 7/8	—	—	—
'21 ...	36 15/16	80†	31 15/16	72 7/16	22 1/16	140	18 15/16	11 7/16	68 3/4	156 13/16	—	—	—
'22 ...	34 3/5	75 1/2†	39 1/5	75 1/2	15 5/16	123	14 1/3	9 5/6	46 5/8	117 1/2	—	—	—
'23 ...	36 1/4	66 9/16	42 15/16	77 5/8	13	103	13 7/16	7 1/2	48	131 1/8	—	—	—
'24 ...	40 3/16	79 15/16	42 5/8	81 3/16	13 1/8	101 3/4	13 5/8	6 1/16	49 4/5	122	—	—	—
'25 ...	40 5/8	73 2/3	43 1/4	80 5/16	13 1/8	100	13 3/8	5 3/4	47 15/16	122 15/16	—	—	—
'26 ...	37 5/8	79 9/16	32 1/8	63 7/16	13	100	13 1/4	5 5/8	48 23/32	107	—	—	—
'27 ...	34 1/4	102 5/8	31 3/4	64 5/16	13	100	12 9/16	5 1/2	45 7/16	107 9/16	—	—	—
'28 ...	35 3/4	80 7/16	29 7/8	66 15/16	11 11/16	100	10 31/32	5 1/2	45 7/8	111 5/32	—	—	—
'29 ...	34 21/32	72	35 15/32	74 1/8	12 3/4	100	10 1/4	5 1/2	44 3/4	107 21/32	—	—	—
'30 ...	25 1/4	52 1/8	36 5/8	61 15/16	12 3/4	100	9 27/32	5 1/2	44 1/2	102 3/16	—	—	—
'31 ...	19 7/16	53 9/16	18 3/16	38 7/8	11 1/4	100	9 1/8	5 1/2	37 19/32	83 3/8	—	—	—
'32 ...	17 5/8	57 3/16	17	38 5/16	10 1/2	100	8 5/8	5 1/2	35 1/2	75 1/16	—	—	—
'33 ...	15 11/16	53 15/16	20 3/4	39 3/4	10 1/4	100	8 11/32	5 1/2	31 7/8	75 19/32	—	—	—
'34 ...	13 17/32	62 3/16	21 13/32	42 3/8	10 3/32	100	7 13/16	5 1/2	31 1/2	79 13/32	—	—	—
'35 ...	19 3/8	61 13/32	24 13/32	43 1/4	10 1/2	100	7 5/8	5 1/2	32 1/4	73 5/32	—	—	—
'36 ...	19 31/32	70 3/16	28 1/2	48 5/32	10 3/16	100	7 5/8	5 3/4	37 9/16	78 27/32	—	—	—
'37 ...	22 9/16	95 23/32	31 1/8	54 11/32	10 25/32	100	7 25/32	5 3/4	58 3/8	103 7/32	—	—	—
'38 ...	14 7/8	68 19/32	26 7/16	46 27/32	10 7/8	100	8	5 3/4	61 3/32	94 5/16	—	—	—
'39 ...	14 5/16	73 15/16	29 1/8	49 23/32	11 1/16	100	8 1/8	5 3/4	57 9/16	107 9/32	—	—	—
Average 1904–13	31 1/4	43 3/4	26 5/8	49 1/2	6 5/8	60	10 5/8	3	38	56	—	—	—
1890–99	24 1/2	35	19 1/4	38	5 1/8	53	8 1/4	4 3/8	40	45	—	—	—
'78–87	32 1/2	40	23	46	6 7/8	62	12 1/2	6	47	47	—	—	—
'67–77	39	50	30	60	12 1/2*	92	14	7 1/4	60	54	—	—	—
	Index Numbers (or Percentages) of Prices, the Average of 1867–77 being 100												
1873 ...	97	86	105		*122	109	110	92	111		1,163	2,974	5,01
1919 ...	178	400	258		138	128	177	126	325		2,405	5,763	9,27
'20 ...	179	400	272		203	164	177	200	335		2,684	6,851	11,29
'21 ...	95	160	116		177	152	135	158	198		1,600	3,985	6,99
'22 ...	89	151	127		122	134	102	128	143		1,361	3,430	5,89
'23 ...	93	133	134		104	112	96	103	157		1,284	3,488	5,80
'24 ...	103	160	138		105	111	97	84	151		1,325	3,792	6,26
'25 ...	104	147	137		105	109	96	79	150		1,314	3,713	6,14
'26 ...	96	159	106		104	109	95	78	137		1,254	3,395	5,66
'27 ...	88	205	107		104	109	90	76	134		1,302	3,337	5,50
'28 ...	92	161	108		94	109	78	76	138		1,286	3,232	5,39
'29 ...	89	144	122		102	109	73	76	134		1,225	3,082	5,17
'30 ...	65	104	110		102	109	70	76	129		1,071	2,524	4,35
'31 ...	51	108	63		90	109	65	76	106		934	2,138	3,71
'32 ...	45	114	61		84	109	62	76	97		893	2,100	3,60
'33 ...	40	108	67		82	109	60	76	94		875	2,159	3,56
'34 ...	35	124	71		80	109	56	76	97		877	2,214	3,67
'35 ...	50	123	75		84	109	54	76	92		912	2,339	3,78
'36 ...	51	140	85		81	109	54	79	102		964	2,453	3,98
'37 ...	58	192	95		86	109	56	79	142		1,112	2,852	4,61
'38 ...	38	137	81		87	109	57	79	136		954	2,504	4,10
'39 ...	37	148	88		88	109	58	79	145		991	2,690	4,27

* Petroleum average, 1873–77. † Nominal.

MISCELLANEA.

CONTENTS

Population Mathematics—III.

By E. C. Rhodes, D.Sc.

[" Population Mathematics—I," containing Chapters I and II, paras. 1–54, appeared in *J.R.S.S.*, 1940, Part I, pp. 61–89; " Population Mathematics—II," containing Chapters III–VI, paras. 55–101, appeared in *J.R.S.S.*, 1940, Part II, pp. 218–245. References to these are made in the following paper.]

Chapter VII.

102. So far we have been mainly concerned with the form of the birth function on the assumption that the fertility schedule, $\phi(x)$ remained the same over a long period of time. From the form of the birth function we were able to deduce the form of the population function, assuming a constant life-table—*i.e.*, unchanging mortality over a long period of time.

The equation $P(t) = \int_0^A B(t-x)l(x)dx$ enables us to consider the interlaced birth and population functions when the restriction relating to the fertility schedule is removed, but retaining the assumption that the function $l(x)$ remains the same as time changes.

103. We can deduce appropriate forms of $P(t)$ by inserting in the above equation particular functions for $B(t)$.

Thus, suppose $B(t) = B_n t^n$, where n is a positive integer. We have :

$$P(t) = \int_0^A B_n(t-x)^n l(x)dx$$

$$= B_n\left(t^n L_0 - nt^{n-1}L_1 + \frac{n \,.\, n-1}{2} t^{n-2}L_2 \;.\;.\;.\right)$$

where $L_s = \int_0^A x^s l(x)dx.$

If $B(t) = B_0$, a constant, $P(t) = B_0L_0$.
If $B(t) = B_1t$, $P(t) = B_1(tL_0 - L_1)$.
If $B(t) = B_2t^2$, $P(t) = B_2(t^2L_0 - 2tL_1 + L_2)$.
If $B(t) = B_0 + B_1t + B_2t^2$, then $P(t) = B_0L_0 - B_1L_1 + B_2L_2 + t(B_1L_0 - 2B_2L_1) + t^2B_2L_0$.

We see that if $B(t)$ is a polynomial of the nth degree in t, then $P(t)$ is also a polynomial of the nth degree.

104. Interesting illustrations of the connection between the birth function and the population function may be obtained from data for England and Wales.

Census populations (Female) for England and Wales and corresponding values obtained from the equation $P(t) = 10{,}366{\cdot}61 + 131{\cdot}5783t + 0{\cdot}7130844t^2$.

(Population figures in thousands.)

Years	1821	1831	1841	1851	1861	1871	1881	1891	1901
Population ...	6,150	7,126	8,137	9,146	10,290	11,653	13,335	14,942	16,799
Values of t ...	− 40	− 30	− 20	− 10	0	10	20	30	40
$P(t)$	6,244	7,061	8,020	9,122	10,367	11,754	13,283	14,956	16,771

If we are permitted to represent the changes in these population figures by means of the formula

$$P(t) = 10{,}366{\cdot}61 + 131{\cdot}5783t + 0{\cdot}7130844t^2,$$

then the values of B_0, B_1, B_2 which will give us the birth function may be obtained on the assumption of constancy of the life-table function $l(x)$. For we shall have B_0, B_1 and B_2 given by the equations:

$$B_2L_0 = 0{\cdot}7130844.$$
$$B_1L_0 - 2B_2L_1 = 131{\cdot}5783.$$
$$B_0L_0 - B_1L_1 + B_2L_2 = 10{,}366{\cdot}61.$$

105. Lotka gives values of L_0, L_1, L_2 computed from a variety of life-tables in "Applications de l'analyse au phénomène démographique," *Journal de la Société de Statistique de Paris*, Nov. 1933. The following are quoted from that paper:

Values of L_0, L_1, L_2 from Life-tables (Female)

	English			American (whites)		
	1838–54	1871–80	1881–90	1901	1910	1919–20
L_0... ...	41·8752	44·6244	47·1866	51·1019	53·6412	57·5178
L_1... ...	1352·762	1466·602	1567·981	1740·587	1845·173	2014·934
L_2... ...	63298·28	69005·92	73947·0	84027·48	89253·3	99179·1

Taking the 1838–54, 1871–80 and 1881–90 values of L_0, L_1, L_2 in turn, we arrive at the following values of B_0, B_1, B_2:

Values of L_0, L_1, L_2	1838–54	1871–80	1881–90
Values of B_0	358·87	339·02	322·04
,, B_1	4·2424	3·9989	3·7928
,, B_2	0·017029	0·015980	0·015112

From the formula $B(t) = B_0 + B_1t + B_2t^2$ we obtain the following sets of values of $B(t)$ corresponding to census years, and they may be compared with the recorded female births.

Year	England and Wales. Recorded female births (nearest hundreds)	Values of $B(t)$ from the formula, using the various life-tables		
		1838–54	1871–80 (nearest hundreds)	1881–90
1821	—	216,400	204,600	194,500
1831	—	246,900	233,400	221,900
1841	—	280,800	265,400	252,200
1851	300,900	318,100	300,600	285,600
1861	340,400	358,900	339,000	322,000
1871	391,800	403,000	380,600	361,500
1881	433,500	450,500	425,400	403,900
1891	448,500	501,500	473,400	449,400
1901	455,900	555,800	524,600	497,900

106. We notice that, if the 1838–54 mortality experience were supposed to be continually in operation, the number of births necessary to maintain the population as recorded at censuses would be higher than the recorded births as judged by the figures from 1851 onwards. If the 1871–80 mortality experience were assumed to be in operation, then the number of births conforming with actual populations would be near the recorded births from 1851–81, but lower, and higher than the recorded births in 1891 and 1901. Finally, if the 1881–90 experience were assumed throughout, the births obtained from the formula would be much lower than the recorded births from 1851 to 1881, nearly the same in 1891 and higher in 1901.

107. These figures are interesting, since they give the number of births required on the bases of constant mortality, and by comparison with recorded births the effect of constantly changing mortality may be seen.

108. Another illustration is furnished by the population figures for England and Wales from 1891 to 1931. We may represent these figures by means of a second degree polynomial fairly well. The equation for $P(t)$ in this case is:

$$P(t) = 18{,}676{\cdot}06 + 147{\cdot}6600t - 2{\cdot}384286t^2,$$

where t is measured from the year 1911 in one year units.

The following table gives the original data and the values of $P(t)$ from the formula.

England and Wales, Female populations (in thousands)

Year	1891	1901	1911	1921	1931
Recorded	14,942	16,799	18,625	19,811	20,819
Values of t ...	-20	-10	0	10	20
Values of $P(t)$ ...	14,769	16,961	18,676	19,914	20,675

The values of B_0, B_1, B_2 will be given by :

$$\begin{aligned} B_2L_0 &= -2{\cdot}384286 \\ B_1L_0 - 2B_2L_1 &= 147{\cdot}6600 \\ B_0L_0 - B_1L_1 + B_2L_2 &= 18{,}676{\cdot}06. \end{aligned}$$

109. If we now use the values of L_0, L_1, L_2, given in para. 105, obtained from life-tables for the years 1901, 1910 and 1919–20 from American experience, we obtain three sets of values of B_0, B_1, B_2.

Values of L_0, L_1, L_2	1901	1910	1919–20
Values of B_0	432·35	411·63	367·07
,, B_1	$-0{\cdot}28889$	$-0{\cdot}30520$	$-0{\cdot}33711$
,, B_2	$-0{\cdot}046658$	$-0{\cdot}044488$	$-0{\cdot}041453$

The following table gives the values of $B(t)$ from the formula for the different sets of B_0, B_1, B_2, and comparisons with the recorded female births for the census years 1891–1931.

Year	England and Wales Recorded female births (nearest hundreds)	Values of $B(t)$ from formula, using the various life-tables (nearest hundreds)		
		1901	1910	1919–20
1891	448,500	419,500	400,000	357,200
1901	455,900	430,600	410,200	366,300
1911	432,200	432,300	411,600	367,100
1921	413,900	424,800	404,100	359,600
1931	308,500	407,900	387,700	343,700

110. If the 1901 mortality experience were supposed to be in operation throughout, the number of births required for the census populations would be lower than actually recorded in 1891 and 1901, the same in 1911 and higher in 1921 and 1931, considerably more in the later year. If the 1910 or the 1919–20 mortality experience is used, the number of births is less in 1891–1921 and greater in 1931 than the recorded births.

111. The progress with time of the births is similar, under each

mortality assumption. The equation for $B(t)$ shows that each birth function attains a maximum in the decade 1901–10. With the mortality experience of 1901 and that of 1910 the maximum number of births occurs in 1908. The maximum for the 1919–20 mortality experience occurs in 1907. The recorded maximum number of female births occurred in 1903, disregarding the exceptional high level of 1920. The maximum value of the polynomial expression for the population function will occur in 1942. This is consistent with other forecasts of the maximum value of the population.

112. The problem of dealing with other possible birth functions and population functions is complicated by the fact that it is difficult to replace $l(x)$ by any simple mathematical expression which will apply throughout the range of x from O to A. In order to illustrate the possible forms of connected population and birth functions, it is worth while assuming as a simple form of the life-table function, $l(x) = 1 - \frac{x}{A}$. This does not fit adequately any particular life-table, but it does crudely indicate the changes which take place in $l(x)$ when x changes.

113. As illustrations of the results of using this function for $l(x)$ we take first $A = 84$, and, second, $A = 96$, which will give us results somewhat similar to those obtained in para. 105 for the mortality experience of 1838–54 and 1881–90.

We have, now :

$$L_0 = \int_0^A l(x)dx = \int_0^A \left(1 - \frac{x}{A}\right) dx = \frac{A}{2},$$

which gives $L_0 = 42$ (when $A = 84$) and 48 (when $A = 96$).

When we take $B(t) = B_0 + B_1t + B_2t^2$, we get :

$$P(t) = B_0\frac{A}{2} - B_1\frac{A^2}{6} + B_2\frac{A^3}{12} + \left(B_1\frac{A}{2} - B_2\frac{A^2}{3}\right)t + B_2\frac{A}{2}t^2$$

If we use the equation $P(t) = 10{,}366{\cdot}61 + 131{\cdot}5783t + 0{\cdot}7130844t^2$, as in para. 105, we get :

	$A = 84$	$A = 96$
B_0	341·20	311·30
B_1	4·0836	3·6920
B_2	0·016978	0·014856

The following values of $B(t)$ are obtained, to the nearest hundred

Year	$A = 84$	$A = 96$	Year	$A = 84$	$A = 96$
1821	205,000	187,400	1871	383,700	349,700
1831	234,000	213,900	1881	429,700	391,100
1841	266,300	243,400	1891	479,000	435,400
1851	302,100	275,900	1901	531,700	482,800
1861	341,200	311,300			

The general progression of these figures is similar to that of the corresponding figures in para. 105.

For purposes of illustration we could use this life-table function, $l(x) = 1 - \frac{x}{A}$, instead of using values of L_0, L_1, L_2 obtained from life-tables computed from actual mortality experience.

114. Let us then assume $l(x) = 1 - \frac{x}{A}$. In the general case we have:

$$P(t) = \int_0^A B(t - x)\left(1 - \frac{x}{A}\right)dx.$$

Let us write $t - x = u$ and we get:

$$P(t) = \int_{t-A}^{t} B(u)\left(\frac{u + A - t}{A}\right)du$$

$$= \left[\frac{(u + A - t)^2}{2A}B(u)\right]_{t-A}^{t} - \int_{t-A}^{t} \frac{(u + A - t)^2}{2A}B'(u)du$$

$$= \frac{A}{2}B(t) - \left[\frac{(u + A - t)^3}{6A}B'(u)\right]_{t-A}^{t} + \int_{t-A}^{t} \frac{(u + A - t)^3}{6A}B''(u)du.$$

Proceeding in this manner, we get:

$$P(t) = \frac{A}{2!}B(t) - \frac{A^2}{3!}B'(t) + \frac{A^3}{4!}B''(t) - \frac{A^4}{5!}B'''(t) + \ldots$$

115. By substituting any particular function for $B(t)$ in the above we may get the corresponding function for $P(t)$. For purposes of computation it is better to replace the above equation by one more suitable. We can get a fairly good approximation by assuming:

$$\frac{2}{A}P(t) = c_1B(t - A\alpha_1) + c_2B(t - A\alpha_2) + C_3B(t - A\alpha_3).$$

If we expand the right-hand side of this, we get:

$$\frac{2}{A}P(t) = (c_1 + c_2 + c_3)B(t) - (c_1\alpha_1 + c_2\alpha_2 + c_3\alpha_3)B'(t) + \frac{c_1\alpha_1^2 + c_2\alpha_2^2 + c_3\alpha_3^2}{2!}B''(t) \ldots$$

Then, if we arrange that $\frac{S(c_1\alpha_1^r + c_2\alpha_2^r + c_3\alpha_3^r)}{r!} = \frac{2}{r+2!}$ for $r = 0, 1,$ $2, 3, 4, 5$, $c_1B(t - A\alpha_1) + c_2B(t - A\alpha_2) + c_3B(t - A\alpha_3)$ will be the same as $\frac{2}{A}\left(\frac{AB(t)}{2!} - \frac{A^2}{3!}B'(t) + \frac{A^3}{4!}B''(t) \ . \ . \ .\right)$ up to and including the sixth term of the expansion.

116. The finding of c_1, c_2, c_3, α_1, α_2, α_3 does not present any difficulties. $\alpha_1, \alpha_2, \alpha_3$ are the roots of $35\alpha^3 - 45\alpha^2 + 15\alpha - 1 = 0$. We get:

$$\alpha_1 = 0{\cdot}0885880,\ \alpha_2 = 0{\cdot}7876596,\ \alpha_3 = 0{\cdot}4094666.$$
$$c_1 = 0{\cdot}4018636,\ c_2 = 0{\cdot}1396538,\ c_3 = 0{\cdot}4584926.$$

So we may take as a good approximation to $\frac{2}{A}P(t)$ the expression

$$0{\cdot}4018636B(t - 0{\cdot}0885880A) + 0{\cdot}1396538B(t - 0{\cdot}7876596A) + 0{\cdot}4584926B(t - 0{\cdot}4094666A).$$

117. As a check on this, if we assume $B = B_0e^{\lambda t}$, we get:

$$P(t) = \frac{A}{2!}B_0e^{\lambda t} - \frac{A^2}{3!}B_0\lambda e^{\lambda t} + \frac{A^3}{4!}B_0\lambda^2e^{\lambda t} \ . \ . \ .$$
$$= \frac{B_0}{A\lambda^2}e^{\lambda t}(e^{-A\lambda} - 1 + A\lambda).$$

The approximation is $c_1B_0e^{\lambda(t - Aa_1)} + c_2B_0e^{\lambda(t - Aa_2)} + c_3B_0e^{\lambda(t - Aa_3)}$. Therefore $\frac{A}{2}(c_1e^{-\lambda Aa_1} + c_2e^{-\lambda Aa_2} + c_3e^{-\lambda Aa_3})$ should be equivalent to

$$\frac{A\lambda - 1 + e^{-A\lambda}}{A\lambda^2}.$$

Let us assume $A = 100$, $\lambda = 0{\cdot}03$, then we should have:

$$c_1e^{-3a_1} + c_2e^{-3a_2} + c_3e^{-3a_3} = \frac{2 + e^{-3}}{4{\cdot}5}, \text{ approximately.}$$

Actually, the left-hand side is 0·455452 and the right-hand side is 0·455508. The approximation is correct to 0·012 per cent. If we assume $B = B_0e^{-\lambda t}$, we shall have another case for comparison, and here we should get, taking $A = 100$, $\lambda = 0{\cdot}03$ again:

$$c_1e^{3a_1} + c_2e^{3a_2} + c_3e^{3a_3} = \frac{e^3 - 4}{4{\cdot}5}, \text{ approximately.}$$

The left-hand side is 3·57378 and the right-hand side is 3·57452. The approximation is correct to 0·022 per cent.

We may, with some confidence, use this approximation for practical purposes.

118. We will now assume that the birth function is the logistic function—*i.e.*, assume that $B(t) = \frac{1}{1 + e^{-\lambda t}}$, t being measured in years. Let us take $\lambda = 0{\cdot}03$, $A = 100$. We have the following values for $B(t)$ and $P(t)$ for various values of t.

Values of t	$B(t)$	$P(t)$	Values of t	$B(t)$	$P(t)$
− 90	0·0630	1·471	40	0·7685	27·39
− 80	0·0832	1·959	50	0·8176	30·64
− 70	0·1091	2·597	60	0·8581	33·71
− 60	0·1419	3·425	70	0·8909	36·52
− 50	0·1824	4·484	80	0·9168	39·02
− 40	0·2315	5·819	90	0·9370	41·19
− 30	0·2891	7·470	100	0·9526	43·03
− 20	0·3543	9·467	110	—	44·55
− 10	0·4256	11·82	120	—	45·78
0	0·5000	14·51	130	—	46·77
10	0·5744	17·50	140	—	47·54
20	0·6457	20·71	150	—	48·14
30	0·7109	24·04			

119. Since

$$P(t) = 50(c_1 B(t - 100\alpha_1) + c_2 B(t - 100\alpha_2) + c_3 B(t - 100\alpha_3))$$

and each B function is a logistic, $P(t)$ tends to 50 when t tends to infinity.

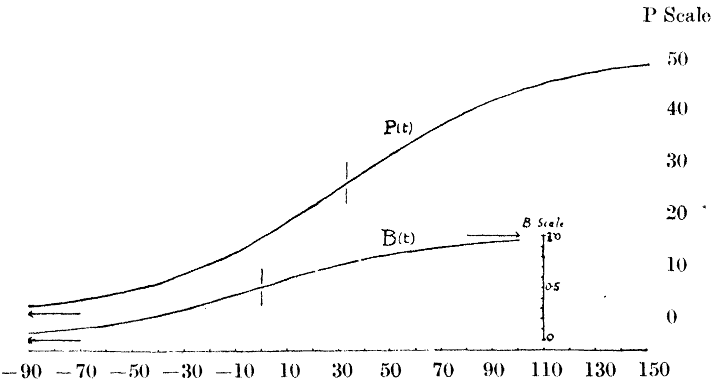

FIG. 1.—Birth and Population Functions.

The accompanying diagram (Fig. 1) shows the values of $B(t)$ and $P(t)$. $P(t)$ is seen to change in a manner reminiscent of that of

a logistic function. There is a point of inflexion where t is about 33, and for this value of t, $P(t)$ is 25.

120. But $P(t)$ is not a logistic function. If $P(t)$ were a logistic function and its equation

$$P(t) = \frac{50}{1 + e^{k(\beta - t)}},$$

taking the maximum value as 50, then we should have:

$$k(\beta - t)\log_{10}e = \log_{10}\left(\frac{50}{P(t)} - 1\right),$$

and $\log_{10}\left(\frac{50}{P(t)} - 1\right) - \log_{10}\left(\frac{50}{P(t+10)} - 1\right) = 10k\log_{10}e$
$=$ constant for all values of t. We can test this by writing down the values of $\log_e\left(\frac{50}{P(t)} - 1\right)$ for different values of t, and obtaining the successive differences.

Values of t	$\log_{10}\left(\frac{50}{P(t)} - 1\right)$	Differences	Values of t	$\log_{10}\left(\frac{50}{P(t)} - 1\right)$	Differences
− 90	1·5185	—	40	$\bar{1}$·9168	0·1166
− 80	1·3896	0·1289	50	$\bar{1}$·8004	0·1164
− 70	1·2613	0·1283	60	$\bar{1}$·6840	0·1166
− 60	1·1335	0·1278	70	$\bar{1}$·5670	0·1170
− 50	1·0065	0·1270	80	$\bar{1}$·4491	0·1179
− 40	0·8804	0·1261	90	$\bar{1}$·3300	0·1191
− 30	0·7553	0·1251	100	$\bar{1}$·2096	0·1204
− 20	0·6316	0·1237	110	$\bar{1}$·0877	0·1219
− 10	0·5093	0·1223	120	$\bar{2}$·9645	0·1232
0	0·3884	0·1209	130	$\bar{2}$·8399	0·1246
10	0·2689	0·1195	140	$\bar{2}$·7142	0·1257
20	0·1506	0·1183	150	$\bar{2}$·5862	0·1280
30	0·0334	0·1172			

The differences are not constant with time.

121. We can, however, find a logistic function which approximates to $P(t)$ by fitting a straight line to the values of $\log_{10}\left(\frac{50}{P(t)} - 1\right)$ in the above table. The line $y = 0{\cdot}0120703\,(33{\cdot}56777 - t)$ gives a good fit to these values and corresponds to the logistic expression

$$\frac{50}{1 + e^{0\cdot0278929(33\cdot56777 - t)}}$$

which has a point of inflexion when $t = 33{\cdot}56777$.

The values of this expression for various values of t may be compared with the values of $P(t)$.

Values of t	Values of $P(t)$	Values of logistic	Differences
− 90	1·471	1·562	+ 0·091
− 80	1·959	2·042	+ 0·083
− 70	2·597	2·661	+ 0·064
− 60	3·425	3·455	+ 0·030
− 50	4·484	4·463	− 0·021
− 40	5·819	5·730	− 0·089
− 30	7·470	7·297	− 0·173
− 20	9·467	9·205	− 0·262
− 10	11·82	11·48	− 0·34
0	14·51	14·08	− 0·43
10	17·50	17·09	− 0·41
20	20·71	20·34	− 0·37
30	24·04	23·76	− 0·28
40	27·39	27·22	− 0·17
50	30·64	30·61	− 0·03
60	33·71	33·79	+ 0·08
70	36·52	36·68	+ 0·16
80	39·02	39·21	+ 0·19
90	41·19	41·38	+ 0·19
100	43·03	43·18	+ 0·15
110	44·55	44·66	+ 0·11
120	45·78	45·85	+ 0·07
130	46·77	46·79	+ 0·02
140	47·54	47·53	− 0·01
150	48·14	48·11	− 0·03

The greatest difference shown is one of 6·2 per cent., for $t = -90$, the largest difference in the later stages of development is 3 per cent. for $t = 0$. If the logistic curve were superimposed on the curve showing $P(t)$ in Fig. 1, the two curves would be indistinguishable. For practical purposes we may perhaps regard the logistic curve as the same as that of $P(t)$.

122. It is interesting to note that the developing constant of this logistic curve is 0·0278929, which differs from that of the birth function, which was 0·03.

Further, the point of inflexion of the logistic curve occurs when $t = 33{\cdot}56777$. We observed in para. 119 that the point of inflexion of the $P(t)$ curve occurred when t was about 33 years.

123. We see, by example, that when the birth function is a logistic the population function is similar to a logistic and may be approximately replaced by a logistic. Alternatively, if the population function is exactly a logistic, the birth function is approximately a logistic.

124. Let us now return to the equation $P(t) = \int_0^A B(t-x)l(x)dx$.

On the basis of what we have already done, let us now expand

$$B(t-x) = B(t) - xB'(t) + \frac{x^2}{2!}B''(t) - \ldots$$

We get:

$$P(t) = L_0B(t) - L_1B'(t) + \frac{L_2}{2!}B''(t) - \frac{L_3}{3!}B'''(t) + \ldots,$$

where $L_n = \int_0^A x^n l(x)dx$, as before.

Thus, in the general case if we know $B(t)$ we can find $P(t)$, when the L's are known.

We can, from the above, obtain $B(t)$ in terms of $P(t)$ and its derivatives. Let us assume:

$$B(t) = a_0P(t) + a_1P'(t) + a_2P''(t) + a_3P'''(t) + \ldots$$

Then we have:

$$\begin{aligned} P(t) = {} & L_0(a_0P(t) + a_1P'(t) + a_2P''(t) + a_3P'''(t) + \ldots) \\ & - L_1(a_0P'(t) + a_1P''(t) + a_2P'''(t) + a_3P^{IV}(t) + \ldots) \\ & + \frac{L_2}{2}(a_0P''(t) + a_1P'''(t) + \ldots) \\ & - \frac{L_3}{6}(a_0P'''(t) + a_1P^{IV}(t) + \ldots) \ldots \end{aligned}$$

giving $1 = L_0a_0$, $0 = L_0a_1 - L_1a_0$, $0 = L_0a_2 - L_1a_1 + L_2\frac{a_0}{2}$,

$$0 = L_0a_3 - L_1a_2 + \frac{L_2}{2}a_1 - \frac{L_3}{6}a_0, \text{ etc.}$$

Hence

$$a_0 = \frac{1}{L_0},\ a_1 = \frac{L_1}{L_0^2},\ a_2 = \frac{L_1^2}{L_0^3} - \frac{1}{2}\frac{L_2}{L_0^2},$$

$$a_3 = \frac{L_1^3}{L_0^4} - \frac{L_1L_2}{L_0^3} + \frac{L_3}{6L_0^2}, \text{ etc.}$$

In this way, for a given $P(t)$, $B(t)$ may be obtained when we are given the life-table function $l(x)$ from which the L's may be obtained.

125. We may obtain an approximation to $B(t)$, which may serve in practice, as a variant of the method used in para 115.

Let us assume that $B(t)$ is approximately the same as $d_1P(t + t_1)$. Substitute in the equation $P(t) = \int_0^A B(t - x)l(x)dx$.

We have:

$$P(t) = d_1\int_0^A P(t + t_1 - x)l(x)dx, \text{ approximately.}$$

This gives:

$$\begin{aligned} P(t) &= d_1\int_0^{A_1}\left(P(t) + (t_1 - x)P'(t) + \frac{(t_1 - x)^2}{2}P''(t) + \ldots\right)l(x)dx \\ &= d_1L_0P(t) + d_1(t_1L_0 - L_1)P'(t) + \\ &\qquad d_1\int_0^A\left(\frac{(t_1 - x)^2}{2}P''(t) + \ldots\right)l(x)dx. \end{aligned}$$

Now, if we take $d_1 = \frac{1}{L_0}$ and $t_1 = \frac{L_1}{L_0}$—*i.e.*, taking t_1 as the mean of the $l(x)$ distribution—then we may write :

$$P(t) = P(t) + \frac{\mu_2}{2}P''(t) + \frac{\mu_3}{6}P'''(t) + \ . \ . \ ., \text{ approximately,}$$

where μ_2, μ_3 . . . are the moments of the $l(x)$ distribution with respect to the mean. Thus the error introduced by assuming that

$$B(t) = \frac{1}{L_0}P\left(t + \frac{L_1}{L_0}\right)$$

is equivalent to an error in $P(t)$ of an amount

$$\frac{\mu_2}{2}P''(t) + \frac{\mu_3}{6}P'''(t) + \ . \ . \ .$$

126. Let us take as an illustration the values of $B(t)$ and $P(t)$ already given in para. 118. For these values we had

$$L_0 = \frac{A}{2} = 50 \,.\, \frac{L_1}{L_0} = \frac{A}{3} = 33\tfrac{1}{3}.$$

From the values of $P(t)$ given in para 118 we can obtain the corresponding values for $t = 3\frac{1}{3}$, $13\frac{1}{3}$, $23\frac{1}{3}$, etc. We have :

Values of t.	Values of $P(t)$	Values of $P(t)$ ÷ 50.	Values of t	Values of $B(t)$	Difference.	Difference, per cent.
− 56·67	3·74	0·0748	− 90	0·0630	+ 0·0118	+ 18·7
− 46·67	4·90	0·0980	− 80	0·0832	+ 0·0148	+ 17·8
− 36·67	6·33	0·1266	− 70	0·1091	+ 0·0175	+ 16·0
− 26·67	8·10	0·1620	− 60	0·1419	+ 0·0201	+ 14·2
− 16·67	10·21	0·2042	− 50	0·1824	+ 0·0218	+ 11·9
− 6·67	12·68	0·2536	− 40	0·2315	+ 0·0221	+ 9·6
3·33	15·48	0·3096	− 30	0·2891	+ 0·0205	+ 7·1
13·33	18·55	0·3710	− 20	0·3543	+ 0·0167	+ 4·7
23·33	21·81	0·4362	− 10	0·4256	+ 0·0106	+ 2·5
33·33	25·16	0·5032	0	0·5000	+ 0·0032	+ 0·6
43·33	28·49	0·5698	10	0·5744	− 0·0046	− 0·8
53·33	31·69	0·6338	20	0·6457	− 0·0119	− 1·8
63·33	34·68	0·6936	30	0·7109	− 0·0173	− 2·4
73·33	37·39	0·7478	40	0·7685	− 0·0187	− 2·4
83·33	39·79	0·7958	50	0·8176	− 0·0217	− 2·7
93·33	41·84	0·8368	60	0·8581	− 0·0213	− 2·5
103·33	43·57	0·8714	70	0·8909	− 0·0195	− 2·2
113·33	44·99	0·8998	80	0·9168	− 0·0170	− 1·9
123·33	46·14	0·9228	90	0·9370	− 0·0142	− 1·5
133·33	47·06	0·9412	100	0·9526	− 0·0144	− 1·2

127. Here we have tested this approximation throughout nearly the whole effective range of a growth curve. The errors are relatively large in the early part of the range where the number of births is small, but in the second half of the range, after the point of inflexion is passed, the errors involved are in the neighbourhood of 2 to 3 per

cent., and diminish when the growth curves tend towards their asymptotic values.

As a crude approximation, this formula

$$B(t) = \frac{1}{L_0} P\left(t + \frac{L_1}{L_0}\right)$$

appears to give fairly accurate results when we are dealing with growth curves in the second stage of their development (for values of t greater than that corresponding to the point of inflexion).

128. As another illustration, we may take the birth and population functions given by Kuczynski in *Fertility and Reproduction*, pp. 82–3. For these functions Kuczynski quotes $L_0 = 58{\cdot}354$, $\frac{L_1}{L_0} = 36{\cdot}007$ (p. 87, *loc. cit.*).

Let us assume

$$B(t) = \frac{1}{58{\cdot}354} P(t + 36), \text{ approximately.}$$

We have the following results.

Year	$P(t)$	$P(t) \div 58{\cdot}354$	Year	$B(t)$	Difference, per cent.	Year	$P(t)$	$P(t) \div 58{\cdot}354$	Year	$B(t)$	Difference, per cent.
2065	5,007,509	85,813	2029	85,594	0·26	2096	4,482,764	76,820	2060	76,587	0·30
2066	4,989,380	85,502	2030	85,095	0·48	2097	4,466,807	76,547	2061	76,290	0·34
2067	4,971,233	85,191	2031	84,598	0·70	2098	4,450,872	76,274	2062	75,996	0·37
2068	4,953,097	84,880	2032	84,184	0·83	2099	4,434,971	76,001	2063	75,708	0·39
2069	4,934,988	84,570	2033	83,851	0·86	2100	4,419,106	75,729	2064	75,424	0·41
2070	4,916,920	84,260	2034	83,519	0·88	2101	4,403,288	75,458	2065	75,145	0·42
2071	4,898,907	83,952	2035	83,183	0·92	2102	4,387,523	75,188	2066	74,872	0·42
2072	4,880,975	83,644	2036	82,844	0·97	2103	4,371,816	74,919	2067	74,605	0·42
2073	4,863,137	83,339	2037	82,582	0·92	2104	4,356,173	74,651	2068	74,342	0·42
2074	4,845,398	83,035	2038	82,314	0·88	2105	4,340,599	74,384	2069	74,083	0·41
2075	4,827,772	82,732	2039	82,118	0·75	2106	4,325,098	74,118	2070	73,827	0·40
2076	4,810,284	82,433	2040	81,912	0·64	2107	4,309,673	73,854	2071	73,572	0·38
2077	4,792,938	82,136	2041	81,696	0·54	2108	4,294,324	73,591	2072	73,319	0·37
2078	4,775,735	81,841	2042	81,466	0·46	2109	4,279,053	73,329	2073	73,069	0·36
2079	4,758,682	81,549	2043	81,222	0·40	2110	4,263,859	73,069	2074	72,820	0·34
2080	4,741,772	81,203	2044	80,958	0·30	2111	4,248,739	72,810	2075	72,573	0·33
2081	4,725,012	80,975	2045	80,755	0·25	2112	4,233,692	72,552	2076	72,325	0·32
2082	4,708,386	80,687	2046	80,531	0·19	2113	4,218,715	72,295	2077	72,077	0·30
2083	4,691,890	80,404	2047	80,288	0·14	2114	4,203,803	72,040	2078	71,828	0·29
2084	4,675,498	80,123	2048	80,025	0·12	2115	4,188,954	71,785	2079	71,581	0·28
2085	4,659,204	79,844	2049	79,745	0·12	2116	4,174,164	71,532	2080	71,335	0·28
2086	4,642,996	79,566	2050	79,448	0·15	2117	4,159,429	71,279	2081	71,088	0·27
2087	4,626,860	79,290	2051	79,206	0·11	2118	4,144,746	71,028	2082	70,841	0·26
2088	4,610,775	79,014	2052	78,951	0·08	2119	4,130,111	70,777	2083	70,592	0·26
2089	4,594,729	78,737	2053	78,679	0·08	2120	4,115,523	70,527	2084	70,341	0·26
2090	4,578,710	78,464	2054	78,397	0·09	2121	4,100,978	70,278	2085	70,090	0·27
2091	4,562,698	78,190	2055	78,103	0·11	2122	4,086,477	70,029	2086	69,839	0·27
2092	4,546,702	77,916	2056	77,804	0·14	2123	4,072,018	69,781	2087	69,588	0·28
2093	4,530,703	77,642	2057	77,500	0·18	2124	4,057,600	69,534	2088	69,338	0·28
2094	4,514,713	77,368	2058	77,195	0·22	2125	4,043,223	69,288	2089	69,087	0·29
2095	4,498,733	77,094	2059	76,890	0·27						

129. Throughout this range of 60 years, the approximate formula gives results which are correct to less than 1 per cent., and for the major part of the range the approximation is correct to less than half of 1 per cent.

As an indicator of the connection between the birth function and the population function this crude formula works fairly well.

The formula may of course be written

$$P(t) = L_0 B\left(t - \frac{L_1}{L_0}\right).$$

130. With some reserve we may test this formula on the figures for female population and female births in England and Wales. We

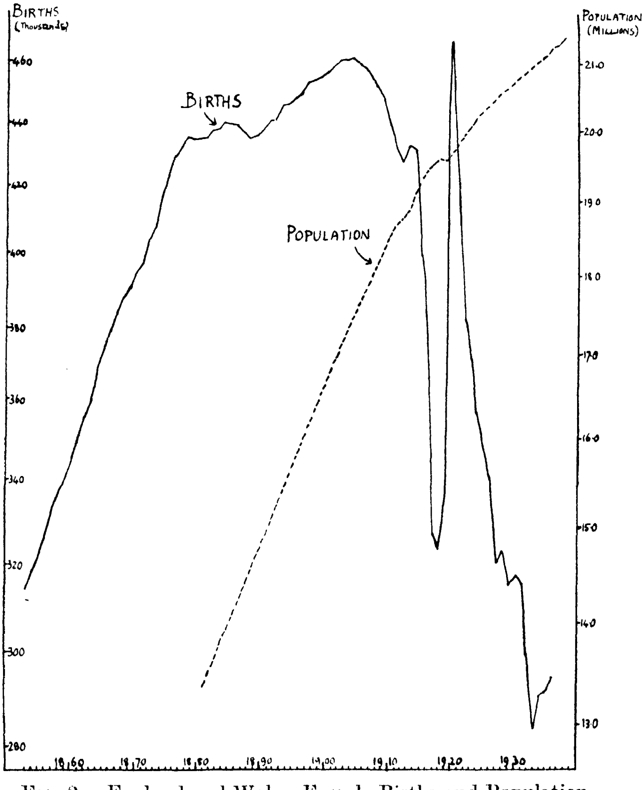

FIG. 2.—England and Wales, Female Births and Population.

do not expect to get evidence that this approximate formula works with exactness, owing to the fact that a constant life-table certainly does not apply. The results however are of interest.

When we look at a graph showing the female births on a logarithmic scale we observe a break in the steady upward trend somewhere in the decade 1875–85. We also observe a maximum in the year 1903. Before the first break the trend of births appears to have

been fairly steady, apart from year-to-year variations, with practically the same proportionate rise each year. After this break the births still rise, but at a slower rate. From about 1904 the births decline until the post-war rise in 1920.

Fig. 2 shows the 5-year moving averages of the births up to 1908, thereafter the recorded births.

131. The population figures show a fairly steady rise at about the same annual rate until about 1915, then a continued rise at a slower rate. It is reasonable to suppose that the break in the birth curve in the decade 1875–85 is related to the break in the population curve in the year 1915, there being a lag of 30–40 years. This corresponds to known values of $\frac{L_1}{L_0}$ from existing life-tables. For the England and Wales female life-table for 1881–90, already quoted, para. 105, $\frac{L_1}{L_0} = 33{\cdot}2$. For the United States white female experience of 1919–20, $\frac{L_1}{L_0} = 35{\cdot}0$.

132. It seems worth while, therefore, computing the ratio

$$\frac{P(t)}{B(t - t_0)}$$

for a period of years, giving t_0 various values in the neighbourhood of 30 to 40, to see whether, for any value of t_0, this ratio appears to be reasonably constant. If it is, we shall have the appropriate L_0 and the appropriate t_0 will give $\frac{L_1}{L_0}$.

Owing to the fluctuations in the birth figures from year to year, a five-year moving average was used instead of the original births. Further, in order to be able to test this as far back in time as possible, the annual recorded births in the years 1851–60 were increased by 2·9 per cent., and those in the years 1861–70 were increased by 1·8 per cent., the proportions estimated by Farr, *35th Annual Report of the Registrar-General*, 1872, p. v, in order to make allowances for non-registrations in those periods.

133. In the following table, the births (5-year moving average) are given from 1853 to 1908, the estimated population from 1883 to 1938, and values of the ratio $P(t) \div B(t - t_0)$ for $t_0 = 30$, 36, and 45. Over the whole period this ratio shows a rising tendency when $t_0 = 30$ and a falling tendency when $t_0 = 45$. For $t_0 = 36$, there appears to be no general trend upwards or downwards. Similar figures were computed for $t_0 = 35$, 37, 38, 39, 40, and for $t_0 = 36$, the ratios appeared to have least variation.

Year	Births (5-year average). (000)	Population (000,000)	Ratio $P(t) \div B(t-t_0)$ $t_0 = 30.$	$t_0 = 36.$	$t_0 = 45.$	Year	Births (5-year average). (000)	Population (000,000)	Ratio $P(t) \div B(t-t_0)$ $t_0 = 30$	$t_0 = 36$	$t_0 = 45$
1853	314·0	—	—	—	—	1896	449·4	15·89	42·1	46·1	—
1854	318·3	—	—	—	—	1897	453·0	16·08	42·1	45·8	—
1855	322·0	—	—	—	—	1898	453·5	16·27	42·1	45·8	51·8
1856	326·5	—	—	—	—	1899	454·8	16·46	42·2	45·6	51·7
1857	332·0	—	—	—	—	1900	456·6	16·65	42·3	45·4	51·7
1858	336·7	—	—	—	—	1901	459·0	16·84	42·4	45·2	51·6
1859	339·9	—	—	—	—	1902	460·7	17·02	42·2	44·9	51·3
1860	344·4	—	—	—	—	1903	460·8	17·19	42·2	45·0	51·2
1861	350·8	—	—	—	—	1904	461·2	17·37	41·7	44·9	51·1
1862	355·2	—	—	—	—	1905	459·0	17·55	41·5	45·0	51·0
1863	361·0	—	—	—	—	1906	458·1	17·73	41·3	45·0	50·5
1864	366·9	—	—	—	—	1907	454·9	17·92	41·5	45·2	50·5
1865	372·4	—	—	—	—	1908	451·5	18·10	41·6	44·8	50·1
1866	378·7	—	—	—	—	1909	—	18·29	42·1	44·9	49·9
1867	382·0	—	—	—	—	1910	—	18·48	42·5	44·4	49·6
1868	386·5	—	—	—	—	1911	—	18·66	42·9	44·1	49·3
1869	389·8	—	—	—	—	1912	—	18·75	42·9	43·7	49·2
1870	394·1	—	—	—	—	1913	—	18·88	43·1	43·8	48·9
1871	396·8	—	—	—	—	1914	—	19·07	43·3	43·8	48·9
1872	403·6	—	—	—	—	1915	—	19·27	43·8	44·4	48·9
1873	407·8	—	—	—	—	1916	—	19·42	44·3	44·7	48·9
1874	416·6	—	—	—	—	1917	—	19·53	44·7	44·9	48·4
1875	422·9	—	—	—	—	1918	—	19·59	45·1	44·8	48·0
1876	428·9	—	—	—	—	1919	—	19·56	44·9	44·7	47·0
1877	431·5	—	—	—	—	1920	—	19·66	45·0	44·7	46·5
1878	434·9	—	—	—	—	1921	—	19·83	45·1	45·1	46·2
1879	434·4	—	—	—	—	1922	—	19·96	45·3	45·5	46·3
1880	434·4	—	—	—	—	1923	—	20·08	45·0	46·0	46·2
1881	434·6	—	—	—	—	1924	—	20·22	45·3	46·5	46·5
1882	436·9	—	—	—	—	1925	—	20·31	45·3	46·6	46·8
1883	438·0	13·68	43·6	—	—	1926	—	20·39	45·4	46·7	46·9
1884	440·0	13·84	43·5	—	—	1927	—	20·48	45·2	46·5	46·9
1885	439·7	14·00	43·5	—	—	1928	—	20·59	45·4	46·7	47·0
1886	438·7	14·16	43·4	—	—	1929	—	20·64	45·4	46·3	46·9
1887	436·8	14·32	43·1	—	—	1930	—	20·73	45·4	46·5	47·1
1888	434·6	14·49	43·0	—	—	1931	—	20·83	45·4	46·5	47·5
1889	435·6	14·65	43·1	46·7	—	1932	—	20·92	45·4	46·6	47·9
1890	436·9	14·82	43·0	46·6	—	1933	—	20·99	45·5	46·3	48·3
1891	440·1	14·99	42·7	46·6	—	1934	—	21·05	45·6	46·4	48·3
1892	440·7	15·17	42·7	46·5	—	1935	—	21·14	46·1	46·5	48·4
1893	445·9	15·35	42·5	46·2	—	1936	—	21·25	46·4	46·5	48·3
1894	446·1	15·53	42·3	46·1	—	1937	—	21·33	46·6	46·4	48·3
1895	448·3	15·71	42·2	46·2	—	1938	—	21·42	47·4	46·5	48·0

The values of these ratios for $t_0 = 30$, 36, and 45 are shown in the accompanying graph (Fig. 3).

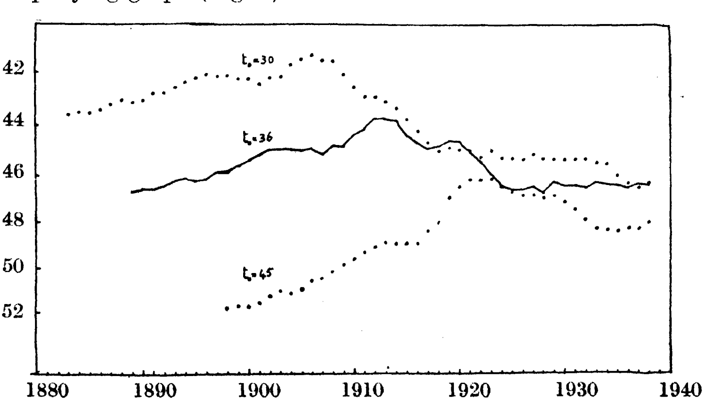

FIG. 3.—Values of $P(t) \div B(t - t_0)$

134. The most appropriate value for t_0 appears to be 36. The average of the ratios $P(t) \div B(t - t_0)$ for $t_0 = 36$ is 45·6. This gives the corresponding value of L_0.

If we now use this value of L_0 and compute 45·6 $B(t - 36)$, we shall get figures corresponding to the population for each value of t. These may be compared with the estimated populations. They are shown in the table below.

Year	45·6 × $B(t-36)$. (mn.)	Population. (mn.)	Difference, per cent.	Year	45·6 × $B(t-36)$. (mn.)	Population. (mn.)	Difference, per cent.
1889	14·32	14·65	− 2·3	1914	19·83	19·07	+ 4·0
1890	14·51	14·82	− 2·1	1915	19·81	19·27	+ 2·8
1891	14·68	14·99	− 2·1	1916	19·81	19·42	+ 3·4
1892	14·89	15·17	− 1·9	1917	19·82	19·53	+ 1·5
1893	15·14	15·35	− 1·4	1918	19·93	19·59	+ 1·7
1894	15·36	15·53	− 1·1	1919	19·98	19·56	+ 2·1
1895	15·50	15·71	− 1·4	1920	20·07	19·66	+ 2·1
1896	15·70	15·89	− 1·2	1921	20·05	19·83	+ 1·1
1897	16·00	16·08	− 0·5	1922	20·01	19·96	+ 0·2
1898	16·20	16·27	− 0·5	1923	19·92	20·08	− 0·8
1899	16·46	16·46	0	1924	19·82	20·22	− 1·8
1900	16·73	16·65	+ 0·5	1925	19·87	20·31	− 2·2
1901	16·98	16·84	+ 0·9	1926	19·92	20·39	− 2·3
1902	17·27	17·02	+ 1·5	1927	20·07	20·48	− 2·0
1903	17·42	17·19	+ 1·3	1928	20·10	20·59	− 2·5
1904	17·62	17·37	+ 1·5	1929	20·33	20·64	− 1·5
1905	17·77	17·55	+ 1·3	1930	20·34	20·73	− 1·9
1906	17·97	17·73	+ 1·4	1931	20·45	20·83	− 1·9
1907	18·10	17·92	+ 1·0	1932	20·49	20·92	− 2·1
1908	18·41	18·10	+ 1·7	1933	20·66	20·99	− 1·6
1909	18·60	18·29	+ 1·7	1934	20·68	21·05	− 1·8
1910	19·00	18·48	+ 2·8	1935	20·74	21·14	− 1·9
1911	19·28	18·66	+ 3·3	1936	20·82	21·25	− 2·1
1912	19·56	18·75	+ 4·3	1937	20·93	21·33	− 1·7
1913	19·68	18·88	+ 4·2	1938	21·01	21·42	− 2·0

135. The greatest difference between the computed population from the formula and the estimated population is 4·3 per cent. in 1912. The general trend of the population is fairly well given by the formula.

When we remember the fact that this formula is only supposed to be an approximation, and that it is by no means certain that a constant mortality experience is the correct assumption to make for this period, and that the population figures are also influenced by migration, the agreement between the figures derived from the formula and the population estimated by the Registrar General may be regarded as quite good.

136. We observe from the table in para. 133 and the accompanying diagram that from 1924–38 the ratio between $P(t)$ and $B(t - t_0)$ lies between 46·3 and 46·7 and appears to be fairly constant. The

average is 46·5. If we use this value of L_0 and $t_0 = 36$ for this period, and continue to use these values for the future, we have the following results.

Year	Births (5-year average) (000)	Year	Population (from Formula) (mn.)	Estimated Population	Difference, per cent.	Year	Recorded births (000)	Year	Population (Formula)
1888	434·6	1924	20·21	20·22	− 0·0	1909	448·0	1945	20·83
1889	435·6	1925	20·25	20·31	− 0·3	1910	439·7	1946	20·45
1890	436·9	1926	20·31	20·39	− 0·4	1911	432·2	1947	20·10
1891	440·1	1927	20·46	20·48	− 0·0	1912	427·7	1948	19·88
1892	440·7	1928	20·49	20·59	− 0·5	1913	432·7	1949	20·12
1893	445·9	1929	20·73	20·64	+ 0·4	1914	431·9	1950	—
1894	446·1	1930	20·74	20·73	+ 0·0	1915	399·4	1951	—
1895	448·3	1931	20·84	20·83	+ 0·0	1916	383·4	1952	—
1896	449·4	1932	20·89	20·92	− 0·1	1917	327·0	1953	—
1897	453·0	1933	21·06	20·99	+ 0·3	1918	323·5	1954	—
1898	453·5	1934	21·09	21·06	+ 0·2	1919	336·2	1955	—
1899	454·8	1935	21·14	21·14	+ 0·0	1920	466·8	1956	—
1900	456·6	1936	21·23	21·25	− 0·1	1921	413·9	1957	—
1901	459·0	1937	21·34	21·33	+ 0·3	1922	380·7	1958	17·70
1902	460·7	1938	21·42	21·42	0	1923	370·8	1959	17·24
1903	460·8	1939	21·42	—	—	1924	356·7	1960	16·58
1904	461·2	1940	21·44	—	—	1925	347·4	1961	16·15
1905	459·0	1941	21·34	—	—	1926	340·3	1962	15·82
1906	458·1	1942	21·30	—	—	1927	320·4	1963	14·90
1907	454·9	1943	21·15	—	—	1928	323·1	1964	15·02
1908	451·5	1944	20·99	—	—	1929	315·0	1965	14·65
						1930	317·4	1966	14·76
						1931	308·5	1967	14·35
						1932	299·6	1968	13·93
						1933	283·7	1969	13·19
						1934	290·8	1970	13·52
						1935	291·2	1971	13·54
						1936	294·7	1972	13·70

137. We see that assuming $t_0 = 36$ and $L_0 = 46{\cdot}5$ gives very good correspondence between the estimated population and the formula for the years 1924–38. We note that when this formula is used on the births subsequent to 1902, the female population from the formula attains its maximum in 1940 at 21,440,000, corresponding to the maximum births in 1904. After 1908 the actual births have been used instead of the 5-year moving average, partly on account of the disturbance in the birth-figures during and immediately after the War years. The subsequent figures for the female population up to 1972 have been computed. They show a decline to about 13,500,000 in 1970. Thus, on this assumption, the female population in 1970 will be about 35 per cent. less than the present population.

138. Somewhat different results for future populations are obtained if we use, instead of the value of L_0 computed from the birth and population statistics, the value for the life-table based on

England and Wales experience for females in 1930–32. L_0 is 62·8 and $\frac{L_1}{L_0}$ is 36·49. These figures are from unpublished material of Mr. D. V. Glass. Let us still use $t_0 = 36$ in our computations, but now take $L_0 = 62{\cdot}8$. The resulting figures for the population, obtained by multiplying the births 36 years earlier by 62·8, are as follows, starting with the births in 1922.

Year	Population (millions)	Year	Population (millions)
1958	23·9	1966	19·9
1959	23·2	1967	19·4
1960	22·4	1968	18·8
1961	21·8	1969	17·8
1962	21·4	1970	18·3
1963	20·1	1971	18·3
1964	20·6	1972	18·5
1965	19·8		

On this assumption of more favourable mortality experience the female population of England and Wales would, in 1970, be about 15 per cent. lower than at the present time.

Appendix

Population growth curves

139. The rate of increase of a population in a unit of time is the ratio of the increase, in the unit of time, of the population to the population at the commencement of this time interval. If a certain population has a size P at a time t, and a size $P + dP$ at a time $t + dt$, the increase in the time dt is dP, and the increase in unit time is $\frac{dP}{dt}$. The rate of increase is $\frac{1}{P}\frac{dP}{dt}$. If this rate of increase is denoted by R, we can state the connection between the population, the time and the rate of increase in the form

$$\frac{1}{P}\frac{dP}{dt} = R.$$

This may also be written in the form $\log P = \int R dt + \text{Constant}$, where the size of the constant can only be determined by a knowledge of the size of the population at some given period of time.

Theoretically, the above equation giving $\log P$ will enable us to get the mathematical connection between P and t when the form of R is known.

140. The simplest case occurs when R is a constant, equal to r (say). In this case we have $P = Ae^{rt}$, where A is a constant. Thus, with a constant rate of increase, the population varies like the exponential function. When t is very large and negative, P is very small, when t is very large and positive, P is very large indeed. There is, theoretically, no limit to the size of the population with a constant rate of increase.

141. When we come to a consideration of cases where R, the rate of increase, may change with time, it is preferable to think of R's changes in relation to the changes in P. Instead of R being constant, we may regard R as gradually decreasing as t and P increase. One of the simplest forms of decreasing functions of P is $r(1 - kP)$, where k is a constant. In this case the differential equation for P becomes:

$$\frac{1}{P}\frac{dP}{dt} = r(1 - kP).$$

On integration, this gives:

$$P = \frac{\frac{1}{k}}{1 + \frac{1}{kA}e^{-rt}},$$

where A is a constant. When t is very large and negative P is very small. When t is very large and positive, P approaches a value $\frac{1}{k}$. This upper limit to the size of the population may be denoted by L instead of $\frac{1}{k}$. Thus the equation becomes $P = \dfrac{L}{1 + \frac{L}{A}e^{-rt}}$. Let us suppose that the value of P is $\frac{L}{2}$ when t has the value β, then we have P in the form:

$$P = \frac{L}{1 + e^{-r(t-\beta)}}.$$

This is generally called the *Logistic* curve of population growth. The differential equation may be written in the form:

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{L}\right)$$

This differential equation shows that P is continuously increasing throughout the whole range of t, since both P and $1 - \frac{P}{L}$ are positive. The second derivative gives:

$$\frac{d^2P}{dt^2} = r\frac{dP}{dt}\left(1 - \frac{2P}{L}\right).$$

Thus $\frac{d^2P}{dt^2}$ is positive when $P < \frac{L}{2}$, is zero when $P = \frac{L}{2}$ and is negative when $P > \frac{L}{2}$. This critical value of L occurs when $t = \beta$. The curve has a point of inflexion where $t = \beta$.

142. The logistic curve was obtained by assuming that the rate of increase declined with time in the manner indicated by the formula $R = r(1 - kP)$. We may obtain other growth curves by assuming $R = rf(P)$, where $f(P)$ stands for any function of P. In particular, if we assume that R decreases with increasing population, we should assume that $f(P)$ was a decreasing function of P. The simple case $f(P) = 1 - \frac{P}{L}$ led to the logistic curve.

143. Another simple form of $f(P) = \sqrt{1 - \frac{P}{L}}$ leads to another well-known curve. Here the differential equation becomes

$$\frac{1}{P}\frac{dP}{dt} = r\sqrt{1 - \frac{P}{L}}.$$

We get, on integration, $P = L\ \text{sech}^2\frac{r}{2}(t - \gamma)$, where $P = L$ when $t = \gamma$.

This curve shows P increasing continuously for values of t from large negative values to γ, when P attains its maximum value L. Thereafter P declines. The curve is symmetrical about the point $t = \gamma$, $P = L$. If the equation is written in the form

$$\frac{1}{\sqrt{P}} = \frac{1}{\sqrt{L}} \cosh . \frac{r}{2}(t - \gamma)$$

we see that the inverse of the square root of the population is graphically represented by the well-known catenary.

144. Another simple form of $f(P)$, $f(P) = \sqrt{\frac{1}{P} - \frac{1}{L}}$, which decreases with increasing P leads to a differential equation,

$$\frac{1}{P}\frac{dP}{dt} = r\sqrt{\frac{1}{P} - \frac{1}{L}}.$$

We get on integration :

$$P = \frac{L}{2}\left(1 - \cos\frac{r}{\sqrt{L}}(t - \delta)\right).$$

Thus the population growth curve is the simple cosine curve, the population is zero when $t = \delta$ and increases to a maximum L when $t = \delta + \frac{\pi\sqrt{L}}{r}$. This curve is applicable when populations increase from zero to a maximum in finite time.

It is apparent that there is no limit to the number of growth curves which may be derived from the equation $\frac{1}{P}\frac{dP}{dt} = R$, where R is supposed to take successively different forms which decrease when P increases.

145. Where population data are given for successive values of t it is possible to determine, approximately at any rate, whether a certain growth curve is appropriate or not.

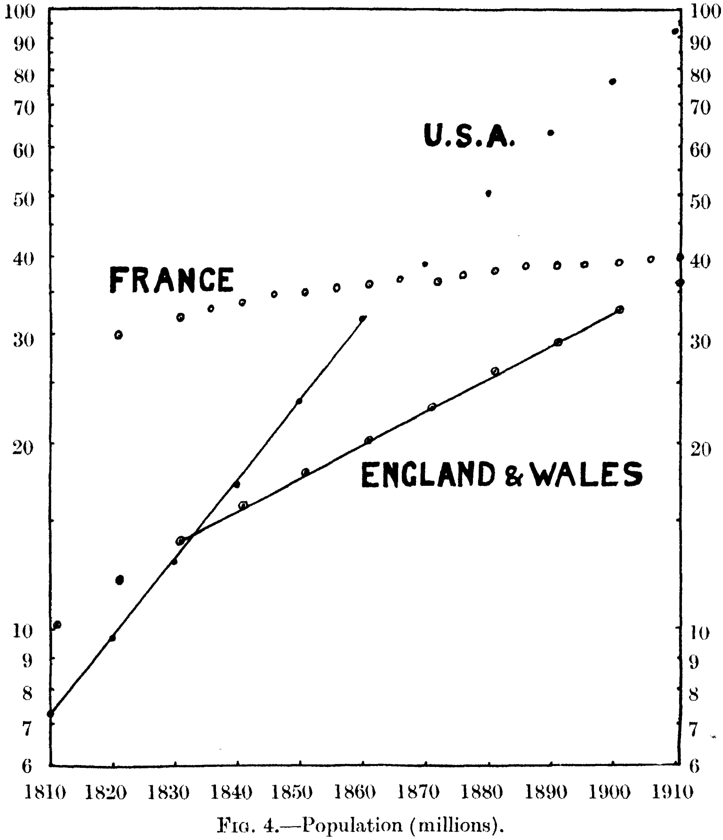

FIG. 4.—Population (millions).

Thus, with the simple exponential curve, if we are given values of P for $t = t_0, t_1, t_2, t_3, \ldots$, the values of P being $P_0, P_1, P_2, P_3, \ldots$, then since $P = Ae^{rt}$, $\log_{10}P = \log_{10}A + rt \log_{10}e$, and we should have, when we plot the values of $\log_{10}P$ against the corresponding values of t, a series of points lying on a straight line. The slope of the line would determine the value of r. If this exponential relationship is inappropriate, the fact would be indicated by these points

obviously suggesting some other relationship between $\log P$ and t than a linear one.

In the diagram (Fig. 4) the population figures at successive enumerations for England and Wales, United States and France are plotted on a logarithmic scale. A glance at this diagram is sufficient to show that the exponential growth curve is inappropriate for any one of these series. Yet for certain periods—*e.g.* England and Wales, 1831–1901, and United States, 1810–60—the points appear reasonably to lie upon straight lines, and for these periods the exponential function might reasonably describe the way in which the population changed.

146. For the logistic curve a different procedure is adopted. Here we have :

$$P = \frac{L}{1 + e^{-r(t-\beta)}}.$$

We may write, for two values of P at times $t = a$, $t = a + h$,

$$\frac{L}{P(a)} = 1 + e^{-r(a-\beta)}, \quad \frac{L}{P(a+h)} = 1 + e^{-r(a+h-\beta)}.$$

We get :

$$\frac{1}{P(a+h)} = \frac{e^{-rh}}{P(a)} - \frac{e^{-rh} - 1}{L}.$$

Thus, if pairs of values $\frac{1}{P(a)}, \frac{1}{P(a+h)}$ are plotted on rectangular axes, the corresponding points will lie upon a straight line of slope e^{-rh}, so long as the population figures are known at equal intervals of time (h). This test is appropriate for census data. Fig. 5 is the diagram for the census data of the United States from 1800 to 1910. The data are given in the following table :

	Population (millions)	Reciprocal		Population (millions)	Reciprocal
1800	5·308	0·1884	1860	31·443	0·0318
1810	7·240	0·1381	1870	38·558	0·0259
1820	9·638	0·1038	1880	50·156	0·0199
1830	12·866	0·0777	1890	62·948	0·0159
1840	17·069	0·0586	1900	75·995	0·0132
1850	23·192	0·0431	1910	91·972	0·0109

The plotted points in Fig. 5 are the following, using x and y to represent the co-ordinates.

x_s	0·1884	0·1381	0·1038	0·0777	0·0586	0·0431	0·0318	0·0259	0·0199	0·0159	0·0132
y_s	0·1381	0·1038	0·0777	0·0586	0·0431	0·0318	0·0259	0·0199	0·0159	0·0132	0·0109

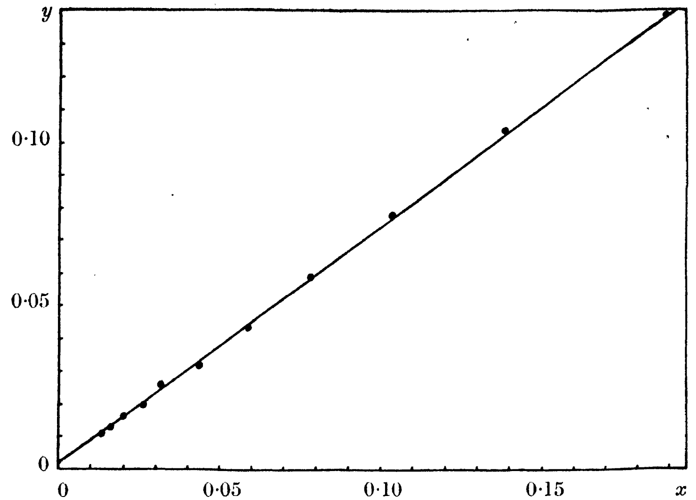

FIG. 5.—United States, 1800–1910. $x = \frac{1}{P(a)}$, $y = \frac{1}{P(a+h)}$.

The equation of the linear connection between x and y may be taken as

$$y - \bar{y} = \sqrt{\frac{S(y_s - \bar{y})^2}{S(x_s - \bar{x})^2}}(x - \bar{x}),$$

where $\bar{x}$, $\bar{y}$ are the average values of x_s, y_s; respectively. This equation is used instead of that which emerges by fitting a straight line on the assumption that errors occur in the values of y_s, the x_s's not being liable to error, or that which emerges when the errors are assumed to occur in the x_s's, the y_s's being exact. Our equation depends on the assumption that both the x_s's and the y_s's are liable to error. The equation of the line is :

$$y = 0{\cdot}73149x + 0{\cdot}001349.$$

We have therefore :

$$e^{-rh} = 0{\cdot}73149, \quad \frac{1 - e^{-rh}}{L} = 0{\cdot}001349.$$

From these we get $L = 199{\cdot}0$. Thus the upper limit of the logistic curve is 199 millions. We also have, since $h = 10$ (years), $r = 0{\cdot}03127$.

Finally, the value of β may be obtained in the following manner. We can now tabulate the values of $\frac{L}{P}$ and since $\frac{L}{P} - 1 = e^{-r(t-\beta)}$, we have:

$$\log_{10}\left(\frac{L}{P} - 1\right) = r\beta \log_{10} e - rt \log_{10} e.$$

Thus

$$r\beta \log_{10} e = \log_{10}\left(\frac{L}{p} - 1\right) + rt \log_{10} e$$

Year	Values of t	$\left(\frac{L}{p} - 1\right)$	$\log_{10}\left(\frac{L}{p} - 1\right) + rt \log_{10} e$
1800	− 50	36·499	0·8833
1810	− 40	26·493	0·8799
1820	− 30	19·652	0·8860
1830	− 20	14·471	0·8889
1840	− 10	10·661	0·8920
1850	0	7·583	0·8798
1860	10	5·330	0·8626
1870	20	4·162	0·8909
1880	30	2·969	0·8799
1890	40	2·162	0·8780
1900	50	1·619	0·8883
1910	60	1·164	0·8808
			Average 0·8825

We take $r\beta \log_{10} e = 0{\cdot}8825$. Hence $\beta = 65$ (years). The point of inflexion in the curve, when the population is $\frac{L}{2}$—*i.e.*, 99·5 millions—occurs at the year 1915 (1850 is the zero for t in the above calculations).

We thus get $L = 199{\cdot}0$ (millions), $r = 0{\cdot}03127$, $\beta = 65$ (years). The values obtained from the curve may be compared with the recorded population series.

U.S. Census populations (millions).

	Recorded	Logistic	Difference, per cent.
1800	5·308	5·317	− 0·17
1810	7·240	7·198	+ 0·58
1820	9·638	9·712	− 0·76
1830	12·866	13·043	− 1·36
1840	17·069	17·412	− 1·97
1850	23·912	23·064	+ 0·56
1860	31·443	30·243	+ 3·97
1870	38·558	39·161	− 1·54
1880	50·156	49·930	+ 0·45
1890	62·948	62·503	+ 0·71
1900	75·995	76·615	− 0·81
1910	91·972	91·773	+ 0·22

The agreement between the two sets of figures shows how well the logistic formula describes the changes in the recorded population from 1800 to 1910.

Grateful acknowledgement is made to Dr. A. J. Lotka, whose many publications have been consulted at each stage of the present work.

It may interest readers to know that a collection of the more important publications of Dr. Lotka on this subject are in the Library of the Society.

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On the Method of Maximum Likelihood

By M. G. Kendall

1. In his recent paper on "The Present Position of Mathematical Statistics," Dr. Bartlett (1940) made some remarks about maximum likelihood which prompted me, in a written contribution to the discussion, to ask how he fitted the likelihood principle into the theory of statistical inference. Dr. Bartlett's reply is necessarily abbreviated; but short as it is, it reveals very substantial differences between us. From subsequent discussion with Dr. Bartlett and others I find that there is an extraordinary amount of difference of opinion as to the proper place which likelihood should occupy in statistical theory and the considerations by which its use should be justified, and it accordingly seems that a useful purpose will be served by an examination of the subject. It is only fair to give notice at the outset that my own views differ as much from other people's as their views appear to differ among themselves; but I have done my best in the following paper to be objective.

2. As a preliminary remark, I should like to make clear my view (from which I understand Dr. Bartlett to dissent) that a discussion of this subject is not of the type which is usually stigmatized by the adjective "academic." The principle of maximum likelihood dictates a course of action, and a man who accepts it may pursue quite a different line from one who does not. We are not here indulging in an argument whether the true is more beautiful than the good, or how many angels can stand on the point of a needle, but are debating the grounds for acceptance or rejection of a rule of conduct. There is as much to be gained by examining the basis of the principle of maximum likelihood as by examining the basis of the principle of Archimedes. An undue enthusiasm for either can lead to very incorrect behaviour.

3. It is as well, perhaps, to begin by a definition of the matter under discussion. Suppose we have a continuous frequency distribution $f(x, \theta)dx$, in which x is the variate and the parameter θ is unknown. A random sample of n values of x is drawn from this population, and it is desired to estimate θ. We have, for the joint probability of the occurrence of values $x_1, x_2, \ldots x_n$

$$dF = f(x_1, \theta) f(x_2, \theta) \ldots f(x_n, \theta)\, dx_1, dx_2 \ldots dx_n \quad (1)$$

$$= L dx_1\, dx_2 \ldots dx_n \quad (2)$$

If f refers to a discontinuous distribution we have a similar equation, but the differentials on the right no longer appear.

The function L is called the Likelihood and the principle of

maximum likelihood states that we shall take as our estimate of θ that function of the observed x's which maximizes L (if such a one exists); that is to say, the estimate of θ is a solution (if it exists) of

$$\frac{\partial L}{\partial \theta} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Since L does not vanish except at extreme values, this is in general equivalent to

$$\frac{\partial}{\partial \theta}(\log L) = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

a form more suitable for explicit solution. Similar equations apply when more than one parameter is to be estimated, but I shall not consider this more general case, which does not appear to raise any new point of principle.

If the distribution f is such that θ can only have discrete values, the differential equation (3) is, of course, not applicable, but θ is still to be estimated as the value which maximizes L.

4. The next point on which it is desirable to have an explicit understanding is: what are we aiming at in estimating the parameter θ? There are two possible objectives here. It is common ground that nothing categorical can be said about the value of θ from the sample. But A may have it as his object to get as close as possible to the real value of θ on this particular occasion; whereas B may be satisfied with the more general assurance of being right over a large number of similar cases—or rather, with having a method which will give better results than other methods *in the long run*. This, it seems to me, is an important distinction. I doubt whether an inquirer of type A can show a single convincing reason why any method should be better than any other at any one trial. The viewpoint of B is, I feel sure, what most of us have in mind in examining the merits of a method of estimation—can it be relied on in the long run, or on the whole?

5. Such phrases as "in the long run" imply that we are considering the act of estimation undertaken on this particular occasion as one member of a series or an aggregate of such acts. In other words, the act is itself a member of a population. It is important to consider what sort of a population this is. We may suppose that as we go through our statistical career the necessity for making estimates will continually present itself; the aggregate of our actual acts of estimation might then be our population; or we may consider all the other people who have to make estimates, and pool all our acts of estimation to form a larger population; or we may even construct a hypothetical number of people confronted with our data, and so construct a hypothetical population of their hypothetical

acts of estimation. Whatever we do, the population forms a background to our method of estimation against which the reliability of the method is judged.

6. The definition of the maximum likelihood principle in section 3 was expressed in somewhat sophisticated mathematical terms, but a certain amount of the mathematical scaffolding can be removed. To put it colloquially, the principle says that when there is a choice of possible values of an estimate, we shall select the one which gives the greatest probability to the observed event. One would like to go further and enunciate the principle as stating that we should choose the *hypothesis* which gives the greatest probability to the observed event. This, however, is only permissible if we impose a restriction on what is meant by a hypothesis. I will digress for a moment on this interesting point.

7. A hypothesis in the wider scientific sense includes any proposition which has meaning and which can be supposed to be either true or untrue. It is a hypothesis that Claudius was murdered by Agrippina, or that life exists on Mars. A statistical hypothesis, in the current usage of the term, relates only to a proposition about the numerical value of a parameter. The two examples just given are not statistical hypotheses and I do not think they can be made so. The distinction is important, but is not so restrictive on the scope of statistical hypothesis as might as first sight appear. The verification of nearly all (some would claim all) scientific laws comes down, in the last analysis, to a determination whether a particular parameter is zero or not.*

8. On the understanding that we are speaking of statistical hypotheses asserting that a parameter has a definite value, we may then enunciate the principle of maximum likelihood as follows: if an event can be explained on the basis of each of a set of mutually exclusive hypotheses, we are to accept that hypothesis (if any) which gives the greatest probability to the observed event. This is very far from being equivalent to saying that we are to accept the hypothesis with the greatest probability. It is possible to believe

* As an illustration of the point here made, it may be noted that Professor E. S. Pearson and Dr. Neyman found their theory of statistical tests on a distinction between two sorts of error: (1) we may reject a hypothesis when it is true, (2) we may accept it when it is false. But

I accept p when p is false
= I reject not-p when not-p is true;
and if q = not-p, this equals
I reject q when q is true,

i.e. there is a logical identity between the two sorts of error. The Neyman–Pearson distinction can, I think, only be made for statistical hypotheses, and then is, in effect, between asserting a numerical equality and asserting a numerical inequality.

in the principle of maximum likelihood while denying any meaning to the expression " the probability of a hypothesis." In fact Fisher introduced the principle largely to replace the ideas of inverse probability which lead to such an expression.

The question I propose to discuss is: what are the grounds on which the principle of maximum likelihood is to be accepted? I cannot pretend to know what is in the minds of all the authorities who use and advocate the principle, but there are at least five different answers to this question.

9. In the first place one can adopt the principle as a kind of postulate of scientific inference, without seeking to establish it by deduction from more primitive concepts. One of the major arguments for this approach (if arguments were deemed necessary) would, I think, be that in essence the principle states that we are to choose the hypothesis which gives the maximum probability to what has actually happened; and that this is extremely plausible because the most probable event is the one which happens the most frequently. For anyone who has not met this argument before it is worth while reflecting on the reason why the principle is not a logical consequence of the fact that probable events are most frequent.

10. Secondly, one could take a stand on the properties of maximum likelihood estimates. It would be pointed out that such estimates provide sufficient statistics if any such exist; that they provide statistics with minimum sampling variance; that the likelihood principle is closely allied to minimum χ^2; and so on. All these properties, it would be claimed, are very valuable statistically and in themselves justify the use of the method.

11. Although the distinction between these two approaches is real enough, I have no doubt that some statisticians would rely on them both. Dr. Bartlett does not, being content to rely on the second. " Maximum likelihood estimates," he says (1940, p. 29), " should be judged on their statistical properties." On the other hand, Professor Fisher, if I understand him correctly, would attribute weight to both. Fisher's attitude is particularly worthy of attention because it was he who christened the maximum likelihood principle and is mainly responsible for our present knowledge of its mathematical consequences.

12. In early work Fisher (1912) based what afterwards became the maximum likelihood principle on inverse probability. This he subsequently considered to be a mistake (1921). " There are," he says (1930), " two different measures of rational belief, appropriate to different cases. Knowing the population we can express our incomplete knowledge of, or expectation of, the sample in terms of probability; knowing the sample we can express our incomplete

knowledge of the population in terms of likelihood." The antithesis between probability and likelihood is insisted on by Fisher in several places. I understand him to subscribe to the frequency theory of probability, but so far as I am aware he has not attempted to analyse the concept of likelihood in terms of simpler ideas such as those of frequency. I hope, therefore, that I interpret his attitude correctly in saying that he regards the maximum likelihood principle essentially as a kind of postulate of statistical inference.

13. A third approach would be a development of that described in section 10. For large samples the maximum likelihood estimate is the one with minimum variance. According to Dr. Bartlett (1940, p. 12), Dr. Silverstone, in some unpublished work, has proved a sort of converse result, that if a minimum variance estimate exists, it must be a maximum likelihood solution. Thus reliance on maximum likelihood may be transferred to reliance on minimum variance. The difficulty is, of course, not resolved thereby, but it is shifted into a region wherein more plausible reasons for acceptance can be adduced. In essence, however, these reasons are the same as those of the first approach referred to in section 9.

14. The three approaches described above are usually combined with a more or less forceful repudiation of the principle of inverse probability. Thus they stand in sharp contrast to the approach of Dr. Jeffreys (1939), in whose theory of inference inverse probability plays a pivotal part. To Jeffreys the maximum likelihood method is acceptable merely because it provides an approximation to exact results given by his theory. "The method," he says (1939, p. 146), "in the great bulk of cases gives results which are indistinguishable from those given by the principle of inverse probability, which supplies a justification of it."

15. Fifthly, an approach which I have met (but have not heard argued at any length) is the purely pragmatic one. Put briefly, it is this: that any method such as that of maximum likelihood is to be judged on results. If the method works in practice, it is sound; if it does not, some other method must be sought.

16. Before putting forward my own views I will describe briefly the reasons why I reject the last four of these approaches. My attitude towards the first will be sufficiently clear in the treatment of my own approach.

17. To take the fifth approach (the pragmatic one) first: I dismiss this on two grounds. First, to anyone who claimed that the maximum likelihood principle is to be judged on its results I would reply that, so far as I am aware, no controlled experiments have been conducted to find out whether the principle works in practice or not, still less to compare it with other methods. Second, in many cases

in which the principle is used experimental verification is theoretically impossible. To say that the principle " works " in practice can only mean, I submit, that the estimates which it gives are near the truth, at least in the majority of cases. (It is not necessary to stop to inquire what it meant by " near " or " the majority "—I am prepared to let the pragmatist choose any definition within reason.) Now, in many cases we never do know whether an estimate is near the truth, for the simple reason that we never know the truth. For instance, if we wish to estimate the probability of getting a head when tossing a penny, we can never find the true probability, which is the limit of the proportion of heads in an infinite number of trials. This criticism, it seems to me, is valid against any attempt at experimental verification in sampling from an infinite population, and *a fortiori* from a hypothetical population.

18. Coming now to Dr. Jeffreys' viewpoint : everything depends here on whether one accepts his theory of probability. Although I pay tribute to the consistence and skill with which Dr. Jeffreys has elaborated his theory, he has not yet convinced me that it is free from objection. Perhaps some day he will, but until then I cannot follow him in regarding maximum likelihood as a method of approximation to the principle of inverse probability. I agree to this extent that the two things, properly understood, are not very different; but whereas probability to Dr. Jeffreys is a measure of rational belief, I am still trying, in this paper, to exhibit inferential processes as based on ideas of frequency.

19. The approach of section 10 is not, I think, legitimate in a discussion of fundamentals. The statistical properties considered as justifying the method are merely mathematical deductions from it. All mathematics are tautology, and mathematics cannot produce from the principle anything which was not latent in it. When comparing different systems which explain the facts equally well, we are entitled to choose the one with the simplest mathematical properties; and sometimes we may even forego accuracy of representation in the interets of mathematical tractability. But no mathematical theorem justifies the axioms and postulates from which it was derived. If the mathematical theorem can be related to observed phenomena, that is a different thing; but in that case the test of acceptability is correspondence between mathematical picture and actual fact. It is a matter of experimental evidence, not of mathematical theory.

20. Before considering the approaches of sections 9 and 13, I would like to dispose of one question concerning inverse probability.

Suppose we require to estimate the parameter θ from a sample of n from the population $f(x, \theta)$.

Suppose the possible values of θ to be discontinuous and finite in number. According to the method of Bayes, we would assume that the actual value of θ was chosen at random from a population of θ's represented, say, by a "prior" probability function $p(\theta)$; that is to say, the relative frequency of any particular θ in this population is $p(\theta)$. According to Bayes' theorem, the posterior probability of θ is given by

$$\begin{aligned} P(\theta) &\propto p(\theta)f(x_1, \theta) \; . \; . \; . \; . \; f(x_n, \theta) \\ &\propto Lp(\theta) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (5) \end{aligned}$$

In accordance with Bayes' postulate, we would assume, in the absence of other knowledge, that $p(\theta)$ was constant. Then the value of θ with the greatest posterior probability is the one which maximizes L. In other words, Bayes' theorem and the method of maximum likelihood give the same results.

The important thing to notice here is that the actual value of θ is supposed to have emanated from the possible values by random sampling. On the frequency theory of probability no other supposition is, I think, possible. If one is prepared to take prior probability as a measure of uncertainty of mind, of course, the position is different, and no question of the generation of the actual value of θ need appear. On such an approach Bayes' theorem would be a proposition about attitudes of mind (and this is Dr. Jeffreys' view). But to the adherent of the frequency theory of probability Bayes' postulate can only be interpreted as an assumption about the occurrence of θ in a population of θ's; and what it says is that unless anything is known to the contrary the population is to be assumed rectangular.*

21. If, in reaching equation (5), we had described the population with reference to a parameter ϕ functionally related to θ, Bayes' theorem and the method of maximum likelihood would still give the same results; for the values of θ, and hence of ϕ, are discontinuous, and to assume that all values of θ are equally frequent is to assume the same of ϕ. But if the prior distribution of θ, say $p(\theta)d\theta$, is continuous an apparently different situation results. As before,

$$P(\theta) \propto Lp(\theta)d\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (6)$$

Had we worked with a parameter ϕ, the principle of maximum likelihood would give the same result as for θ, since $\frac{\partial L}{\partial \theta}$ and $\frac{\partial L}{\partial \phi}$ vanish together, the latter being equal to $\frac{\partial L}{\partial \theta}\frac{\partial \theta}{\partial \phi}$. But inverse probability

* As a matter of convenience I will call the population rectangular even if it is discontinuous, meaning that all values occur equally frequently.

would only give the same answer if we assumed that the prior distribution of ϕ, say $g(\phi)$, was constant. This is inconsistent with the former assumption that $p(\theta)$ was constant, for

$$p(\theta)d\theta = p\{\theta(\phi)\}\frac{\partial\theta}{\partial\phi}d\phi$$

and hence

$$g(\phi) = p\{\theta(\phi)\}\frac{\partial\theta}{\partial\phi}.$$

This was one of the grounds on which Fisher (1924) criticized inverse probability. If we assume a certain continuous form for the prior distribution of one parameter, we cannot in general assume the same form for a second parameter functionally related to the first.* But it seems to me that to an adherent of the frequency theory of probability the criticism is not so much one of inverse probability as of the use of continuous frequency distributions; and I proceed to expand this point.

22. The sort of population which occurs in practice is invariably discontinuous. Even when we have data, such as height measurements or rainfall records, in which the values of the variate can in theory be any of a continuous range, the universe is discontinuous and finite, being limited on the one hand by the impossibility of measuring to more than a certain degree of accuracy, and on the other hand by the impossibility of making an infinite number of measurements. In practice it is found that certain populations can be represented mathematically to a satisfactory degree of approximation by a continuous function. We write

$$dF = f(x)dx$$

expressing that, wherever we choose x, the frequency between two values $x - \frac{1}{2}dx$ and $x + \frac{1}{2}dx$, close together, will be $f(x)$. This approximation holds only as long as we do not make dx too small. But we find that as the number in the population increases the approximation is accurate enough for smaller and smaller values of dx, and so we proceed to the limit. The process is so familiar to mathematicians that it passes almost without comment.

23. Now, what does it mean to take a random sample from such a population? We can, I think, only assign a meaning to such an expression by considering a limiting process. Let us in fact imagine a process of selection which is random with respect to the variate-values which are the centres of a series of intervals, and let us imagine

* This is certainly true if, as is here supposed, prior probability is a relative frequency. Dr. Jeffreys avoids this difficulty by taking his prior probability as an expression of ignorance, which is not subject to the laws of transformation of frequency-distributions.

the intervals tending to zero limit. If the process remains random however small the intervals become, we may regard it as tending to a sort of "continuous" random selection. In practice such a selection would not be realized, but there appears to be nothing contradictory in the idea, and it may serve to describe the external world in the same way that some processes of finite summation can be adequately represented by integration.

24. But it is important to realize that randomness of this kind is dependent on the nature of the process to the limit. Consider, for instance, the problem of taking points at random on a straight line MN of unit length. We may divide the line into segments of length $1/n$ and take a sample of these segments by a method which gives each segment an equal probability. We may then imagine n, the number of segments, to tend to infinity, and we should, for instance, have the result that the probability of a point lying on the half of the line terminating at M is $\frac{1}{2}$. Now take MP perpendicular to MN and also of unit length, and consider a star of $(n - 1)$ lines through P, together with PM and PN and making angles $\pi/4n$ one with the next. These will cut off n segments on MN, and we may again consider a process which gives each segment an equal probability. If now we proceed to the limit, we see that the probability of a point lying on a given segment is proportional to the angle which that segment subtends at P. For instance the probability that a point lies on the half of the line ending at M will be $\frac{4}{\pi} \tan^{-1} \frac{1}{2}$, which is not the same result as before. There is nothing paradoxical in this. The probabilities are different because the limiting random process was different.

In drawing inferences from samples from continuous universes, therefore, it is necessary that the selection shall be random *for the particular limiting process employed.*

25. When this fact is borne in mind, I think the apparent inconsistency in the principle of inverse probability disappears. θ and ϕ are regarded as having been drawn at random from populations of θ's and ϕ's. If those populations are continuous it is necessary to have regard to the limiting process under which the random selection was carried out. If we replace such statements as "the most probable value" by "the most probable range of values" and consider that range as tending to zero, Bayes' theorem gives consistent results. For example, if we assume that $f(\theta)d\theta$ is a rectangular population, we may say that frequencies of θ in the ranges $d\theta$ are equal, that, as $d\theta$ tends to zero, they remain equal, and that θ is regarded as having emanated from the population by a process of random selection which gave all equal ranges an equal

chance of appearing. Had we considered a parameter ϕ, we should, on the same assumptions about θ, have reached the result that frequencies in ranges $\frac{\partial\theta}{\partial\phi}d\phi$ are equal and such ranges are equally probable. This is, of course, different from supposing that the selection of the parameter was such that ranges of $d\phi$ were equally probable, and Bayes' theorem gives a different answer for such a case. In the first case we assumed that the actual value of θ emanated from the population of θ's by a process which is random in the limit for $d\theta$; in the second case by a process which is random for $d\phi$. The two give different results just as the two cases of direct probability considered in section 24 gave different results, and for the same reason.*

26. I have explained that Bayes' postulate for an adherent of the frequency theory of probability is equivalent to an assumption about the equidistribution of parameters in populations from which actual parameters under estimate emanate. My own view can now be stated shortly: that the use of the principle of maximum likelihood is ultimately based on precisely similar assumptions. In the following defence of this statement my attitude towards the approaches referred to in sections 9 and 13 will, I hope, be clear.

27. I remark in the first place that if the line of reasoning of section 25 is accepted, both inverse probability and maximum likelihood will always give the same results if proper regard is had to the way in which Bayes' postulate for a continuous universe of parameters is to be interpreted; and it would be surprising if in the one case we could dispense with assumptions which are necessary to support the other.

28. It is best, however, to go right back to the principle of maximum likelihood and examine it on its merits. Why should we accept that hypothesis which gives the greatest probability to what has actually happened, considering that something which has happened is certain? This is the point at which it is useful to bear in mind the distinction drawn in sections 4–6 between type A, the man who claims to be right on this particular occasion, and type B, the man who is content to be right in the long run. A's position

* A great deal more remains to be said about the difficulties associated with continuous populations, but I defer a full discussion for the present. It may however be noted that by a suitable choice of the form of the population from which the sample is derived the principle of maximum likelihood itself can be made to give apparently inconsistent results. A change of variate in a continuous distribution will not leave the maximum likelihood solution invariant. Thus different choices of parent populations will give different estimates of a parameter. It is to be remembered that in many cases the form of the population is at choice, for any continuous distribution can be transformed into any other continuous distribution.

seems to me indefensible. He says, in effect: on hypothesis H_1 the observed event is more probable than on hypothesis H_2, and therefore H_1 is right. Translated into frequency this is equivalent to saying that if H_1 was right, and he took a large number of samples, the event would be more frequent than if H_2 was right; and therefore H_1 is right. But he has not taken a large number of samples and does not intend to do so; and he is therefore not allowed to appeal to the theory of probability for help. At one trial anything may happen.

29. As I said in section 5, I think most statisticians are of type B, and consider any particular act of estimation as one of a population of acts. What guarantee or reasonable ground for belief is there that in this population the method of maximum likelihood will give the right answer in the majority of cases, or in even one case, or that it will be right more often than blind guesswork? There must be some reason for a belief that it will. To assume the principle as a postulate is all very well for purposes of a formal exposition, but belief in the truth of the principle has nevertheless to be justified, or, at the very least, we have to consider whether the belief can be justified before accepting the principle as an Article of Faith. After prolonged introspection I cannot escape the conclusion that one's attitude to the principle, like one's attitude to probability, is ultimately based on ideas of frequency. I do not mean that the principle of maximum likelihood can be logically deduced from frequencies in aggregates on the lines of the attempts to deduce the theory of probability in that way; but whether we take probability as a relative frequency or an attitude of mind, I think that the foundation for the conception, psychologically speaking, is the relative frequency with which events happen in the world at large. Similarly, I am unable to separate a belief in the principle of maximum likelihood from a belief that events in the universe happen in a certain way. The populations of happenings in this case are of a higher type than those considered in the theory of probability, being populations of populations. Bayes' postulate deals explicitly with the point. The principle of maximum likelihood, as it seems to me, makes equivalent assumptions, but leaves them tacit. We suppose that in our populations of acts of estimation a definite state of affairs will be realized in the long run—that is to say, that our estimates will be nearer the truth than the estimates given by alternative methods. The basis for this supposition is not, I think, the notion of probability as ordinarily understood (relating to populations of samples from a constant aggregate), but a truly inverse notion about the way in which the aggregates from which samples are taken are distributed in the universe. The populations

under estimate are supposed to be distributed in such a way that if we follow the principle of maximum likelihood, we shall be right in the greater number of cases. One of the ways in which the populations could be distributed to justify this assumption would be that indicated by Bayes' postulate, and though there may be others, I think this is the one which is at the back of our minds. To deny that the principle is founded on such conceptions seems to me to be tantamount to asserting that it must be accepted as a measure of belief on intuitive grounds. Perhaps in the long run this avoids a lot of trouble, and it certainly avoids a lot of argument; but what it does not avoid is the intellectual uneasiness which attaches to any code of procedure based on faith or intuition rather than on reason.

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REVIEWS OF STATISTICAL AND ECONOMIC BOOKS

CONTENTS

1.—*Statistical Mathematics.* By A. C. Aitken. (University Mathematical Texts.) Edinburgh and London: Oliver and Boyd. 1939. $7\frac{1}{4}'' \times 4\frac{3}{4}''$. vii + 153 pp. 4*s*. 6*d*.

In his paper given to this Society (see this *Journal*, Vol. CII, pp. 532–64), Dr. Wishart has emphasized the need for a treatise on Mathematical Statistics as opposed to the existing "Method Text-Books" dealing with the applications of statistical methods to business, economics, biology, agriculture, psychology, etc. This distinction between two types of text-books is very useful. It is commonplace that "Method Text-Books" are best understood by the majority of their readers if the mathematical apparatus is cut down to a minimum. Their authors will therefore welcome a mathematical treatise for purposes of reference; they will feel justified in giving applications of methods rather than proofs and will be relieved from the ungrateful task of explaining mathematical arguments by mysterious hints or cumbersome elementary arithmetic.

With his *Statistical Mathematics* Dr. Aitken has made a first step towards producing such a mathematical treatise. It will be welcome to all those who know, or are prepared to learn, a reasonable amount of mathematics, viz. elementary algebra up to the binomial theorem, simple infinite series, and elementary calculus. Each chapter is an example of clear and concise mathematical argument, and readers of a certain standard of maturity in mathematical reading will no doubt appreciate the book more than beginners. The author does not claim that in the 148 small-sized pages he has tackled the task described by Dr. Wishart in his paper. Dr. Aitken has confined himself to the mathematical foundations of statistics, leaving, maybe

wisely, a good deal of the theory to the discrepant needs of the applications.

The deductive line of approach has been adopted. Starting from fundamental conceptions of probability, classical large-sample theory is first developed. Orthodox proofs have been modernized by the use of generating functions, and in particular by factorial moment generating functions. Some elementary but important points are fittingly stressed, and greatly facilitate the understanding of the more advanced part of the theory. We should mention section 11, giving the effect on moments and semi-invariants of a change in origin and scale, and also section 37, dealing with transformations of the shape of a distribution function by a mathematical change of variable. Section 20, where Sheppard's Correction is derived, gives a clear and concise discussion of its validity, which is shown to be conditioned by the accuracy of an approximate quadrature. The numerical processes given, such as the computation of moments by summation, practical curve fitting with standard curves, practical routine of fitting a polynomial, the solution of normal equations, periodic regressions, etc., all show a masterly technique based in many cases on Dr. Aitken's original researches. The seventh and last chapter deals briefly with the fundamental sampling distributions, viz. Student's t, the χ^2 and z distributions, and the probability integral of the correlation coefficient.

If a criticism may be made, it would concern omissions (due perhaps to lack of space) of important modern small-sample results. The problem of estimation is dealt with in a last section covering half a page. The relation between R. A. Fisher's analysis of variance and the classical theory of dispersion due to Lexis, Charlier and Coolidge is well brought out. But this merely touches the interesting aspect of the analysis of the binomial variance and corresponding methods of estimation which have been developed recently through new results applicable, mainly, to problems in genetics.

These limitations in scope, however, which are probably dictated by the standard size of the University Mathematical Texts, should in no way detract from the value of this admirable little book.

H. O. H.

2.—*Mathematics of Statistics.* By J. F. Kenney. New York: Van Nostrand. London: Chapman and Hall. 1939. 2 Vols. Part I, 12*s.* 6*d.* Part II, 11*s.*

It is perhaps more than a mere coincidence that a number of books on Mathematical Statistics have made their appearance just at the time when the need for such a treatise was stressed by Dr. Wishart in a paper read to this Society. J. F. Kenney's *Mathematics of Statistics* is another step towards the completion of a modern text on the subject.

Scope and style of this work have been adapted to the needs of the student in his early stages, and are closely linked with a definite system of teaching at American Universities. There is a detailed and sometimes too extended discussion of elementary statistical aspects, supplemented by numerous instructive examples and

exercises. The subject-matter is given in two volumes. Part I deals with elementary concepts and is intended for students with a knowledge of higher School Algebra only. Part II, on the other hand, requires some acquaintance with higher mathematics and gives the more advanced theory. This part is, up to a point, independent of Part I, and the subdivision of the subject-matter has been accomplished with great skill.

In Part I the inductive line of approach has been adopted, and the starting-point is a practical and simple problem: namely, the tabulation of data. Elementary conceptions, such as frequency distributions, graphical representations, averages, and measures of dispersion, are introduced from the need of summarizing data. In this the subject-matter has very much in common with American method text-books. However, great care is taken to explain fully the simple mathematical background of each operation (even such simple matters as the use of the summation sign Σ are taught). As a whole, the text is clear and instructive. There is, however, an unfortunate tendency to state simple relations in terms of formal theorems of mathematical appearance. To give an example, Theorem VII on p. 37 exhibits the effect on the mean of a change in scale and origin by a mathematical formula which might frighten the beginner, whereas a practical example such as the transformation of degrees Fahrenheit to degrees Centigrade would have made matters clear.

An adequate discussion of the comparative merits of averages and measures of dispersion can, of course, only be given in the light of the theory of estimation, but the author gives a number of practical points which are of interest.

The section on correlation deals with large-sample-relations between the correlation- and regression-coefficients, correlation ratios, etc. Tests of the "reliability" of regressions are referred to the normal curve instead of to the t-distribution. Although it is pointed out that such a test is only approximate, it would have been better to anticipate here the *application* of the t-test whilst postponing its *derivation* to Part II.

Two minor points of criticism must be added. The term "number of variates" is used in the sense of "number of observations," or "number of items in the sample." In view of a possible confusion in genuine "multi-variate problems," it would have been better to adhere to the usual terminology. A similar inaccurate use of professional terms makes the section on calculating machines of little value.

Part II gives the application of the theory of probability to statistics. In an orthodox, deductive line of approach the fundamental theorems of Bernoulli, Laplace and Poisson are introduced from first principles. One misses a logical link with the descriptive, inductive Part I, and would have liked to see the theory of probability introduced as an appropriate hypothesis on which statistical tests are to be based.

In dealing with the Poisson distribution, infinite summation is avoided and approximate results are obtained by a summation over a finite number of categories. An appeal to text-book mathematics

would have been more convenient and could have been added very well to the numerous similar appeals of Chapter II, giving theorems carefully selected to facilitate the teaching of the subsequent sections. A chapter on frequency curves (giving the normal and Pearson type curves, hypergeometric and Charlier series) is followed by a discussion of correlation-theory in terms of a two-variate distribution function. There is some unavoidable repetition of subject-matter which was given in Part I in terms of the sample correlation. The text extends to multi-variable correlation theory, but many of the theorems are of academic interest only.

The most important sections are those on small-sample theory, where the reader is guided with considerable skill through the distribution problems of the fundamental modern statistics. Fisher's geometrical argument is given as a proof of the joint distribution of $\bar{x}$ and s^2. Unfortunately, the author denotes by s^2 the mean of the squared deviations (and not the best estimate of σ^2, which he denotes by $\hat{\sigma}^2$). It is followed by a derivation of the t- and z-distributions, a discussion of Fisher's approximate distribution of the transformed sample correlation, and a proof for the χ^2 test of "goodness" of fit from properties of the multinomial.

The last chapter deals with statistical inference and the theory of estimation, which are rightly left to the end, as the most difficult part. The essential difficulties concerning *a priori* probability assumptions are well brought out and their partial solution by the fiducial argument and "confidence belts" is clearly explained.

The text will be welcome to students and all those who, with little mathematical equipment but a good deal of patience at their disposal, wish to learn something about the mathematical foundations of statistics. H. O. H.

3.—*Statistical Technique in Agricultural Research: a simple exposition of practice and procedure in biometry.* By D. D. Paterson. New York and London: McGraw Hill, 1939. 9″ × 6″. ix + 263 pp. 19*s.* 9*d.*

This book is an attractive and useful work for the agricultural research worker, and includes most of the modern statistical technique he is likely to require. It includes chapters on analysis of variance, goodness of fit and contingency tables, correlation, regression, and field experiments; and as it concludes with an appendix of statistical tables, is a self-contained manual for the active research investigator.

Mr. Paterson is Reader in Agriculture at the Imperial College of Tropical Agriculture, Trinidad, and he might consequently be expected to be concerned rather with the practice of statistical method than with its theory. He may thus perhaps be excused some theoretical lapses, such as the suggestion on page 16 that when the number of observations in the sample is large, it can be assumed that their distribution is normal; or the statement on page 42 that Fisher's z is normally distributed. Nevertheless, the reader is entitled to expect accuracy in a text-book, and must be on his guard against errors of this kind. Further, the author can be less excused for lapses which more obviously affect his experimental lay-outs and

analyses. A particularly glaring example is the analysis of data on page 53 without the observations first being subjected to a transformation. On page 230 a curious criticism is made of the $2 \times 2 \times 2$ design, treatment differences isolated in single degrees of freedom being imagined to lead to less sensitive, rather than more sensitive, tests of significance.

The reviewer considers the comparatively late position of the subject-matter of Chapter IV (Diagrams) anomalous, nor can he understand the omission of the method of multiple regression in Chapter VI. In spite, however, of these criticisms, the book is recommended to any statistical research worker. M. S. B.

4.—*Graphic Presentation.* By Willard C. Brinton. New York: Brinton Associates. 1939. 9″ × 6″. 512 pp.

This book contains almost 500 pages of charts collected from many sources. There is almost no padding; each chart is accompanied by only a brief explanation, so that the total number of charts is enormous. It is a pleasure to turn from the orthodox presentation of a few diagrams and graphs in one chapter of a text-book of statistics with long but rather stilted explanations, justifications and criticisms to something fresh. But this something fresh is not a substitute for text-books, the definitions do not always elucidate—for instance an Ogive chart is " a frequency distribution in which 'more than' or 'less than' data are presented. One scale of the grid represents percentages and the other scale represents 'more than' or 'less than' values." This work must be a fruitful source of ideas for those whose duty it is to present statistics in a new and interesting way for the jaded appetites of boards of directors. The 676 illustrations have been selected from thousands of clippings which the author could not resist saving during the 25 years that have elapsed since publication of his *Graphic Methods for Presenting Facts* in 1914.

A first reaction to the book is one of repulsion at the apparent complications of some of the charts, but one can expect half-page reproductions of map charts, progress, relationship and chronology charts to appear confusing. In contrast, some of the flow-charts appear interesting even in the reproductions. It is interesting to see the thin black line representing reparations paid by Germany and retained by Britain. It is interesting to see the relative importance of various sources of capital for construction in the United States, and in the same diagram the forms of construction resulting from the investment.

Examples are given of maps, bar charts, sector charts, time series, frequency, Ogive and Lorenz curves, and ratio charts, as well as some examples of three-dimensional models. There are about 50 pages of suggestions on the preparation of charts, including chapters on the use of the camera and lantern slides. There are pictorial graphs and cartoons, as when the burglar adjusts appropriately the office chart of assets before leaving.

An ultimate conclusion is that there is nothing fundamentally new, but that the number of permutations of a few basic diagrams is very large, and many of these give an impression of newness. H. S. B.

5.—*Population Policies and Movements in Europe.* By D. V. Glass. Oxford: Clarendon Press. 1940. 9″ × 6″. vi + 490 pp. 25*s*.

Whatever the future may have in store for us, there is no doubt in the mind of most of us that there will be a future. Already, during this past year of war, various authoritative persons have been revelling in plans for future organisation of one kind and another. Whatever the faults of the past, it serves inevitably as the foundation of the future. This book, giving a summary of up-to-date population policies and movements in Europe, presents a picture of the keystone of the future building, Population.

It is a big book, but one can see that it had to be a big book. One can hardly explain Italian or German methods for reviving the family without showing the previous trends which have led to the new policies of those countries. One can hardly give an account of the trends in population movements in the past century without showing to what extent these trends have been similar in different countries. The explanation of these changes brings in a variety of ideas and technical processes. It is needful to include an account of these processes. So, this book is necessarily a large book. In many respects it should be regarded as a book of reference. There is a wealth of bibliographical material.

Mr. Glass starts off by giving us an account of the changes in population size and structure in England and Wales during the past hundred years. This is illustrated by statistics of all kinds from the national records. He gives an account of the movement for the control of population size and brings us up to date with a picture of a population about to attain its maximum. We see that in the past few years there has been net immigration into the country. We finish the chapter with the feeling that the country had reached the stage when problems of future population were beginning to be the concern of the State. The next chapter is devoted to a *résumé* of the history of State intervention in matters of population. One gathers that many laws may have been passed without much effect.

Then we are given accounts of the movements in France and Belgium towards Family Allowances, direct economic aid to workers with large families. There are some interesting statistics giving the proportion of family allowances to basic earnings. An attempt is made to assess the successs of the various schemes which have been introduced by the pro-natalist movement. Mr. Glass doubts if there is any statistical evidence that the population measures of these two countries have been sufficiently influential to cause a rise in fertility. He goes on to attempt to compute what is the requisite allowance per child necessary to influence fertility. The computations give results in excess of allowances actually allotted in practice.

A chapter on Italy follows. Here again, Mr. Glass tries to estimate from the available statistical data what has been the effect of the Italian policies. Up to 1938, " The fall in fertility showed no visible check." It is admitted that the fall in fertility might have

been greater had it not been for the population measures put into operation since the advent of the new regime.

The German experience is next analysed. Here, undoubtedly, there has been an increase in births, attributed to the special measures of the new order in Germany. It is, of course, impossible to exclude the possibility that the change in Germany is due to the "psychic rebirth" of the nation. The materialist can perhaps find more substantial reasons. Mr. Glass thinks that the suppression of illegal abortion has been a major factor.

An examination of the population measures introduced in the Scandinavian countries follows.

One is left with the feeling that very little positive result has been achieved in all these countries after the expenditure of a considerable amount of money.

In the next chapter our author presents us with a reasoned forecast of the population of England and Wales in the very near future and a discussion of the consequences of a stationary and a declining population.

An Appendix deals with all kinds of technical topics, the Life Table, the computation of Reproduction Rates, and the like. Finally, there are copious notes for each chapter and excellent indices.

Mr. Glass is to be congratulated on the book, which should be on the shelf of every student of population problems. E. C. R.

6.—*The Cameroons and Togoland.* A Demography Study. By Robert R. Kuczynski. Oxford University Press. Issued under the auspices of the Royal Institute of International Affairs. xviii + 579 pp. 30*s*. net.

The above work is defined as an attempt to assemble all the essential facts relating to the collection of population statistics and to the demographic situation of a prescribed African area over an extended period of time, the available data being regarded as more ample than for other African areas at a similar stage of development. The Cameroons and Togoland were selected for this study, since three great Powers, each following a different colonial policy, were concerned with their administration, viz., Germany from 1884 to 1914, and later France and Great Britain under Mandate of the League of Nations.

Census returns, records of births and deaths, the data assembled during the course of special surveys, miscellaneous statistical records, Governmental reports, the special reports submitted to the Permanent Mandates Commission of the League of Nations, and the announcements of official representatives at the sessions of the Commission, are all taken in turn and examined in great detail; at the end of each section of the work under review the author records his conclusions. In this connexion it may be stated that with a single exception—British Togoland, the smallest of the territories dealt with in this study—the author finds it impossible to decide whether populations have increased or decreased. Of British Togoland (population approximately 300,000) he says (p. 579), "There

is not the least doubt that the population of the Territory has increased since the grant of the Mandate to the British," though how far this is due to the combination of such factors as immigration, fertility and mortality, he is unable to speak with certainty. Other matters discussed at length include the fertility of native women, infant mortality, morbidity and mortality experiences, whether polygamy adversely influences the birth-rate, and the social services provided in the areas concerned.

Dr. Kuczynski condemns the Mandatory Governments for their failure to produce dependable statistics, speaks of the "ignominious failure" (p. 272) of vital registration efforts in some areas of the British Cameroons, and is indignantly critical of French methods of census-taking in the areas under their administration.

This is all very well, but readers of this study familiar with conditions in Tropical Africa may incline to the belief that the author's criticisms and conclusions suffer from the defect that they are based *solely* upon an expert analysis of the raw data representing end results of various activities, without any personal experience or knowledge of the varying conditions of life, the customs, prejudices, and superstitions of the people, and the numerous languages and dialects, etc., met with in the territories producing the recorded facts. However expert as a professional statistician an investigator may be, if he is entrusted with the critical examination of data relating to primitive peoples in remote places, yet lacks personal knowledge of the territories concerned, their peoples, their social organization, and other features characterizing their conditions of life, his criticisms and conclusions may prove both unfair and misleading. Dr. Kuczynski perhaps too readily condemns the quality of the information available for his investigation, yet he might easily have selected several areas in Tropical Africa wherein administrators have been conspicuously successful in establishing dependable census and registration systems in the face of difficulties undreamed of by workers whose experiences have been confined to statistical activities in Western Europe.

P. G. E.

7.—*Indian Agricultural Statistics. An Introductory Study.* By P. J. Thomas and N. Sundararama Sastry. University of Madras. 1939. 144 pp.

This book is an outcome of the economic enquiry conducted in 1934 on behalf of the Government of India by Dr. Bowley and Professor D. H. Robertson in co-operation with a number of Indian economists, including one of the authors. In drawing up a scheme for an economic census of India the investigators had to examine the nature and value of existing economic statistics in that country; and since agricultural statistics "form the backbone of the whole statistical framework of India," special attention was naturally given to them. During the course of the enquiry Professor Thomas realized the great importance of crop forecasting in all phases of India's economic life and, finding that no detailed account of forecasting methods was readily available, collaborated with Mr. Sastry in producing this book to fill the gap.

Crop forecasting is the most troublesome, and therefore the most interesting, major problem of agricultural statistics. In one form or another it raises nearly all the difficulties which have to be surmounted in the collection of information by the State. On the one hand we have to organize a series of reports from a large area and take account of such factors as human fallibility in judgment, human proneness to delay in the submission of returns and the constitutional pessimism of crop reporters (which seems to be a universal phenomenon); on the other hand we have to call into play quite recondite theoretical results in the selection of samples and the formation of averages. All the world over statisticians are wrestling with these problems, and the book will, one hopes, be read with interest wherever the agricultural oracle has to be consulted.

Although crop forecasting occupies a very prominent position in the book, other topics are discussed. There is a very interesting introductory historical chapter on the Government organization of statistics in India; an account of the present methods of collecting statistics; a description of present Government publications on agricultural statistics; and finally two chapters on the national income of India and the problem of food supply for her vast population. The book concludes with some appendices on crop-cutting experiments in India and on the crop reporting services of the U.S.A. and England and Wales, and 23 tables of Indian agricultural statistics.

Some of the figures given in the tables at the end of the book will arouse deep compassion in the British agricultural statistician. The United Kingdom takes a census of crop acreages and livestock populations every year—indeed, in war-time, every three months—and gets crop reports monthly. Estimates of yield and production can thus be made with some considerable accuracy, although most statisticians still think they are capable of improvement. But here are some of the Indian forecasts :

Area under groundnut according to different forecasts in the same year. (000 *acres.*)

	1st forecast	2nd forecast	Final forecast
1933–34	2,080	5,670	8,115
1938–39	2,593	6,879	8,500

Comparison of approximate commercial cotton crop with crop forecasts. (000 *bales.*)

	1932–33		1935–36	
	Approx. crop	Forecast	Approx. crop	Forecast
Punjab	887	656	1,796	1,600
Rajputana	148	68	235	91

If ever figures spoke for themselves, surely these do. Once the importance of crop forecasts in the economic life of India is admitted, the case for wholesale improvement in her statistical service is overwhelming.

It is easy to suggest improvements—aerial photography has interesting possibilities—but it is far from easy to persuade Govern-

ments to adopt them. "The world economic crisis of 1929-30," say the authors, "served as an eye-opener to the Government of India in many respects. The depression seriously affected the trade and revenues of India and measures had to be devised for counteracting it. At every stage the need for accurate statistics was keenly felt. . . . But the Government found that their statistical material was not adequate for drawing conclusions and formulating policies." A Statistical Research Branch was therefore created. The Government, however, were still not satisfied, and invited Dr. Bowley and Professor Robertson to make an enquiry. "The experts submitted a report in which they recommended the appointment of a permanent economic staff of four members including two economists and a statistician. They also recommended that an economic census should be carried out every five years and suggested the main lines on which it shall be organized."

"These recommendations were long under consideration. The scheme of an economic census was regarded as too costly and was abandoned for the time being. The idea of an economic general staff was welcomed by many, but even this involved additional expenditure and was eventually abandoned." The Government did, however, decide to appoint an Economic Adviser whose duties include the interpretation of statistics. The authors expect that the method of collecting and interpreting statistics will be materially improved in the near future. One sincerely hopes so. The task is tremendous, but it is not impossible.

One suggestion might be made for subsequent editions—the inclusion of a glossary of Indian words. Any reader who is unfamiliar with Indian local government is apt to have his attention diverted by such sentences as "The tahsildar, in his turn, gives an annawari estimate for the whole taluk. . . ." To be quite fair, one should explain that this is not a representative sample of the style of the book (which, indeed, is very well written), and that the native terms are mostly explained somewhere in the text. A glossary would nevertheless save the non-Indian reader some trouble and relieve the strain on his memory. M. G. K.

8.—*Alcohol and the Nation: a Contribution to the Study of the Liquor Problem in the United Kingdom from 1800–1935.* By George B. Wilson, Ph.D., B.A. London: Nicholson and Watson. 1940. xii + 456 pp. 63*s*.

An early reviewer of this book sought to belittle it by arguing that Dr. Wilson had done little more than bring together a mass of statistics which any interested person could obtain for himself from other sources. That criticism is in no way justified. Even if, contrary to the fact, the book had been nothing more than a compendium of statistics the student of social affairs would still be everlastingly grateful to Dr. Wilson for having made the compilation, and thereby saved the student an immense amount of time and labour hunting up the statistics for himself.

The book deals with various aspects of "the drink problem," and it contains a comprehensive appendix of 42 tables of statistical

material relating to spirits, beer, and wine (production, consumption, imports, exports, and sale by clubs and licensed premises); to the materials used in manufacture; to the public revenue derived from the taxation of these liquors; to the national expenditure thereon; and to the mortality and drunkenness associated with their consumption. To complete the statistical picture of "alcohol" the appendix contains tables relating to alcohol used for non-potable purposes, while in the text of the volume will be found statistics about "sweets" (mainly British-made wines), and a rather disappointingly small account of cider.

The reader will be grateful not only for the wide scope of this material, but particularly for the fact that in general the statistics cover long periods of time, some of them going back to the eighteenth century and many to the beginning of the nineteenth. Even the reviewer referred to in the opening sentence above, granted he has all the original sources at his elbow (which it is safe to say is not the case), must be glad that he need no longer wade through a succession of volumes to obtain a continuous series of statistics stretching over many decades.

There are so many tables, and so much labour has clearly been involved in their compilation, that it is not surprising if Dr. Wilson should occasionally slip up. The Irish Free State (now Eire) dropped out of the United Kingdom statistics in 1922, but the tables sometimes fail to warn the reader of this fact, and occasionally averages are given for the quinquennium 1920–24 which are presumably a mixture of 1920–21, including the Free State, and 1922–24, excluding that State. Occasionally, too, the references in the text to the tables in the appendix are wrongly numbered.

One of the most interesting tables is that showing the national expenditure on alcoholic drinks from 1910 to 1935, the figures in which are Dr. Wilson's own estimates. Earlier estimates by other investigators, going back to 1820, are given in the text of the volume. The highest of all these estimates indicates a national expenditure on drink of £470 million, or £10 per head of the whole population, in the boom year 1920, just after the Great War—a figure nearly three times the pre-war estimate, thanks to large war and post-war increases in the rates of duty on spirits, beer and wine. The latest estimate in the table is for 1935 and totals under £240 million. Dr. Wilson's estimate for 1938 was £257 million. Since the present war began the rates of duty on drink have again begun to soar, and one wonders what new heights the estimates of national expenditure will reach.

The tabular appendix occupies little more than one quarter of the book. The preceding text contains an immense amount of information garnered from a multitude of sources and presented in a way that excites and holds the readers' interest. The book opens with some concise technical data about alcohol and the alcoholic drinks, and this is followed by a discussion, subject by subject, of the statistics contained in the appendix. There are historical surveys, not only of the liquors themselves, but of the tied-house system, the licensing system, temperance reform, alcoholic mortality, and drunkenness.

There is also a most interesting discussion of many of the factors which have influenced the demand for alcoholic liquors.

Dr. Wilson has devoted a long life to the cause of temperance reform, and in less practised hands the subject of this thesis might easily have suffered from biased treatment. Only very rarely has the present reviewer detected even the slightest deviation from the path of strict objectivity, as, for example, when Dr. Wilson refers to a reduction in the beer duty in 1923 as enabling "the brewer by brewing weaker beer to escape a substantial proportion of the tax." The reduction was in fact made for the benefit of the consumer, and we should not have described it quite like that! But this is an exceptional lapse, and in general Dr. Wilson maintains his reputation for fair presentation of the facts about the liquor trade.

In a Foreword to the book Lord Stamp states that "expenditure on alcohol absorbs some seven or eight per cent. of the national income, and a much higher percentage of the income of the mass of the people." Whether or not the precise percentage quoted is accurate, this statement describes a social fact of great importance and there are few people living capable of dealing with it informatively, comprehensively, and dispassionately. Dr. Wilson is one of them, and this book worthily enshrines the results of his experience, knowledge, and research. A. D. W.

9.—*The Minimum Wage. An International Survey.* Geneva: The International Labour Office; London: P. S. King. 1939. $9\frac{1}{2}'' \times 6''$. viii + 257 pp. 5*s*.

As the analysis given in the report on "Minimum Wage-Fixing Machinery" published by the International Labour Office in 1927 is now largely out of date, a new series of monographs has been prepared and this volume covers the experience of Australia, New Zealand, Peru, the United States of America, Belgium, France, Czecho-Slovakia, Great Britain and Ireland (I have purposely altered the unimaginative alphabetical arrangement in the book). The analysis is confined to the discussion of legally enforceable minimum rates, whilst collective agreements, apprenticeship regulations and the so-called "fair-wage clause" in public contracts are not discussed.

The whole volume is a valuable survey of the legal position. It is less valuable on the question of enforcement. For Britain figures are given of the number of inspectors, of the number of farms and factories inspected, of the number of underpaid workers found and of prosecutions. This is very satisfactory, and one can make guesses at the amount of evasion. In other cases the reports are much less satisfactory. In too many the only evidence of enforcement given is that fines may be imposed without any statistics to show that in fact fines are imposed. In extreme cases this may mean no more than saying that in England people may be compelled to attend church. We read that in the State of Washington civil action on the part of the employee is the only means of ensuring payment of the minimum rate. An employee dependent on his work will not take such a step lightly, even if he has the money resources available. In Peru indigenous workers with a contract of employment for a year

or longer can leave the farm if their wages are not paid in cash. This again suggests that enforcement is a risky business for the employee. In fact, the regulations themselves suggest that enforcement in the Andes must be a dead letter, for the minimum rate fixed at the beginning of each year must be equal to the average wages, which suggest that the minimum rate is also the average, and hence the maximum ; or alternatively that wages must rise continuously year by year.

The above examples are chosen to illustrate some dangers of minimum-wage legislation, and are neither intended to be criticisms of the specific countries which happen to be mentioned nor of the compilers of the monographs. H. S. B.

10.—*The Incidence of Income Taxes.* By Duncan Black, M.A., Ph.D. London: Macmillan. 1939. $8\frac{1}{2}'' \times 5\frac{1}{2}''$. xxi + 316 pp. 12*s.* 6*d.*

Believing, not without reason, that prevailing conceptions of tax incidence need reconsideration, the author of this work has been fired with the ambition to remove current confusions and misconceptions, particularly with regard to general income taxes, although incidentally he devotes much attention also to partial income taxes.

Notable use is made of the diagrammatic method. Most of the diagrams are useful and illuminating, but here and there are examples of what unsympathetic critics might be tempted to call the unnecessarily repetitive, the unhelpfully elucidatory, and the unintentionally inconclusive. An example of the first-named may be found on p. 71, and of the second in Appendix 2, which employs algebra with Greek letters and three diagrams to demonstrate what to many readers might otherwise have been more immediately apparent. The third type is seen when the author, chasing a hare started some years ago in America, arms himself with indifference curves. These purport to prove beyond all doubt that, if wage rates are increased, a man may wish to work either more hours per day or less hours per day. But he appears to ignore the real point at issue—namely, whether the various shapes of indifference curves are probable or possible. It is perhaps significant that his two curves on p. 162 represent a case where a man may have 25 hours of leisure per day. And one notes that, like others who have chased this particular hare during the last few years, he takes no account of one or two simple points including the obvious yet not irrelevant fact that a man cannot live indefinitely without eating.

The author's main ambition is the erection of an "exact theory" of the incidence of general income taxes. He starts by trying to demolish parts of the Colwyn Report—in particular its theory of incidence and, what is less vulnerable, Dr. Coates's Appendix. He makes great use, however, of other sections of the Report, and obtains some very useful bricks from Mr. D. H. Robertson's contemporary criticism of the Report. As architect he appoints De Viti de Marco, the author of *First Principles of Public Finance*, from which we are led to hope so much. This much-lauded work

(which, like poetry, but for a different reason, apparently lost much of its glory in translation) may be remembered by statisticians for having provided an outstanding example of the so-called statistics and statistical methods (cf. in particular the diagram on p. 200) which an economist may accept as proof of a preconceived theory. And in other directions one may wonder whether as a guide De Viti is so reliable, so helpful, or so original as we are told. The fact that revenue obtained from taxation is spent by the Government was not first discovered in Italy in the twentieth century—some of its implications occupied Chinese theorists centuries before Christ. And if more modern theorists have decided to analyse Public Revenue and Expenditure separately, it does not follow either that they are unaware of any interdependence or that their policy is entirely indefensible. Methodologically, at any rate, it offers substantial, some may think, overwhelming advantages. Dr. Black, however, seems to think otherwise, and hoped that by following his Italian blue print he might erect a more exact theory of tax incidence. But his hopes are not realized, and one is left with a general impression of anything but exactness. Nevertheless he deserves our thanks for laying stress on points often overlooked, for laying bare the vaguenesses surrounding prevailing concepts of incidence, and for his admirable treatment of some of the rarely analysed problems of partial income taxes. C. O. G.

11.—*Essays in Monetary Theory*. By D. H. Robertson. London: P. S. King & Son. 1940. $8\frac{3}{4}'' \times 5\frac{1}{2}''$. ix + 234 pp. 11*s*. 6*d*.

On many occasions in recent years the non-academic student of monetary and trade-cycle matters has trembled lest the progress in this most important field of economic theory should be dissipated in terminological dispute and confusion. That this has not been so is due in no small measure to the efforts of a small group of economists here and in the United States who have assumed the task (in Professor Robertson's own words) of "disentangling issues of words from issues of substance and . . . of setting forth opposing viewpoints as fairly as possible without pronouncing between them." The efforts of Professors Robertson, Haberler and Hansen have been notable in this connection.

These circumstances will lessen the regret that Professor Robertson has not seen fit for many years to give us a statement of his views "in more architectural form." Instead, he has brought within the covers of this book a welcome selection of his lectures and writings. The first twelve essays deal mainly with the monetary controversy of the last decade; these are followed by six reviews, the well-known notes in the *Economic Journal* of 1924 and 1929 on the Terms of Trade and the Transfer Problem, a short essay on the future of international trade and, finally, by that delightful broadcast talk "A Visit to the Laccadive Islands."

Since Professor Robertson's general standpoint and most of these essays will already be familiar to readers of the *Journal*, comment will be confined mainly to the issues raised in the first and longest item in the book: "Mr. Keynes and the Rate of

Interest," which can usefully be read in conjunction with the latter part of essay XI: "A Survey of Modern Monetary Controversy." The first essay is based largely on a course of lectures at the London School of Economics in 1939.

Professor Robertson has little difficulty in showing in Section I that there is no conflict of substance between the usual approach to rate of interest as the price of loanable funds and Mr. Keynes's "statical" approach to rate of interest as expressing the relation at a moment of time between the amount of money permitted to be in existence and a schedule of people's desires to hold money in the light of other possible uses for their resources. Whilst admitting the validity of Professor Robertson's general criticisms of Mr. Keynes's approach in Sections I and II, one is led to wonder whether a part of the controversy could not have been avoided by dropping the usual simplifying assumption of a single rate of interest and accepting that we have to deal in real life with an incredibly complicated *structure* of interest rates, and therefore with the question not only of what determines the absolute height of the structure of rates, but also of what determines the *shape* of the structure both at a moment of time and over a period. In normal times the structure of interest rates used to behave in an organic manner, and there was ample reason to assume a single rate in considering how the fundamental forces of thrift and productivity were translated more or less smoothly into an appropriate addition to the real wealth of the community. Unfortunately, for some years now it has become apparent that we have to consider not only *how* interest rates adjust savings and investment, but *whether* they do. We have to recognize that the whole interest-rate structure has become much less organic under the pressure of many forces, such as the "pegging" by Governmental action of certain interest rates, the tendency to invest for capital profit rather than for income, political insecurity, broad social changes, and so on. Further complications arise from the fact that, for some time, an increasing proportion of new investment has been made with little regard to the appropriate rate of interest. Industries have become self-financing and have invested their corporate savings in ways determined by internal factors rather than by profit rates, and public authorities have been influenced in their investment decisions less by considerations of profitability, or even utility, than by the possibilities of cheap borrowing in the gilt-edged market.

As Professor Robertson has emphasized on many occasions, part of the problem arises out of the tendency of the stock market to replace the banking system as the channel for guiding new savings into new investment. Indeed, it is only in the stock market that interest rates in recent years have shown any marked sensitiveness, the rates determined outside that market being very sticky either for institutional reasons or because they related to very long-term contracts. Unfortunately, even the stock market has come to play more and more unsatisfactorily its rôle in the savings–investment complex. Political factors have been partly responsible for this, of course, but there have been deeper forces, such as the growing

tendency in Anglo-Saxon countries to invest for capital profit (or maintenance) rather than for income (a reflection to a large extent of heavier taxation).

The net result of all these and other factors has been that the actual relationship between saving and investment has become increasingly tenuous, with a correspondingly disruptive effect upon the structure of interest rates. It is not surprising, therefore, that to talk about "productiveness and prospectiveness" and thrift has been regarded as an inadequate answer to the question of the determination of interest rates, because we know that, for some years now, the interest rate structure has been dictated largely by other forces than what might be termed "pure interest" considerations. It may be true, as Professor Robertson claims, that Mr. Keynes's theory has over-stressed these other factors to the exclusion of the fundamental long-term forces, but, at least to students of the formation of interest rates on the stock market in the United States and in this country over the last decade, Mr. Keynes's theory has considerable appeal.

In Sections III and IV of this essay, Professor Robertson discusses illuminatingly the questions of the behaviour of interest rates during the course of the cycle and of the relation between short and long rates. The advantages of thinking from the outset in terms of the structure of interest rates instead of the single rate again become evident at this stage, because it can be shown more easily how and why particular interest rates move in different directions during the course of the cycle. For example, the anticipation of a marked expansion of industrial activity accompanied by higher commodity prices will tend to increase the present yield on fixed interest securities and to depress the present yield on equities, owing to the working of the discounting factor in security prices.

In the concluding Section of the essay the author deals briefly with the relation of the "liquidity preference" view of interest rates to the problem of monetary equilibriun in a progressive community. There will be general agreement with Professor Robertson's warning against excessive gloom and with his view that many of our problems are to be attributed to political and institutional factors. Nevertheless, one wonders whether the distortion of the savings–interest rates–investment process is not part of a fundamental change in the whole economic system, and whether the removal of the external difficulties will not bring such profound changes as to shift our problems on to an entirely new plane.

J. E. W.

12.—*The Stock Market, Credit and Capital Formation.* By Fritz Machlup. London: William Hodge & Co. 1940. $8\frac{3}{4}'' \times 5\frac{1}{2}''$. xii + 416 pp. *21s.*

This is a revised and enlarged edition of a book which appeared in German in 1931 in the series of monographs published by the Austrian Institute for Trade-Cycle Research. The central theme is a problem which attracted considerable attention in Germany and in the United States both before and especially after the Great

Depression—the question of the rôle of the Stock Exchange in the body economic and, more particularly, the responsibility of the Stock Exchange for the excesses of the boom and the ravages of the slump. Professor Machlup's discussion of the problem ranges over the widest possible field: from the mechanics of the operation of the Stock Exchange, through the question of the supply of capital to the relations of the supply of credit to the trade cycle.

The first half of the book is devoted to a most detailed refutation of the once-popular argument that increased stock-market activity tends to divert capital from its more legitimate industrial uses, thereby contributing to depression. Professor Machlup shows conclusively the limited conditions under which capital can be said to be "tied up" on the Stock Exchange, and exposes the widely prevalent view that, in the United States, the volume of Brokers' Loans reflects resources diverted into stock-market speculation. In fact, they reflect the excess of the withdrawals of customers' funds over new investments in securities. This section could have strengthened by a more extended reference to the dual functions of the Stock Exchange as a market for securities and as a source of new capital for industry, to the greater rôle played by the Stock Exchange in determining large parts of the interest rate structure and, especially, to the dangers of the tendency in recent years for the public in the Anglo-Saxon countries to invest for capital profit (or to avoid capital loss) rather than for income. In particular, it might have been shown that the stock market (even more than the banking system) may tend to distort the savings–investment process by providing new resources for investment at interest rates which are not justified by the volume of savings, since the discounting mechanism which helps to determine security prices has an inherent instability. The author also appears to under-estimate the effects of capital gains or losses upon consumption, particularly in certain sections of the economy. In fact, the experience of both the United States and this country in the period 1936–38 showed that capital gains and losses (and indeed the general tone of the Stock market) had quite important and rapid reactions on consumption.

In the following chapters of the book the author moves into the field of monetary and trade-cycle theory. His views are those of the Austrian School as presented in this country by Professor Hayek—any expansionist process not based on an increase of voluntary saving or dishoarding is regarded as necessarily self-reversing. Professor Machlup emphasizes particularly the rôle which surplus balances in the hands of corporations may play in the beginning of the expansion. After discussing the limits to which credit expansion can safely be carried without causing "cumulative–reversive" movements—*i.e.*, the question of quantitative credit control—the author goes at length into the problem of qualitative control—*i.e.*, whether it is safer to grant new credits in respect of working capital only, and whether loans should be granted directly to industry or through the intermediary of the Stock Exchange. His conclusion is that what matters is whether the amount of new credit exceeds the limit. If it does, the result will be inflationary,

no matter what apparent safeguards may be provided as to the way in which the new credit is issued.

Readers of the *Journal* will already be familiar with the general criticisms of the position taken by the Austrian School. Despite these attacks, Professor Machlup holds to the view that "one can avoid the downswing only by avoiding the upswing." He admits that political and social forces may render such a policy impracticable, but he overlooks the considerable historical evidence which points to what Professor Robertson calls the "discontinuous" nature of material progress in our capitalistic society. It is hard to escape the conclusion that Professor Machlup's policy would confine to eternal stagnation a system which, before the war, was making heavy weather even in an atmosphere of cheap and abundant credit.

There is a most useful appendix of financial statistics, relating mainly to the United States, which has been carried up to the middle of 1939. J. E. W.

13.—*Tin Restriction and Tin Price.* By M. J. Schut. Haarlem: De Erven F. Bohn. 1940. $9\frac{1}{2}'' \times 6\frac{1}{4}''$. xii + 115 pp. Hfl. 1.50 in paper covers (written in Dutch, statistical tables and graphs headed in Dutch and English).

This monograph is one of a series sponsored by the Netherlands Economic Institute. Like two other volumes in the same series, it is an attempt to apply to particular problems the technique of multiple correlation as it has been developed by Professor Tinbergen.

In the first four chapters the author attempts to evaluate statistically the inter-relation between the production, price, and consumption of tin, and also between these variables on the one hand and certain economic series on the other. It is not surprising that he obtains quite a number of good fits which show that it is not impossible for certain interesting connections to exist. The author is well aware that it would be wrong to claim more than that for his methods. For example, on pp. 52–53, he discusses the partial correlation equation which "explains" the tin price by movements in stocks, wholesale prices, and a trend factor. The movement in stocks is split up into three factors: their rate of change, their absolute size (including pool stocks), and the pool stocks. A negative partial regression is obtained for the 1921–24 Bandoeng Tin Pool, but on the other hand a positive connection with the level of tin prices appears for the stocks held by the International Pool of 1931–34. The author points out that "it is easy to understand why the two pools may have exerted different influences on the price of tin, in view of the different manner in which the stocks were accumulated." Nevertheless, it seems unlikely that the existence of the Bandoeng pool did in fact put producers in a worse position than that in which they would have found themselves in its absence; the negative correlation may easily be due to the fact that even the pool failed to slow down noticeably the extraordinarily steep post-war fall in tin prices.

A few other somewhat strange results are also reached. Accord-

ing to the formula at the foot of page 26, it would seem that a rise in the United States' tin-plate production by 10,000 tons (that is to say, an increase in the consumption of tin by the tin-plate industry of 150 to 170 tons) would *ceteris paribus* lead to an increase in total tin consumption of 4,300 tons. This formula for the total consumption of tin in the United States in the period 1923–1936 also contains a negative trend term (— 2·21 T, equivalent to a drop of 2,210 tons per annum), motor-car production and tin price being the remaining factors in it. Two pages further on the total consumption of tin in the United States by all industries other than tin-plate is "explained" by the level of tin prices, industrial activity, and building activity. The trend term in this case is positive and amounts to at least 2·6 T, or 260 tons per annum. For these two equations to be consistent it would therefore be necessary for the annual quantity of tin used in tin-plate to show an average decline of at least 2,470 tons per annum, other things being equal. This seems hardly to be borne out by the facts, as this figure, during 1925–31, moved between 23,000 and 28,000 tons, fell to 16,000 tons in 1932, and rose (with a slight relapse during 1934) to a level of 35,000 tons in 1936.

Another doubtful item is the calculation, in Chapter V, of the time-lag between changes in the price of tin and in the production of reclaimed tin. This, the author claims, is found by the weighted average of one and two years, the weights being the respective partial regression coefficients in an equation "explaining" the production of reclaimed tin by the average price during the previous year and the year before that. There is all the less reason for this unwarranted conclusion, as figures of daily tin prices are easily accessible.

The last two chapters (VI and VII) contain an attempt to evaluate the effects of restriction by calculating what would have happened if the constants for the years immediately preceding restriction had been maintained unchanged. Here, however, the author's enquiry is largely vitiated by his simultaneous attempt to see whether tin production is more closely connected with the volume of consumption or with the tin price. Such a procedure may be justified in order to check a calculated elasticity of demand, but to explain the volume of production by one without the other is one of the pitfalls into which the investigator is led only through the empirical approach to economics, so popular on the Continent.

Nevertheless, the book deserves attention as a conscientious attempt to test the usefulness of multiple correlation analysis in a new field. It would undoubtedly have gained in quality by the omission of all doubtful and inconsistent items; these only deserve to be mentioned in footnotes as rejected alternatives. M. R.

14.—*The Road and Rail Transport Problem.* By Brigadier-General Sir H. Osborne Mance, with an introduction by Sir Arthur Salter, M.P. London: Pitman. 7″ × 7″. 166 pp. 7*s.* 6*d.* net.

General Mance's book may be said to come out at an opportune moment; the controversy between road and rail has not been settled, though it is in abeyance during the war because road vehicles depend mainly on imported motive power. Unlike some other

writers on this subject, the author has no axe to grind and can look at the question from an expert but impartial point of view. Those who think that Parliament will have an easy task in " co-ordinating " the country's transport system may learn from this account of the experiments and failures in other countries how difficult the task will be. Thus in Northern Ireland legislation based on the careful report of an English expert failed miserably. Hauliers, bought out under the scheme, have started business again as " ancillaries," and the new Transport Board was hopelessly overburdened. In Eire the Government's efforts had a little more success. The experience of Ireland and of other countries shows that the private or ancillary owner is the chief danger to be encountered in any national transport plan. Almost everywhere the ancillary is left free, with the result that he takes the easy regular traffic and relies on the railway for peak loads and heavy goods.

A large part of General Mance's book is taken up by discussion of the rate question. Ought transport charges to be based on the *value* of the goods carried or on the *cost* of carrying them? Put a little differently, should rates be based on the service to the public or on the expenses of the conveying concern? Railway rates, as everyone knows, are based on " value," the theory being that part of the profit made by carrying expensive goods should reduce the charges for cheaper, heavy, and bulky goods. Road transport, on the other hand, being free from the railway's obligation to carry all goods offered to it, bases its rate on the cost, as near as can be ascertained, of carrying the consignment and thus skims the cream off the railway traffic by quoting lower charges for goods which are rated highly in the railway's classification. General Mance declares emphatically that a " value " rate structure cannot exist alongside a " cost " structure.

In a short but useful Introduction Sir Arthur Salter puts the other question with which General Mance has to deal—*i.e.*, should road and rail compete freely, as they do now, or not? His own view, which the General adopts, is that the answer would best be found in " a financial amalgamation of the railway and main road services, combined with freedom for businesses requiring transport to run their own vehicles." This subject is dealt with in the second part of the book, the first part being a valuable survey of what has actually been done in this and in certain foreign countries. The author gives far more space to goods than to passenger traffic. He approves of the railway companies' policy of going into partnership with the large omnibus concerns. Co-operation in goods traffic is much more difficult, because there are so many more owners of lorries. He has a poor opinion of the Transport Advisory Committee and of the Royal Commission on Transport, but he thinks highly of the Salter Report and of the Chambers of Commerce Report.

General Mance would ask—what are the chief items of cost in transport? and answers, (1) running costs, (2) fixed operating charges, and (3) interest on capital. On the average he reckons that in the case of the railways, assuming their " standard revenue " to have

been earned, the proportions are (1) 35 per cent.; (2) 40 per cent.; and (3) 25 per cent. In the case of road transport he reckons the proportions as (1) 60 per cent.; (2) 30 per cent.; and (3) 10 per cent.

General Mance's monopoly would not be watertight, because there would still remain the competition of private cars, ancillaries, farmers' vehicles, and tradesmen's vans. Even now railway companies have duties from which road hauliers are free—*e.g.*, the obligation to publish rates and to carry all goods offered to them. Moreover, railway rates are prescribed by the State, and are fixed on the value system. On the other hand, hauliers can carry what traffic they like and charge what they like, without publishing their rates or giving the same treatment to all.

Our author devotes a chapter to the question—" Whether rail or road transport is cheaper to the community? " But he takes the financial facts of the moment as decisive, whereas he ought to consider the real cost in terms of effort. What would a general or a contractor do if he had to move men and materials backwards and forwards across a trackless country? Would he build macadamized roads and run rubber-tyred lorries along them, or would he lay down steel rails to carry steel-tyred trucks? On the question of charges General Mance observes that the heavy industries will want the value system maintained so as to keep their present low rates, while the finished-goods industries will hope for reductions if rates are based on costs.

In France an excellent system was being established when war broke out, the main idea being to limit long-distance competition with the railways; road operators were not to charge less than railway rates *plus* cartage. In Germany the value structure was accepted as the basis of charges. A law of 1935 compelled concerns operating over fifty kilometres to join a public corporation; the Transport Board takes the money on a value rate and pays the haulier on a cost rate. General Mance's account of transport in Northern Ireland should be read as an example of what to avoid. In every case he finds that " co-ordination has involved putting an end to unregulated competition by the introduction of some form of control of road transport."

It is strange that our author never once refers to the actual amount of licence and petrol duties paid for lorries, coaches, and vans in any particular country. Surely this is a matter of the first importance. No doubt he writes as an engineer rather than as an economist; still, he might have recognized that the main item of cost in all forms of inland transport is the permanent way. Road transport competes with railway transport in charges because it has its permanent way provided by the community, of course in return for a payment, but no payment is made on account of the capital cost of our highways.

Yet the payments which road vehicles have to make for petrol and licence duties are an important factor in their working costs, and therefore in their charges.

A further point ignored by our author is the inconvenience and danger caused to the public, and especially to the children of the

working classes, by motor vehicles, particularly of the heavier kind. What would happen to road transport if the same stringent rules for the safety of the public were imposed on it which Parliament has imposed on the railway? Should not the casualties due to motor vehicles be reckoned as part of the cost of road transport?

It is recognized that the "ancillary user" must be brought under control, but it is not clear how the ancillary is to be "brought into line with the rates structure" if he remains free to compete with public transport under General Mance's scheme for a combined monopoly of road and rail. The owner-driver presents an almost insuperable difficulty. One principle which may be deduced from the facts given in this book is that a distinction should be drawn between long-distance and short-distance transport. Perhaps this is best done by charging a higher duty for long-distance licences, hauliers and ancillaries being treated alike. The distinction might well be applied also to passenger vehicles. A further suggestion would make the licence duties *progressive*, like the income tax, on the ground that the damage done to roads and bridges is far more than doubled when the weight of a vehicle is doubled.

J. E. A.

15.—*Other New Publications*.*

Hill (*Polly*). The Unemployment Services. London: Routledge. 1940. $7\frac{1}{4}'' \times 4\frac{3}{4}''$. xiv + 226 pp. *7s. 6d.*

[This book is concerned, states the authoress, "with the changes that should be made in the unemployment insurance scheme and the Unemployment Assistance Board in a Labour Government's next term of office," but "the proposals made are appropriate only to the period of transition to socialism." Thus the services are "considered against the background of a capitalist system not radically different from the present one." Evidently, then, the proposed changes could be carried out under a government of any colour, and the facts on which they are based, here logically marshalled, adequately documented, and concisely set down, offer, or perhaps challenge, consideration by the responsible authorities. For any one interested in the subject, the book is a compendium of information relating to the unemployment services, their application, implications, and financing, and to the circumstances of the people to whom the services are directed. The last chapter discusses Unemployment Expenditure and the Trade Cycle, and the appendices consist of statistical tables. The book was undertaken as a report for the Fabian Society which has, however, "no collective responsibility for the views expressed."]

Linfield (*H. S.*), Ph.D. State Population Census by Faiths: Meaning, Reliability and Value. New York: Hasid's Bibliographic and Library Service. 1938. $9'' \times 5\frac{3}{4}''$. 72 pp.

[A small but detailed work of reference to the actual facts about the enumeration in censuses of persons according to faith. The countries which include such a question in the schedule are shown and appropriately grouped, and examples are given of the summation of the results in the latest census returns of Germany and certain other countries. The nature of the questions asked varies in different countries according to the motive of the enquiry. In some countries this is

* *See also* "Additions to Library," p. 440 *et seq.*

mainly financial, *e.g.*, to facilitate the assessment of the amount of the grants to the State church, so that the person's legal affiliation is the answer required; elsewhere the question concerns religious conviction (or non-conviction). The identification of race and religion, notably with the Jews but also with some other minority populations (for instance, the Copts) is among the many factors which throw doubt upon the "meaning, reliability and value" of the information obtained, discussed by the author in his fifth and last chapter. Appendices, which occupy nearly half the brochure, give: the text of the question relating to religion in the schedules of the several countries which include it, and the summation of the replies in the returns (by sex, age, district, civil state, etc.); a complete list of the faiths, including "pagan," "agnostic," and "no religion," enumerated in each country; detailed tables showing the countries which do and do not ask the question, with their population, and, for the former countries, the "majority faith" with the number confessing it, and the number of minority groups and their faiths. A bibliography of sources completes the information.]

Robert (Paul). La politique d'isolement économique: causes, modalités, conséquences de l'autarcie. Le problème pour la France. Paris: ed. Domat-Monchrestien. 1940. 9" × 7½". 98 pp.

[The sub-title summarizes the questions which the author has endeavoured to answer in this brochure in order to arrive at a considered verdict on "a policy which men condemn in congresses and conferences but follow unhesitatingly when they are at the head of governments." M. Robert in turn distinguishes and briefly describes the economic, monetary and political causes which have led to the adoption of self-sufficiency, its modes of operation—by restriction applied to trade, financial transactions, and migration; its effect on trade, employment, price level, finance, and politics. He then discusses the policy preferable for France and her empire. The appendices consist of tables, compiled from official sources, showing the amount and distribution of French foreign and colonial trade in 1938, and the progress in recent years of colonial production.]

"Taxation" Manual. Compiled by barristers and experts under the direction of *Ronald Staples*, editor of "Taxation." 3rd edition. London: Taxation Publishing Company, Limited, and Pitman. 1940. 8¼" × 5¾". 328 pp.

[This manual may without exaggeration be said to give exhaustive information about income-tax, including surtax and national defence contribution and covering both law and practice. Moreover, the information is well arranged and clearly conveyed, without waste of words, and is up to date, in that it includes the provisions for 1941. The book will, in short, provide an answer, easily understood, to any question relating to income tax that any one is likely to want to ask. The explanations do explain, and a good index enables any particular point to be looked up at once. An appendix deals fully with wear and tear rates of allowances, and the preliminary matter includes a table of the cases referred to in the text, arranged in alphabetical order.]

University of Liverpool: Social Science Department. New Handbook of Social Statistics. (New Merseyside Series, No. 11.) Liverpool: University Press. 1940. 8½" × 5¼". 40 pp. 1s. net.

[The first Handbook of Social Statistics (no. 8 in this series) was published in 1938 and relates to 1937. The new one gives the position immediately before the outbreak of war, and some later figures. The text is almost entirely new, but the tables preserve the form of those in earlier Survey

reports so that the figures may be compared. The contents cover population movements, trade of the Port, and various aspects of the unemployment position, including the cost to the taxpayer. The chapters on elementary education and housing included in the earlier handbook have been omitted to make room for more material relating to unemployment, of greater present importance because of the changing conditions engendered by the war. Chapter V, Registered Unemployment by Sex and Occupation, compares the numbers unemployed in 1938, 1939, and 1940 in each of the four Merseyside boroughs, classifying men, women, boys, and girls by occupations. Chapter VI, The Burden on the Ratepayer, brings together the subjects previously treated under the headings of Unemployment, Public Assistance, and Rates. Tables show the total amounts paid as unemployment benefit and allowances, the number of persons in receipt of Public Assistance (March 1939 and March 1940), the rates payable in the four boroughs and the allocation of the proceeds between the principal services.]

Wyatt (*T. W.*) and *Jones* (*D. Caradog*). Post-War Poverty. Unemployment can be prevented. Birkenhead: John Woolman. 1940. $7\frac{1}{4}'' \times 5''$. 22 pp. *6d.*

[In a postscript to this pamphlet Mr. J. R. Bellerby sketches the probable phases of a post-war period in this country. He sees, as after the last war, first, a brief depression caused by the discharge of men from the forces and from government work; secondly, a rapid recovery owing to the work of reconstruction and replenishment; thirdly, a pause, occurring when the supply of consumption goods approximately balances the demand, which leads to the fourth period, when consuming power is outstripped by supply and public works may be resorted to as a remedy. The fifth period follows the exhaustion of these resources, and it is with the methods of averting the contingent depression that Mr. Caradog Jones and Mr. Wyatt are concerned. They would destroy the idea that money cost is a real cost and deflation a cure, and the idea that any cost incurred in promoting employment is not a real economy. They plead for full utilization of the vast capacity for production developed during war, and maintain the possibility of preventing unemployment and inflation of prices by careful planning of production together with stimulation of demand suitably timed—for instance, by a gradual instead of a lump payment of gratuities—and the encouragement in every way of spending at the right moments: the essential thing being to keep supply and demand "in step." The authors wish to make it quite clear that they believe their aims can be achieved "under the present economic system, where private enterprise has free play subject only to appropriate government control."]

STATISTICAL NOTES

1. British Official Statistics

We give below a table summarizing the oversea trade of the United Kingdom for the years ended July 1939 and 1940. The successive invasions of Denmark and Norway in April and the Netherlands and Belgium in May, followed by the entry of Italy into the war in June and the collapse of France, have steadily reduced the area with which trade is now practicable, and the area is also effectively reduced by the closing of the Baltic—though some trade may still be practicable with Finland and Sweden via Petsamo—and by the diversion of shipping from the Mediterranean. Trade in April, except as regards re-exports, was at its highest level

Movements and Classes	Twelve Months ended July, 1939	Twelve Months ended July, 1940	Increase (+) or Decrease (–)
Imports, c.i.f.—	£'000	£'000	£'000
Food, drink and tobacco	415,119	440,419	(+) 25,300
Raw materials and articles mainly unmanufactured ...	238,996	324,753	(+) 85,757
Articles wholly or mainly manufactured	239,218	287,558	(+) 48,340
Other articles	8,492	6,320	(–) 2,172
Total Imports	901,825	1,059,050	(+)157,225
Exports, f.o.b.—			
United Kingdom Produce and Manufactures—			
Food, drink and tobacco	37,289	37,397	(+) 108
Raw materials and articles mainly unmanufactured	58,208	48,960	(–) 9,248
Articles wholly or mainly manufactured	367,819	347,291	(–) 20,528
Other articles	13,430	9,167	(–) 4,263
Imported Merchandise—			
Food, drink and tobacco	12,333	9,893	(–) 2,440
Raw materials and articles mainly unmanufactured ...	29,390	18,166	(–) 11,224
Articles wholly or mainly manufactured	13,627	6,917	(–) 6,710
Other articles	640	323	(–) 317
Total Exports	532,736	478,114	(–) 54,622

for many years, and the effect of the successive reductions in the trading area is shown by the figures for subsequent months.

Month	Imports	Exports	Re-exports
	£ mill.	£ mill.	£ mill.
April	110·0	48·3	4·4
May	105·6	45·5	3·2
June	90·8	36·2	2·0
July	87·0	31·2	1·8

Imports in July were the lowest this year, exports the lowest since October 1939, and re-exports the lowest on record, with the exception of September 1918.

Trade with individual countries was published before the war by quarterly periods only, and such figures are no longer published, so that the extent to which the decline since April may have been due to the practical cessation of trade with Europe, apart from Eire, Spain and Portugal, can only be estimated. During the second quarter of 1939 imports from and exports and re-exports to this area were about 31, 31 and 58 per cent. of the respective totals. Applying these proportions to the corresponding figures for July 1939 gives £24·2 million, £12·4 million and £2·2 million. Trade in that month with countries outside Europe, together with Eire, Spain and Portugal, may therefore have been about £54·1 million for imports, £28·0 million for exports and £1·5 million for re-exports. These figures may be roughly comparable with those given above, since, although some trade with the Baltic countries, the Balkans and Russia may still be practicable, there will in present circumstances have been a considerable reduction in trade with Palestine and Egypt, which now has to go via the Cape, and the longer route has also to be followed in trade with India, East Africa, the Far East, and to a considerable extent Oceania. We thus have the following comparisons, the trade of March and April being averaged in view of the incidence of the Easter holiday, and the figures for those months in 1939 excluding the estimated trade with Germany, Austria, Czechoslovakia and Poland.

	Average			
	March–April		July	
	1939	1940	1939	1940
	£ mill.	£ mill.	£ mill.	£ mill.
Imports	70·4	109·3	54·1	87·0
Exports	36·2	44·9	28·0	31·2
Re-exports	4·1	4·0	1·5	1·8

The proportionate increase in imports which was about 55 per cent. in March–April has risen to over 60 per cent.; that for exports has fallen from 24 per cent. to about 12 per cent., whereas re-exports, which showed no substantial change in March–April, exceeded appreciably in July the corresponding figure a year earlier. Thus we appear to have maintained our import trade with the area still open, the rather larger rise for July probably being due to higher prices. The figures for exports clearly indicate a falling away from the peak figures in the spring, when the export drive was getting under weigh. The reduction has no doubt been caused by the temporary concentration of the industrial output upon munitions and Service requirements of all kinds.

The excess of imports over exports of merchandise has varied as follows. It will be seen that the effect of enemy action in Europe has been to reduce appreciably the strain upon our overseas resources, and by leading to some belt tightening has not had wholly evil effects.

Month	1939	1940	Increase
	£ mill.	£ mill.	£ mill.
January	31·5	60·2	28·7
February	22·6	55·8	33·2
March	31·5	63·5	32·0
April	30·3	57·3	27·0
May	31·2	56·8	25·6
June	38·0	52·6	14·6
July	34·2	54·0	19·8

Imports of cotton, silk and artificial silk waste, rubber, miscellaneous textile manufactures and vehicles (including aircraft) have more than doubled in value this year compared with 1939, while a rise of that order was also recorded for iron ore and scrap. The largest increases were those for grain and flour (£25·0 million), cotton (£23·5 million) and the manufactured oils group (£22·8 million). A reduction of £3·3 million for wood and timber is due to the absence of supplies from the Baltic, while the effect of import restrictions is shown in a number of groups of manufactured articles, notably apparel (reduced from £4,537,000 to £879,000) and silk and artificial silk goods (£2,829,000 to £827,000).

As regards exports, the value in the seven months was £4 million more than a year ago. This change of little more than 1 per cent. in the aggregate conceals many changes among individual groups.

Exports of commodities covering a wide field have done consistently well, notably beverages—the United States being an important market for whisky—chemicals, drugs, etc., paper and cardboard, silk and artificial silk goods, miscellaneous textile manufactures, pottery, glass, etc., electrical goods and apparatus, and cutlery, hardware, etc. Apart from the last two groups, declines were recorded for all the metal groups, those for machinery (£7·5 million) and the vehicles group (£6·5 million) being very substantial. Though exports of cotton and woollen and worsted yarns and manufactures showed a considerable rise for the seven months, there was a substantial reduction in July. Exports of coal showed a fall of £2·4 million of which £2·0 million occurred in July, the coal trade being seriously affected by the stoppage by exports to European markets, notably France.

General *wholesale prices* have continued the advance commenced at the beginning of the war, and the Board of Trade index-number for July 1940 showed an increase of 42½ per cent. over the prices current for August 1939, and of 14·2 per cent. since the end of 1939. Food prices have advanced 48½ per cent. and prices of industrial materials and manufactures 39 per cent. The group showing the largest increase is that of cereals, which has advanced 61 per cent., but the textile groups have also shown substantial increases (cotton 52·9 per cent., wool 55 per cent., and other textiles 42·6 per cent.). The advance of 50 per cent. in the group of foods other than meat and cereals is largely due to the increased duties on sugar and tobacco. Those groups, which contain materials or commodities subject to a considerable control and where prices are fixed or regulated, have shown much smaller advances; the iron and steel group has advanced 27·4 per cent. and the non-ferrous metals 22·9 per cent. The controlled prices for iron and steel products were advanced from July 1st, as were the controlled prices of timber, the high prices of which and also those of paper-making materials have been chiefly responsible for the increase in the Board of Trade index-number for the miscellaneous group of materials of about 60 per cent. since the beginning of the war. As the Board no longer publishes the monthly prices of the articles contained in its index-number, it is not possible to say whether the quotations used in the calculation are or are not "free market quotations." Recently the Board has revised to some extent the index-number for the early months of the current year and the revised figures for these months are given in the following table. The figures given in Part II of the Journal are the uncorrected figures :—

(Average for the year 1930 = 100.)

Date	Total Food	Total not Food *	All Articles	Basic Materials †	Inter-mediate Products	Manu-factured Articles	Building Materials
Aug. 1939	90·4	102·2	98·1	94·5	104·0	108·7	104·1
Jan. 1940	122·3	126·7	125·3	137·0	128·1	123·7	112·2
Feb. ,,	125·7	129·4	128·3	136·6	132·7	127·9	114·1
Mar. ,,	123·7	131·4	128·8	140·4	134·9	129·0	114·3
Apr. ,,	125·7	135·4	132·2	152·1	136·7	130·4	114·8
May ,,	128·2	136·4	133·7	152·2	137·4	131·1	116·1
June ,,	130·3	136·4	134·4	151·5	137·4	131·1	117·4
July ,,	134·4	142·3	139·7	148·9	153·8	136·2	127·0
July 1939	*91·2*	*101·7*	*98·1*	*94·3*	*102·0*	*109·1*	*103·3*

* Industrial materials and manufactures. † Excluding fuel.

The revision is most considerable in the group of cereals, but there are smaller alterations in other groups.

The figures for certain other British index-numbers and the official index-number of wholesale prices for the United States are given below.

Date	*Board of Trade* (1930 = 100)	*Economist* (1927 = 100)	*Statist* (1866–77 = 100)	*The Times* (1913 = 100)	United States (Bureau of Labor, 1926 = 100) *
Aug. 1939 ...	98·1	70·3	90·4	114·5	74·8
Apr. 1940 ...	132·2	93·7	126·0	146·0	78·3
May ,, ...	133·7	94·8	128·0	148·5	78·3
June ,, ...	134·4	95·8	130·0	149·8	77·2
July ,, ...	139·7	96·0	129·5	151·7	77·6
July 1939 ...	*98·1*	*69·3*	*88·7*	*111·9*	*75·3*

* Mean of weekly figures.

It will be seen that the rise in general prices in the United States since the beginning of the war has not been very great, amounting only to about 3·8 per cent. The rise in prices of farm products is however more noticeable, being about 9 per cent.

The value of wholesale trading in textiles as indicated in the index-numbers prepared by the wholesale Textile Association and the Bank of England shows a considerable increase in the second quarter of 1940 as compared with the corresponding period in 1939 both as regards the home and the export trade. The index-numbers for the home trade for April, May, and June 1940 were 145, 131, 97, and 96, 105, 94, for 1939. For the export trade the numbers were respectively, 94, 112, and 105 and 67, 93, and 80. The combined

index-numbers were 142, 128, and 97 in the months of April, May, and June 1940, and 94, 103, and 93 in the corresponding months of 1939. There has, of course, been a large increase in textile prices since the beginning of the war, and the figures for the quantities sold are not available.

During the three months May to July 1940, apart from seasonal fluctuations, there has been a rise of about $2\frac{3}{4}$ per cent. in the *cost of living* in working-class families, as measured by the Ministry of Labour's index-number of the retail prices of articles of working-class consumption. Food prices have increased rather more than 3 per cent., as also have the prices of articles of clothing. There has been a slight increase in fuel prices and an increase of about 4 per cent. in miscellaneous items of expenditure. Some considerable portion of the last-mentioned increase is due to additional duties on tobacco and cigarettes and to certain increases in fares. Prices of sugar remained stationary during the period, but an increase of $\frac{1}{2}d.$ a lb. on all kinds came into effect on August 26th. Since the beginning of the war until August 1st, 1940, the general index-number has registered an advance of about 19 per cent., that for food being nearly 19 per cent., for clothing nearly 40 per cent., for fuel 16 per cent., and for other items 22 per cent. Working-class rents have risen but slightly. It should be borne in mind that retail prices of some of the principal articles of food are controlled and maximum prices fixed. For example, when tea was rationed in July the prices were stabilized at those obtaining on July 1st, 1940.

The index-numbers for the various classes of expenditure are given below for recent months and for September 1st, 1939, the average prices for July 1914 being taken as 100.

Date	Food	Rent and Rates	Clothing	Fuel and Light	Other Items	All Items
Sept. 1st, 1939 ...	138	162	205–210	180–185	180	155
May 1st, 1940 ...	159	164	280	208	210	180
June 1st, „ ...	158	164	285	212	210	181
July 1st, „ ...	168	164	290	212	210	187
Aug. 1st, „ ...	164	164	290	212	219	185
Aug. 1st 1939 ...	*137*	*162*	*205–210*	*180*	*180*	*155*

The improvement in *employment*, which had been continuous and of considerable proportions since February 1940, was arrested

in July, when the rate of unemployment in the insured trades in Great Britain and Northern Ireland rose from 5·2 per cent. on June 17th to 5·6 per cent. on July 15th. The increase was largely confined to females, whose rate of unemployment rose from 6·8 in June to 8·2 per cent. in July, or in numbers by nearly 60,000, but there was some falling off in employment among coal-miners, partly owing to temporary stoppages. There was a general decline in employment in July in the textile trades, which was most marked in the linen industry of Northern Ireland. A decline, partly seasonal, was common to all the clothing trades.

Compared with twelve months ago there was a decline in employment in 19 out of the 105 trade groups into which the Ministry of Labour enquires. In these 19 groups about a million and a half workpeople are ordinarily engaged, and among the more important of them are the printing and paper trades, clothing trades (excluding boot and shoe trades), linen and hosiery industries, tinplate trade, tanning industry and laundry service. In slate-quarrying, tinplate, linen, jute, dressmaking, and hatmaking industries and in dock labour the rate of unemployment is still high, and ranges from 14·2 per cent. in the tinplate trade to 32·2 per cent. in the linen industry.

Employment in agriculture showed some decline in July, but throughout the first seven months of 1940 it has been appreciably better than in 1939, although in Northern Ireland the proportion unemployed in July was still as high as 16·4 per cent.

The percentages unemployed in the insured trades in Great Britain and Northern Ireland are set out below.

Date	Percentage Unemployed in Great Britain and Northern Ireland of Workpeople Insured under				
	General Scheme	Agricultural Scheme	General and Agricultural Schemes		
			Males	Females	Total
Aug. 14th, 1939 ...	8·7	3·9	8·9	7·4	8·5
Apr. 18th, 1940 ...	6·8	4·4	6·3	7·5	6·7
May 20th, „ ...	6·2	3·4	5·7	7·1	6·1
June 17th, „ ...	5·3	2·3	4·6	6·8	5·2
July 15th, „ ...	5·7	2·5	4·6	8·2	5·6
July 10th, 1939 ...	*9·0*	*4·0*	*9·3*	*7·2*	*8·7*

The unemployment statistics have been affected to a small extent by the coming into operation of the Old Age and Widows' Pensions Act, 1940, under which women 60 years of age and over cease

to be insurable against unemployment. The Ministry of Labour estimate that about 8,000 women who in previous months would have been included in the numbers of insured unemployed were excluded from those figures at July 15th, though they were, of course, included in the total number of unemployed persons (insured and uninsured) registered at employment exchanges. The effect of the exclusion is to reduce the percentage unemployed among insured women by 0·2 per cent. At the same time, the practice has been discontinued of including in the totals of unemployed all persons receiving training at Government training centres because of the increasing extent to which persons actually in employment or likely soon to be in employment are admitted to such centres. The change in practice is equivalent to a reduction in the percentage of unemployment among males of less than 0·1 per cent.

The following table gives the number of workpeople (uninsured and insured) aged 16 to 64 on the registers of the Employment Offices of the Ministry of Labour in Great Britain.

Date	Wholly Unemployed	Temporarily Stopped	Persons normally in Casual Employment	Total
Aug. 14th, 1939	968,108	211,978	51,606	1,231,692
Apr. 15th, 1940	840,027	90,182	42,486	972,695
May 20th, „	730,773	102,730	47,319	880,822
June 17th, „	648,314	81,380	37,141	766,835
July 15th, „	636,532	153,242	37,492	827,266
July, 1939	*1,013,636*	*190,364*	*52,424*	*1,256,424*

The figure 827,266 recorded for July 15th, 1940, included 9,454 boys and 16,827 girls between the ages of 14 and 16, and 11,777 boys and 21,259 girls between the ages of 16 and 18. Compared with July 1939, there was a reduction in the numbers on the registers of 504,893 men and of 5,920 boys under 18, while there was an increase of 73,053 women and of 8,602 girls under 18. The increase in the number of women on the register is no doubt largely due to the application for war work or other employment of women not usually engaged in industry.

2. Other Statistics

During the six months February to July, 1940, the statistics of *retail sales* prepared by the Bank of England in conjunction with various retail distributors' associations and co-operative societies show that compared with the corresponding period of 1939 there

was an increase in the money value of sales of 5·2 per cent. Sales of food increased by 6·4 per cent. and those of all other kinds of merchandise by 3·8 per cent. Bearing in mind the considerable increase in the retail prices of food and most other articles, it is clear that sales have declined in quantity to an appreciable extent. The most noticeable decrease has been in the value of sales and consequently of quantities in London (Central, West End, and Suburban), which fell 6·6 per cent. In the rest of Great Britain there was an increase of 10·2 per cent.

The International Institute of Agriculture has now issued for the year 1938–39 its annual *Review of the World Agricultural Situation.* This contains much interesting matter dealing with economic aspects of agriculture before the outbreak of war, and summarizes, for the year in question, the position as regards agricultural production and supply, world trade in the principal products, and agricultural prices.

As a matter of current interest, it may be noted that German agricultural production is estimated to have been 17 per cent. greater in volume in 1937–38 than in 1927–29. This increase, it is stated, has allowed Germany to produce enough to satisfy 82 per cent. of her food requirements as against 71 per cent. in 1928, although the proportion of total demand met from home sources naturally varies considerably from product to product. This increase in agricultural production is attributed to a more intensive cultivation, giving a greater yield per unit of area, but it is also associated with considerable changes in the use made of the cultivated area. For example, increased areas have been successfully diverted from crops in sufficient supply to those in short supply, and from crops of uncertain or insufficient yield to those which are more productive. The use of chemical fertilizers and the sales of machines to farmers are said to show large increases.

The last decade has been in all parts of the world a period of intense governmental activity, which has profoundly changed agricultural markets, re-directed trade, and deeply influenced agricultural production. With the object of throwing light on the various problems thus presented, the International Institute of Agriculture has recently published a volume of 1100 pages with the title of *World Trade in Agricultural Products: its growth; its crisis; and the new Trade Policies.* The plan of this work was conceived by Dr. H. C. Taylor, Director of the Farm Foundation at Chicago, who secured a grant from the Rockefeller Foundation to enable it to be carried out. The actual research was entrusted to two economists,

Miss L. B. Bacon and F. C. Schloemer, with assistance from the various resources of the Institute.

The volume, which may be said to present a general picture of the evolution undergone by the world trade in agricultural products, is divided into two parts, the first dealing with commodities, the second with countries. The commodities selected are wheat, rice, sugar, meat, fats and oils, coffee, tea, tobacco, cotton, wool, silk and rubber. These items cover three-fourths of the world trade in agricultural products. In each case a description is given of the trend of trade during approximately the last 30 years, and particularly in the period 1928–37, with an account of tariff measures and trade agreements, the depression in prices, and other influences affecting the several products. In somewhat the same way, the examination of world trade by countries involves what amounts to a short history of the production, imports and exports of the principal countries, with an account of their agricultural marketing policies and the various forms of governmental control aimed, in some cases, at the promotion of home production and self-sufficiency, in others at the preservation of home agriculture from the disastrous effects of the depression in prices. The whole forms a remarkably exhaustive contribution to the study of the prices and supply movements of agricultural trade.

Under the title *War Effort and Industrial Injuries* (Tract no. 253 of the Fabian Society's Series, 16 pp., 3*d*.), Professor Hermann Levy calls attention to the loss in production caused by industrial injuries, which is not only much increased in war conditions, but also becomes of far greater consequence. Statistics of accidents are only available for 8 million workers, belonging to seven large industrial groups, out of the 16 million or so workers covered by workers compensation. They show an annual total of about 2500 fatal and 460,000 other accidents; 300,000 of these injuries result in absences of over two weeks, 40,000 in absences of over six months. In war time more persons, and many more inexperienced persons; are employed for longer hours and at greater pressure, while precautions are apt to be relaxed. Prevention requires effective supervision with the sincere co-operation of the employers. No less essential are better provisions for treatment and rehabilitation of the injured, which Dr. Donald Norris has stated to be " very imperfectly dealt with by existing arrangements." As was shown by the Memorandum of the International Labour Office to the Royal Commission on Workers' Compensation, quoted by Dr. Levy, many foreign countries are in advance of our own in these respects.

CURRENT NOTES

Sir Alfred and Lady Flux have been in Copenhagen for the past twelve months. The last message from them dated 30 May, 1940, runs: "We have been and are both flourishing. Greetings and best wishes to you and to all our friends." Since then communications have been interrupted but attempts have been made to get another message through to them.

Mr. A. R. Burnett-Hurst, a Fellow of the Society since 1913, has been appointed Principal of the City School of Commerce, Liverpool, on the retirement of Mr. John A. Todd, who had held the post since 1923. Mr. Burnett-Hurst will be remembered as having assisted Professor Bowley in his investigations of conditions in Lancashire and other towns; he was afterwards Professor of Commerce in the University of Allahabad, and later Statistician to the Government of N. Rhodesia.

STATISTICAL AND ECONOMIC ARTICLES IN RECENT PERIODICALS

UNITED KINGDOM—

Advancement of Science, April, 1940—Rates and taxes: *Prof. H. O. Meredith.* Measurement in psychology: *R. J. Bartlett.*

Annals of Eugenics, June, 1940—The estimation of missing values in quasifactorial designs: *E. A. Cornish.* The estimation of the proportion of recessives from tests carried out on a sample not wholly unrelated: *R. A. Fisher.* The detection of linkage: *D. J. Finney.* Proof of Fisher's Rules for ascertaining the sampling semi-invariants of k-statistics: *M. G. Kendall.*

The Banker—

May, 1940—Exchange control in evolution: An export policy for wartime: *W. T. C. King.* Commodity control in wartime.

June, 1940—Are export groups enough?: The banks and hire purchase: *J. Mead.* Financial effects of the latest aggression: *Paul Einzig.*

July, 1940—Blockade can still defeat Germany: *Walter Hill.* The banking half year.

August, 1940—Wartime control of interest rates: Limitations of price control: *Donald Tyerman.*

Bankers' Magazine—

June, 1940—The progress of banking in Great Britain and Ireland during 1939, Part I.

August, 1940—The June bank statements: Financing the war: French banking and currency.

Economic Journal, June–Sept., 1940—Changes in wage rates and earnings in 1939–40: Small savings: *John F. L. Bray.* The position and prospects of gold: *G. Findlay Shirras.* American gold policy and allied war economics: *Prof. S. E. Harris.* Rubber and foreign exchange: *P. T. Bauer.* The weight of consignments in transport: *Lord Stamp.* An approximation to a geographical multiplier: *M. C. Daly.*

Economica, May, 1940—The valuation of the social income: *J. R. Hicks.* Socialist calculation—the competitive "solution": *F. A. v. Hayek.* The Bank and its treasure: *J. K. Horsefield.* A reconsideration of the Austrian theory of industrial fluctuations: *L. M. Lachmann.*

Journal of the Institute of Actuaries, April, 1940—Permanent sickness insurance: *E. J. Heath.* A general theory of mortality: *C. D. Rich.*

Public Administration, July, 1940—The administration of food distribution in war-time: *R. N. Spann.* Women in employment: *G. H. Stuart-Bunning.*

UNITED KINGDOM—*Contd.*

Review of Economic Studies, June, 1940—Recent industrial concentration, especially in the Midlands: *A. J. Wensley* and *P. Sargant Florence*. Capital theory and the trade cycle: *T. Wilson*. The Keynesian equations and the balance of payments: *M. Bronfenbrenner*. A theoretical account of futures markets: *J. C. R. Dow*. A symposium on the theory of the forward market:

Journal of the Royal Agricultural Society, Vol. 101, *Part I*, 1940—Farm economics: *C. S. Orwin*.

Proceedings of the Royal Society of Edinburgh, Vol. LX, Part I, 1939–40—The maximal range of error in gross reproduction rates: *Enid Charles*.

Sociological Review, January–April, 1940—The fertility and mortality of the population of Palestine: *Rita Hinden*.

INDIA—

Indian Journal of Economics, January, 1940—Conference Number: Papers read and discussed at the Twenty-Third Conference of the Indian Economic Association; including several on the Scope of Economics, by *J. K. Mehta, A. I. Qureshi and others:* and on Labour Problems in India, by *B. N. Rohatgi, A. C. Bhatia, S. K. Rudra and others.*

Sankhyā, March, 1940—A statistical analysis of the shift of expenditure in a prohibition area: *P. J. Thomas*. Trends of foreign trade of India during the past sixty years: *N. S. R. Sastri* and *N. T. Mathew*. A sample survey of the acreage under jute in Bengal: *P. C. Mahalanobis*. A study of forty-three years of rainfall in Calcutta, 1893–1935: *S. S. Bose*.

AUSTRALIA—

Economic Record, June, 1940—War economy—the nature of the problem: *E. R. Walker*. History of exchange control in Australia: *C. V. Janes*. Australian war finance and banking policy: *T. W. Swan*.

Wheat Studies—

April, 1940—Bulk handling in Australia: *Joseph S. Davis*.

May—World wheat survey and outlook, May, 1940: *V. P. Timoshenko* and *Holbrook Working*.

SOUTH AFRICA—

S. African Journal of Economics, March, 1940—Monoexport in Africa: *C. J. Robertson*. Economists' protest: the operation of the wheat marketing scheme, 1938–1939; memorandum of objections: *T. H. Kelley, and others*. Note on the national income of the Union of South Africa, 1927–28, 1932–33, 1934–35: *S. Herbert Frankel* and *S. D. Neumark*.

UNITED STATES—

Transactions of the Actuarial Society of America, May, 1940—Population, birth and mortality trends in the United States: *Robert J. Myers.*

Annals of the American Academy of Political and Social Science—

May—Marketing in our American economy—whole number:

July—When War ends; a consideration of the conditions necessary to bring about a lasting peace (whole number). By various authors, including *W. E. Rappard, M. J. Bown, J. T. Sholwell, G. Cassel, Viscount Cecil of Chelwood, L. Einaudi, L. Villari, G. S. Patterson.*

American Economic Review, June, 1940—Schumpeter's business cycles: *Simon Kuznets.* Railroad costs and volume of traffic: *Herbert Ashton.*

Proceedings of the American Philosophical Society, February, 1940—Symposium on the totalitarian state:

Econometrica—

April, 1940—Long cycles in residential building; an explanation: *J. B. D. Derksen.* A new method of trend elimination: *M. Kalecki* and *B. Tew.*

July—Clement Colson: *René Roy.* Price and wage policies and the theory of employment: *Richard M. Bissell, Jr.* Employment, investment, and the multiplier: *Montgomery D. Anderson.* A note on Pareto's Theory of Production: *H. Neisser.*

Journal of Experimental Education, March, 1940—Statistical studies of a college freshman testing program: *Harold A. Edgerton.* Corrections to correlation coefficients on account of homogeneity in one variable: *Teobaldo Casanova.* Experimental design and statistical treatment in educational research: *Eugene Shen.*

Harvard Business Review—summer number, 1940—Inflation and price control: *Sidney Weintraub.* Price trends of industrial chemicals: *Melvin T. Copeland.*

Milbank Memorial Fund Quarterly—

April, 1940—Certain characteristics of urban housing and their relation to illness and accidents—summary of findings of the national health survey: *Rollo H. Britten, J. E. Brown* and *Isidore Altman.* Economics of the family relative to number of children: *Frank Lorimer* and *Herbert Roback.* Studies of rheumatic disease: *Ross L. Gauld* and *Frances E. M. Read.*

July—Outstanding population trends affecting problems of social welfare: *Warren S. Thompson.* Population trends and future problems of child welfare: *Katharine F. Lenroot* and *Robert J. Myers.* Factors underlying individual and group differences in uncontrolled fertility: *Regine K. Stix.*

United States—*Contd.*

Monthly Labor Review—

March, 1940—Transportation and recreation expenditures of wage earners and clerical workers.

June—Trends of manufacturing employment, 1929 to 1937: *A. F. Hinrichs* and *Harry Brenner.* Labor policy in Germany: Wages of Poles and Germans in Poland.

Journal of Political Economy, April, 1940—Population movements and economic equilibrium in the United States: *Joseph J. Spengler.* Public ownership versus state purchasing—the case of printing: *Estal E. Sparlin.* Economic changes and industrial unrest in the United States: *Dale Yoder.* Economists on industrial stagnation: *M. H. Dobb.*

Quarterly Journal of Economics, May 1940—The variation in wage-ratios: *E. L. Thorndike.* German corporate profits: 1926–1938: *Maxine Y. Sweezy.* Further measurements of marginal productivity: *Paul H. Douglas* and *Grace Gunn.* The measurement of tax shifting—economics and law: *Wirth F. Ferger.* The causes of price inflexibility: *Jules Backman.* The teaching of economics in public high schools: *J. M. O'Leary.*

Review of Economic Statistics, May, 1940—The agricultural situation, March 1940: *John D. Black* and *Nora Boddy.* Graphical survey of economic developments: Employment in relation to technical progress: *Hans Staehle.*

Social Research, May, 1940—The gold problem: *Fritz Lehmann.*

France—

Journal de la Société de Statistique de Paris, May, 1940—Jeu de la concurrence entre producteurs—courbes d'offres et de transactions: *R. Hénon.* Annuaire des statistiques du travail: *A. Barriol.*

Hungary—

Journal de la Société Hongroise de Statistique, 1939. *No.* 4—Le rôle du calcul des différences finies en statistique: *Ch. Jordan.*

Italy—

Economia, April, 1940—La preparazione agricola alla guerra totale: *Arrigo Serpieri.* L'oggetto del commercio con l'estero in regime d'autarchia: *Jacopo Mazzei.*

Giornale degli Economisti, Jan.–Feb., 1940—A proposito di un recente volume sull'incidenza delle imposte: *M. Fasiani.* Aspetti della recente riforma fiscale: *E. D'Albergo.*

Revista de Economia y Estadistica, 1939, *No.* 4—La naturaleza de la actividad económica: *Giuseppe U. Papi.* Un supuesto remedio a las crisis parciales de superproducción: *Mario Pugliese.*

Rivista di storia economica, March, 1940—Storia di un investimento di capitale. La Compagnie Universelle du Canal Maritime de Suez (1859—1938): *Giulio Capodaglio.*

INTERNATIONAL—

International Labour Review—

May, 1940—A comparison of the gainfully occupied population by sex and age in the various countries of the world.

June—Deferred pay—the Keynes plan : *E. J. Riches*. Requisitioning of labour—hours of women and young persons in Great Britain :

Metron, Vol. XIII, No. 4—Ueber eine spezielle Klasse analytischer Geburtenfunktionen : *H. Hadwiger* and *W. Ruchti*. L'indice di cograduazione del Gini nel caso di serie statistische con ripetizioni : *T. Salvemini*. Sulla determinazione dell'indice di cograduazione : *C. Gini*. Distribution of abilities depending upon two or more independent factors : *C. H. Boissevain*. Ressemblance parentale et ressemblance fraternelle : *C. Gini*. Sulla legge di estinzione delle famiglie : *G. Zappa*. The misstatement of women's ages and the vital indexes : *T. L. Smith* and *H. L. Hitt*. Pigmentazione degli occhi e dei capelli e selezione naturale : *A. Costanzo*. Tarifstatistik als Element der Tarifbildung, erläutert an einem Gerichtsgebührentarif : *A. Komischke*.

LIST OF ADDITIONS TO THE LIBRARY

Since the issue of Part II, 1940, the Society has received the publications enumerated below :—

I.—OFFICIAL PUBLICATIONS.

(a) United Kingdom.

Ministry of Labour and National Service.

Schedule of reserved occupations. Revision May 1940. London: H.M.S.O., 1940. 9¾″ × 6″. 134 pp. 1*s*.

The effects of working conditions upon the health of London central busmen: report of the conferences between representatives of the London Passenger Transport Board, the Transport and General Workers' Union and the Medical Research Council, under the Chairmanship of *Sir John Forster*. London: H.M.S.O., 1939. 9¾″ × 6″. iv + 49 pp. 1*s*.

Medical Research Council. Special Report, Series No. 242. Studies in nutrition: an inquiry into the diet of families in the Highlands and Islands of Scotland, by *E. P. Cathcart, A. M. T. Murray* and *J. B. Beveridge*. London: H.M.S.O., 1940. 9¾″ × 6″. 37 pp. 9*d*.

London County Council.

Members' Library catalogue, Vol. I. London history and topography. London: P. S. King, 1939. 8¼″ × 5¼″. 142 pp. 1*s*.

Report of the Education Officer on juvenile delinquency. London. P. S. King, 1937. 7¼″ × 5″. 23 pp. 6*d*.

(b) British Empire.

Australia—

Tasmania. State Finance Committee. The Tasmanian economy in 1938–39: a survey prepared on behalf of the State Finance Committee by *E. Ronald Walker* . . . assisted by *D. L. Anderson*. . . . Hobart, 1939. 11″ × 8½″. vii + 47 pp.

Canada—

Department of Insurance. Report of the Superintendent of Insurance of the Dominion of Canada for the year ended Dec. 31, 1934. Vol. II, Life insurance companies. Ottawa: 1935. 9¾″ × 6½″. ccccxvi + 995 pp. (From Prof. H. Levy.)

Dominion Bureau of Statistics. Seventh census of Canada, 1931. Vol. III, Ages of the people . . . 1935. viii + 1009 pp. $1. Vol. VI, Unemployment. 1934. xxviii + 1319 pp. $1. Ottawa: 9½″ × 6½″. 2 vols. (From Mr. R. H. Coats.)

Eire—

Department of Industry and Commerce. Census of industrial production, 1938. Dublin: Stationery Office, 1940. 10″ × 6¾″. xii + 37 pp. 1*s*.

Jamaica—

Labour Department. Report on the cost of living survey carried out in Kingston, Jamaica, by the Labour Department, August–November 1939. Kingston: Government Printer, 1940. 13″ × 8¼″. 24 pp. 3*d*. (From Mr. F. A. Norman.)

(b) **British Empire**—*Contd.*

New Zealand—

Census and Statistics Department. Population census 1936. Vol. III, Maori census. vi + 44 pp. Vol. IV, Ages and marital status. viii + 70 pp. Vol. V, Orphan children and dependent children. vii + 31 pp. Vol. VI, Religious professions. v + 29 pp. Appendix C, Census of libraries, 1938. iii + 13 pp. Wellington: 1940. 12″ × 9¾″. 5 parts.

Statistics of employment and income, 1937–38: an appendix to the statistical reports for the year 1938. Wellington, 1939. 13¼″ × 8¼″. 158 pp.

Union of South Africa—

Census and Statistics Office. Report on Workmen's Compensation Act statistics 1936 . . . Pretoria: 1939. 12″ × 9½″. 66 pp. 3*s.*

(c) **Foreign Countries.**

China—

Bureau of Foreign Trade, Ministry of Industry. Chinese economic journal. Vol. IV, Jan.–June 1929 to Vol. XVII, July–Dec. 1935. Shanghai: 1929–35. 9¼″ × 6″. 14 vols. (From the I.C.I. Library.)

Estonia—

Bureau Central de Statistique. Enquête budgétaire, Vol. IV. Tallinn, 1940. 10¼″ × 7¼″. 95 pp.

Germany—

Statistisches Reichsamt. Wirtschaft und Statistik, 1927–1935. Berlin: 1927–35. 11¼″ × 8″. 9 vols. (From the I.C.I. Library.)

U.S.A.—

Children's Bureau. Publication No. 245. Juvenile court statistics 1936 and federal juvenile offenders 1936. 155 pp. 20*c.* 246. Proceedings of conference on better care for mothers and babies. 1938. 171 pp. 20*c.* 247. Welfare of families of sugar-beet labourers. 100 pp. 15*c.* 248. Statistical measurement in group work. 103 pp. 15*c.* Washington: Government Printing Office, 1939. 9″ × 6″. 4 vols.

(d) **International.**

International Labour Office—

Studies and Reports, Series N, No. 23. Methods of family living studies: income—expenditure—consumption. By *R. M. Woodbury.* Geneva: 1940. London: P. S. King. 9¼″ × 6″. viii + 144 pp. 4*s.*

II.—AUTHORS AND MISCELLANEOUS.

Assekuranz-Jahrbuch, gegründet 1880 von A. Ehrenzweig. Redaktion *Prof. S. J. Lengyel.* Band 59. Basel: Verlag für Recht und Gesellschaft, 1940. 8″ × 5½″. [8] + 591 pp. (Prof. Lengyel.)

Bartlett (*M. S.*). A note on tests of significance in multivariate analysis. (Reprinted from *Proceedings of the Cambridge Philosophical Society,* April 1939, pp. 180–85.) Cambridge: 10″ × 7″.

—— Complete simultaneous fiducial distributions. (Reprinted from *Annals of Mathematical Statistics,* June 1939, pp. 129–38.) 10″ × 7″.

II.—Authors and Miscellaneous—*Contd.*

Beveridge (*Sir William*). Some experiences of economic control in war-time. Sidney Ball Lecture, February 29th, 1940. London: Oxford University Press, 1940. $7\frac{1}{2}'' \times 5\frac{1}{2}''$. 38 pp. 1*s.*

British Association for the Advancement of Science. Mathematical Tables, Volume VIII. Number-divisor tables, designed and in part prepared by *J. W. L. Glaisher*, extended and edited by the Committee for the Calculation of Mathematical Tables. Cambridge: University Press, 1940. $11'' \times 8\frac{1}{2}''$. x + 100 pp. 15*s.*

Burn (*D. L.*). The economic history of steelmaking 1867–1939: a study in competition. Cambridge: University Press, 1940. $8\frac{1}{4}'' \times 5\frac{1}{4}''$ x + 548 pp., maps and graphs. 27*s.* 6*d.*

Chiplonkar (*M. W.*). Measurements of atmospheric ozone at Bombay. (Reprinted from *Proceedings of Indian Academy of Sciences*, Sept. 1939, pp. 105–20.) $9\frac{3}{4}'' \times 7\frac{1}{4}''$.

Clark (*Colin*). The conditions of economic progress. London: Macmillan, 1940. $8\frac{1}{2}'' \times 5\frac{1}{2}''$. xii + 504 pp. 25*s.*

Dearle (*N. B.*). The labour cost of the world war to Great Britain 1914–1922: a statistical analysis. (Carnegie Endowment for International Peace. Economic and Social History of the World War. Supplementary Volumes.) New Haven: Yale University Press, 1940. (London: H. Milford.) $11'' \times 8\frac{1}{2}''$ ix + 260 pp. 12*s.*

Dicksee (*Lawrence R.*). Auditing: a practical manual for auditors, by the late *Lawrence R. Dicksee* . . . Sixteenth edition by *Stanley W. Rowland*. . . London: Gee & Co., 1940. $8\frac{1}{4}'' \times 5\frac{1}{4}''$. xx + 1171 pp. 25*s.*

Dutt (*Sachindra Nath*). The population studies of Benoy Sarkar. Calcutta Chuckervertty Chatterjee & Co., 1939. $9\frac{1}{2}'' \times 6\frac{1}{4}''$. 28 pp. 4 *annas.*

L'économiste français: journal hebdomadaire; 48e année, 1920. Paris, 1920. $13'' \times 8\frac{3}{4}''$. 2 vols. (I.C.I. Library.)

Glass (*D. V.*). Population policies and movements in Europe. Oxford: Clarendon Press, 1940. $8\frac{1}{2}'' \times 5\frac{1}{4}''$. viii + 490 pp. 25*s.*

Greenhood (*E. Russell*), *Jr.* A detailed proof of the chi-square test of goodness of fit. The Harvard Phi Beta Kappa Prize Essay for 1939. Cambridge, Mass.: Harvard University Press, 1940. (London: Humphrey Milford.) $6\frac{3}{4}'' \times 4\frac{1}{2}''$. xiii + 61 pp. 8*s.*

Hales (*Charles A.*). The Baltimore Clearing House. Baltimore: Johns Hopkins Press, 1940. (London: Humphrey Milford.) $8\frac{1}{2}'' \times 5\frac{1}{2}''$. xiii + 347 pp. $3.

Harvard Economic Society, Inc. The review of economic statistics and supplements. Vol. XV, 1933. Vol. XVI, 1934. Cambridge, Mass., 1933–34. $10\frac{1}{2}'' \times 8\frac{1}{4}''$. 2 vols. (I.C.I. Library.)

Jones (*W. A. Stewart*). Highway administration: powers and duties of local authorities. London: Gee & Co., 1940. $8\frac{1}{2}'' \times 5\frac{1}{2}''$. 20 pp. 1*s.*

Kendall (*M. G.*). Proof of Fisher's rules for ascertaining the sampling semi-invariants of *k* statistics. (From *Annals of Eugenics*, Vol. 10, Part 2, pp. 215–22, 1940.) $10\frac{3}{4}'' \times 8''$. (From the author.)

Liverpool (University of). Social Science Department: Statistics Division. New handbook of social statistics relating to Merseyside. Liverpool: University Press, 1940. $8\frac{3}{4}'' \times 5\frac{1}{2}''$. 40 pp. 1*s.*

London and Cambridge Economic Service. Special memorandum No. 49. The iron and steel industry 1926–1935: an investigation based on the accounts of public companies, by *R. H. Coase*, *R. S. Edwards* and *R. F. Fowler*. Cambridge: Published by the Executive Committee of the London and Cambridge Economic Service, 1940. $10\frac{1}{2}'' \times 8\frac{1}{4}''$. 31 pp. 5*s.*

Marsh (*Leonard C.*). Canadians in and out of work: a survey of economic classes and their relation to the labour market. Toronto: Published for the McGill University by the Oxford University Press, 1940. $8\frac{1}{2}'' \times 5\frac{1}{4}''$. xx + 503 pp. $3.

Martin (*W. J.*). A statistical study of the health of the London school child. (Reprinted from *Annals of Eugenics*, Vol. 10, Part I, pp. 18–41, 1940.) London. $10\frac{3}{4}'' \times 8''$.

Milbank Memorial Fund. Milbank Memorial Fund: thirty-five years in review. New York, 1940. $9'' \times 6''$. 75 pp.

II.—Authors and Miscellaneous—*Contd.*

National Institute of Economic and Social Research. Changes in wage rates and earnings in 1939–1940. (Reprinted from the *Economic Journal*, June–September 1940, pp. 189–94.) London, 1940. 9½″ × 6½″. (From the Institute.)

Niyogi (*J. P.*). The co-operative movement in Bengal. London: Macmillan, 1940. 8½″ × 5¾″. ix + 267 pp. 10*s.* 6*d.*

Österreiches Institut für Konjunkturforschung. Monatsberichte des österreichischen Institutes für Konjunkturforschung. 2, Jahrgang, 1928. Vienna: 1928. 11½″ × 8″. 196 pp. (I.C.I. Library.)

Oxford, Institute of Statistics. Diary. Week ending 25th November 1939—Week ending 18.5.40. 22 parts. Bulletin. No. 1, 1940 to No. 5, 1940. 5 parts. Oxford: The Institute. 13″ × 8″. 27 parts. (Typewritten.)

Preinreich (*Gabriel A. D.*). The present status of renewal theory. Baltimore, Md.: Waverley Press, 1940. 10″ × 7″. 27 pp. (From the author.)

Snyder (*Carl*). Capitalism the creator: the economic foundations of modern industrial society. New York & London: Macmillan, 1940. 9¼″ × 6″. xii + 473 pp. 16*s.*

Swiss Bank Corporation. British exchange regulations. London: Swiss Bank Corporation, 1940. 8½″ × 5½″. 24 pp.

Welwyn Garden City Research Committee. Life and work in Welwyn Garden City, Spring 1939. Welwyn Garden City, 1940. 13″ × 8″. 104 pp. (Typewritten.)

Die Wirtschaftskurve herausgegeben unter mitwirkung der Frankfurter Zeitung, 1927–35. Frankfurt am Main, 1927–35. 8½″ × 5½″. 9 vols. (I.C.I. Library.)

Wolff (*William*). The fundamental principles of a balance sheet. London: Gee & Co., 1940. 8½″ × 5½″. 24 pp. 1*s.*

Wyatt (*T. W.*), *Jones* (*D. Caradog*) and *Bellerby* (*J. R.*). Post-war poverty and unemployment can be prevented. . . . Birkenhead: John Woolman, 1940. 7¼″ × 5″. 24 pp. 6*d.* (From D. C. Jones.)

REGISTRATION OF THE UNITED KINGDOM

No. I.—ENGLAND AND WALES

A.—BIRTHS, DEATHS, *and* MARRIAGES : *Numbers and Annual Rates per* 1,000 *persons living. Deaths under* 1 *year of age : Mortality per* 1,000 *Live Births in the Calendar Years* 1935–1939 *and in the Quarters of those years.*

Years	1935		1936		1937		1938		1939 *	
Estimated Mid-Year Popln. in thousands	40,645		40,839		41,031		41,215		41,215 *	
	Number	Rate	Number	Rate	Number	Rate	Number	Rate	Number	Rate
Live Births ...	598,756	14·7	605,292	14·8	610,557	14·9	621,204	15·1	620,257	15·0
Stillbirths ...	25,435	0·63	25,045	0·61	24,806	0·60	24,729	0·60	24,285	0·59
Deaths	477,401	11·7	495,764	12·1	509,574	12·4	478,829	11·6	499,192	12·1
Marriages ...	349,536	8·6	354,644	8·7	359,160	8·7	361,768	8·7	437,406	10·5
Infant Mortality	34,092	57	35,425	59	34,917	·58	32,473	53	30,927	50·0
Quarters	Live Births in the Quarters of each Calendar Year									
Jan.–Mar. ...	146,363	14·6	148,035	14·6	145,405	14·4	155,187	15·3	153,547	15·1
Apr.–June ...	155,892	15·4	157,652	15·5	163,777	16·0	164,179	16·0	164,401	16·0
July–Sept. ...	155,498	15·2	155,596	15·2	158,590	15·3	158,082	15·2	161,201	15·3
Oct.–Dec. ...	141,003	13·8	144,009	14·0	142,785	13·8	143,756	13·8	141,108	13·5
	Stillbirths									
Jan.–Mar. ...	6,491	0·65	6,378	0·63	6,268	0·62	6,185	0·61	6,324	0·62
Apr.–June ...	6,620	0·65	6,502	0·64	6,619	0·65	6,639	0·65	6,317	0·61
July–Sept. ...	6,304	0·62	6,067	0·59	6,000	0·58	6,072	0·58	5,963	1·57
Oct.–Dec. ...	6,020	0·59	6,098	0·59	5,919	0·57	5,833	0·56	5,681	0·54
	Deaths (excluding Stillbirths) †									
Jan.–Mar. ...	132,657	13·2	153,591	15·1	163,716	16·2	137,897	13·6	154,158	15·2
Apr.–June ...	121,935	12·0	119,540	11·8	118,525	11·6	119,188	11·6	120,433	11·7
July–Sept. ...	100,066	9·8	99,935	9·7	100,301	9·7	102,545	9·9	103,170	9·9
Oct.–Dec. ...	122,743	12·0	122,698	12·0	127,032	12·3	119,199	11·5	121,431	11·6
	Marriages									
Jan.–Mar. ...	51,441	5·2	49,884	4·9	70,700	7·0	51,906	5·1	46,549	4·6
Apr.–June ...	98,388	9·7	100,621	9·9	80,265	7·8	101,785	9·9	102,344	9·9
July–Sept. ...	110,530	10·8	115,445	11·3	121,421	11·7	116,316	11·2	152,358	14·6
Oct.–Dec. ...	89,177	8·7	88,694	8·7	86,774	8·4	90,332	8·7	136,255	13·0
	Infant Mortality									
Jan.–Mar. ...	9,901	68	11,947	81	10,636	73	10,523	68	9,978	65·0
Apr.–June ...	8,693	56	8,583	54	8,835	54	7,933	48	7,828	48·0
July–Sept. ...	6,884	44	6,795	44	6,795	43	6,540	41	6,358	39·0
Oct.–Dec. ...	8,406	60	8,100	56	8,651	61	7,477	52	6,763	48·0

* Provisional. † Excluding deaths of non-civilians after 2 Sept., 1939.

B.—*Special Town Table* :—POPULATION; BIRTH-RATE *and* DEATH-RATE (*Civilians*) *in each Quarter of* 1939 *in certain of the* 126 *Great Towns.*

Towns	Estimated resident population, mid-1938	Annual Rate to 1,000 Living during the thirteen weeks ending: March 31, 1939 (1st quarter) Live Births	March 31, 1939 (1st quarter) Deaths*	June 30, 1939 (2nd quarter) Live Births	June 30, 1939 (2nd quarter) Deaths*
†Great towns ...	21,242,670	15·3	15·0	15·1	11·2
Including—					
London (City and Met. Bs.)	4,062,800	13·4	15·8	14·1	10·5
West Ham C.B. ...	254,900	17·4	14·6	17·5	9·9
Croydon C.B. ...	243,400	13·5	13·7	14·9	10·1
Brighton C.B. ...	146,500	14·0	18·2	14·1	14·8
Portsmouth C.B. ...	258,400	16·4	15·2	16·6	11·7
Bristol C.B.	415,500	15·2	15·7	16·0	11·5
Cardiff C.B.	224,280	14·8	15·6	17·2	11·9
Swansea C.B. ...	161,100	14·6	15·1	17·3	12·2
Wolverhampton C.B.	145,300	17·0	12·5	18·4	9·6
Birmingham C.B. ...	1,041,000	17·3	13·7	18·2	11·8
Norwich C.B. ...	122,300	13·2	15·5	13·9	12·0
Leicester C.B. ...	263,300	17·5	17·7	14·2	11·3
Nottingham C.B. ...	278,300	16·4	16·4	16·8	12·8
Derby C.B.	139,000	14·1	14·4	15·3	11·9
Birkenhead C.B. ...	144,400	17·2	14·7	15·9	11·2
Liverpool C.B. ...	827,400	19·8	16·4	20·1	12·2
Bolton C.B.	169,400	13·0	18·5	15·7	13·7
Manchester C.B. ...	732,900	14·5	15·7	15·3	12·0
Salford C.B. ...	199,400	17·4	17·3	15·6	13·2
Oldham C.B. ...	126,100	12·0	20·0	11·7	13·8
Burnley C.B. ...	88,650	13·1	17·8	12·2	13·8
Blackburn C.B. ...	113,000	12·2	19·3	12·2	14·4
Preston C.B. ...	113,200	15·7	18·2	15·7	11·7
Huddersfield C.B. ...	126,240	12·5	16·6	13·5	13·1
Halifax C.B. ...	97,370	14·8	17·4	15·3	14·0
Bradford C.B. ...	288,700	12·3	18·5	13·4	14·4
Leeds C.B.	494,000	15·1	16·6	15·2	12·8
Sheffield C.B. ...	520,000	16·3	15·1	16·6	11·2
Kingston-upon-Hull C.B.	318,700	17·8	14·1	19·4	12·3
Sunderland C.B. ...	182,400	19·1	15·5	20·5	12·6
Gateshead C.B. ...	117,000	17·2	16·0	18·2	11·2
Newcastle-upon-Tyne C.B.	291,300	16·8	14·9	18·1	12·5

The Registrar-General states that because of evacuation movements the estimated populations do not provide a satisfactory basis for calculation of birth and death rates in the last two quarters of the year.

* Excluding stillbirths.
† The great towns are those with populations exceeding 50,000 persons. Owing to boundary changes the numbers vary from time to time. In this table the totals relate to 126 towns.

No. II.—SCOTLAND

BIRTHS, DEATHS, *and* MARRIAGES: *Numbers and Annual Rates per* 1,000 *persons living. Deaths under* 1 *year of age: Mortality per* 1,000 *Live Births in the Calendar Years* 1935–1939 *and in the Quarters of those years.*

Years	1935		1936		1937		1938		1939	
Estimated Mid-Year Popln. in thousands	4,956		4,972		4,979		4,985		5,010	
	Number	Rate	Number	Rate	Number	Rate	Number	Rate	Number	Rate
Live Births ...	87,928	17·8	88,928	17·9	87,810	17·6	88,627	17·7	86,899	17·4
Stillbirths ...	Not separately recorded									
Deaths	65,331	13·2	66,749	13·4	68,942	13·9	62,953	12·6	64,413	12·9
Marriages ...	37,988	7·7	37,896	7·6	38,351	7·7	38,747	7·8	46,257	9·2
Infant Mortality	6,754	77	7,315	82	7,050	80	6,163	70	5,955	69·0
Quarters	Live Births in the Quarters of each Calendar Year									
Jan.-Mar. ...	21,978	18·0	22,544	18·3	21,589	17·6	22,245	18·1	21,429	17·4
Apr.-June ...	23,259	18·8	23,594	19·1	23,864	19·2	23,999	19·3	23,615	18·9
July-Sept. ...	21,566	17·3	21,464	17·2	21,745	17·3	21,423	17·0	21,549	17·1
Oct.-Dec. ...	21,125	16·9	21,326	17·1	20,612	16·4	20,960	16·7	20,306	16·1
	Deaths (excluding Stillbirths)									
Jan.-Mar. ...	19,177	15·7	20,196	16·4	22,123	18·0	17,448	14·2	18,995	15·4
Apr.-June ...	16,205	13·1	16,214	13·1	15,781	12·7	15,603	12·5	15,672	12·6
July-Sept. ...	13,461	10·8	13,709	11·0	13,616	10·9	13,807	11·0	13,587	10·8
Oct.-Dec. ...	16,488	13·2	16,630	13·3	17,422	13·9	16,095	12·8	16,159	12·8
	Marriages									
Jan.-Mar. ...	7,695	6·3	7,731	6·3	8,093	6·6	7,891	6·4	7,662	6·2
Apr.-June ...	9,106	7·4	9,451	7·7	9,363	7·5	9,428	7·6	9,960	8·0
July-Sept. ...	11,281	9·0	11,403	9·1	11,516	9·2	11,902	9·5	15,620	12·4
Oct.-Dec. ...	9,906	7·9	9,311	7·5	9,379	7·5	9,526	7·6	13,015	10·3
	Infant Mortality									
Jan.-Mar. ...	2,111	96	2,441	108	2,104	97	1,790	80	1,890	88
Apr.-June ...	1,655	71	1,677	71	1,733	73	1,563	65	1,442	61
July-Sept. ...	1,180	55	1,378	64	1,299	60	1,233	58	1,220	57
Oct.-Dec. ...	1,808	86	1,819	85	1,914	93	1,577	75	1,403	69

No. III.—NORTHERN IRELAND

BIRTHS, DEATHS, *and* MARRIAGES : *Numbers and Annual Rates per* 1,000 *persons living. Deaths under* 1 *year of age : Mortality per* 1,000 *Live Births in the Calendar Years* 1935–1939 *and in the Quarters of those years.*

Years	1935		1936		1937		1938		1939	
Estimated Mid-Year Popln. in thousands	1,271		1,276		1,281		1,286		1,295	
	Number	Rate	Number	Rate	Number	Rate	Number	Rate	Number	Rate
Live Births ...	24,742	19·5	25,909	20·3	25,412	19·8	25,742	20·0	25,254	19·5
Stillbirths ...	Not separately recorded									
Deaths	18,592	14·6	18,429	14·4	19,282	15·1	17,649	13·7	17,549	13·6
Marriages ...	8,844	6·96	9,144	7·17	8,623	6·64	8,617	6·68	8,837	6·8
Infant Mortality	2,136	86	1,992	77	1,969	77	1,933	75	1,777	70
Quarters	Live Births in the Quarters of each Calendar Year									
Jan.-Mar. ...	6,105	19·0	6,448	20·2	6,308	19·7	6,286	19·6	6,231	19·3
Apr.-June ...	6,510	20·2	6,793	21·3	6,917	21·6	6,902	21·5	6,859	21·2
July-Sept. ...	6,274	19·5	6,612	20·7	6,444	20·1	6,515	20·3	6,340	19·6
Oct.-Dec. ...	5,853	18·4	6,056	19·0	5,743	17·9	6,040	18·8	5,821	18·0
	Deaths (excluding Stillbirths)									
Jan.-Mar. ...	5,495	17·1	5,875	18·4	6,796	21·2	5,153	16·0	5,350	16·5
Apr.-June ...	4,753	14·8	4,481	14·0	4,514	14·1	4,283	13·3	4,371	13·5
July-Sept. ...	3,845	11·9	3,558	11·2	3,603	11·3	3,799	11·8	3,686	11·4
Oct.-Dec. ...	4,499	14·2	4,512	14·1	4,369	13·6	4,416	13·7	4,142	12·8
	Marriages									
Jan.-Mar. ...	1,582	4·9	1,725	5·4	1,790	5·6	1,599	5·0	1,432	4·4
Apr.-June ...	2,300	7·1	2,454	7·7	1,913	6·0	2,229	6·9	2,209	6·8
July-Sept. ...	2,638	8·1	2,763	8·7	2,688	8·3	2,602	8·1	2,559	7·9
Oct.-Dec. ...	2,324	7·3	2,202	6·9	2,232	6·6	2,157	6·7	2,637	8·1
	Infant Mortality									
Jan.-Mar. ...	726	119	622	96	616	98	668	106	599	11·2
Apr.-June ...	490	75	480	71	496	72	455	66	448	10·2
July-Sept. ...	478	76	401	61	370	57	368	56	346	9·4
Oct.-Dec. ...	435	74	487	80	482	84	436	72	384	9·3

No. IV.—ÉIRE

BIRTHS, DEATHS, *and* MARRIAGES : *Numbers and Annual Rate per* 1,000 *persons living. Deaths under* 1 *year of age : Mortality per* 1,000 *Live Births in the Calendar Years* 1935–1939 *and in the Quarters of those years.*

Years	1935		1936		1937		1938		1939	
Estimated Mid-Year Popln. in thousands	2,971		2,967		2,948		2,937		2,934	
	Number	Rate	Number	Rate	Number	Rate	Number	Rate	Number	Rate
Live Births ...	58,266	19·6	58,115	19·6	56,488	19·2	56,925	19·4	56,097	19·1
Stillbirths ...	Not separately recorded									
Deaths	41,543	14·0	42,586	14·4	45,086	15·3	40,041	13·6	41,730	14·2
Marriages ...	14,336	4·8	14,763	5·0	14,780	5·0	14,893	5·1	15,192	5·2
Infant Mortality	*3,988	68	4,309	74	4,121	73	3,794	67	3,643	65
Quarters	Live Births in the Quarters of each Calendar Year									
Jan.-Mar. ...	14,414	19·4	14,434	19·5	13,554	18·4	14,083	19·2	14,207	20·2
Apr.-June ...	14,919	20·1	15,100	20·4	15,052	20·4	14,697	20·0	14,468	19·7
July-Sept. ...	14,941	20·1	14,917	20·1	15,008	20·4	14,768	20·1	14,204	19·4
Oct.-Dec. ...	13,992	18·8	13,664	18·4	12,874	17·5	13,377	18·2	13,218	18·0
	Deaths (excluding Stillbirths)									
Jan.-Mar. ...	11,389	15·3	12,873	17·4	15,360	20·8	11,442	15·6	13,318	18·2
Apr.-June ...	10,947	14·7	10,821	14·6	11,206	15·2	9,814	13·4	10,383	14·2
July-Sept. ...	8,795	11·8	8,652	11·7	8,617	11·7	8,587	11·7	8,600	11·7
Oct.-Dec. ...	10,412	14·0	10,240	13·8	9,903	13·5	10,198	13·9	9,429	12·9
	Marriages									
Jan.-Mar. ...	3,857	5·2	3,796	5·1	3,303	4·5	3,824	5·2	3,485	4·7
Apr.-June ...	3,056	4·1	3,427	4·6	3,717	5·0	3,435	4·7	3,593	4·9
July-Sept. ...	4,157	5·6	4,183	5·6	4,202	5·7	4,379	6·0	4,505	6·1
Oct.-Dec. ...	3,266	4·4	3,357	4·5	3,558	4·8	3,255	4·4	3,609	4·9
	Infant Mortality									
Jan.-Mar. ...	1,144	79	1,234	85	1,260	91	1,155	81	1,158	82
Apr.-June ...	1,009	68	1,026	68	1,095	71	946	64	920	64
July-Sept. ...	948	63	893	60	863	57	810	55	744	52
Oct.-Dec. ...	887	63	1,156	85	903	69	883	66	821	62

No. V.—GREAT BRITAIN AND IRELAND

SUMMARY OF BIRTHS, DEATHS, AND MARRIAGES *in the years* 1936–1939: *Numbers and Rates per* 1,000 *persons living. Deaths under* 1 *year of age: Mortality per* 1,000 *Live Births.*

(Compiled from the Quarterly Returns of the respective Registrars-General.)

	England and Wales	Scotland	Northern Ireland	United Kingdom	Éire
Area in statute acres (thousands)	37,340	9,462	3,488	50,290	17,254
			1936		
Population (in thousands) ...	*40,839*	*4,972*	*1,276*	*47,081*	*2,967*
Births	605,292	88,928	25,909	720,129	58,115
Birth rates	14·8	17·9	20·3	15·2	19·6
Deaths	495,764	66,749	18,429	580,942	42,586
Death rates	12·1	13·4	14·4	12·3	14·4
Marriages	354,644	37,896	9,144	401,684	14,763
Marriage rates	8·7	7·6	7·17	8·5	5·0
Deaths under 1 year	35,425	7,315	1,992	44,732	4,309
Infant Mortality rates ...	59	82	77	62	74
			1937		
Population (in thousands) ...	*41,031*	*4,979*	*1,281*	*47,289*	*2,948*
Births	610,557	87,812	25,412	723,779	56,488
Birth rates	14·9	17·6	19·8	15·3	19·2
Deaths	509,574	68,942	19,282	597,798	45,086
Death rates	12·4	13·9	15·1	12·6	15·3
Marriages	359,160	38,345	8,623	406,134	14,780
Marriage rates	8·7	7·7	6·64	8·6	5·0
Deaths under 1 year	34,917	7,050	1,969	43,931	4.121
Infant Mortality rates ...	58	80	77	61	73
			1938		
Population (in thousands) ...	*41,215*	*4,985*	*1,286*	*47,485*	*2,937*
Births	621,201	88,604	25,742	735,550	56,925
Birth rates	15·1	17·7	20·0	15·5	19·4
Deaths	478,829	62,953	17,649	559,431	40,041
Death rates	11·6	12·6	13·7	11·8	13·6
Marriages	361,768	38,747	8,617	409,132	14.893
Marriage rates	8·7	7·8	6·68	8·6	5·1
Deaths under 1 year	32,473	6,163	1,933	40,569	3,791
Infant Mortality rates ...	53	70	75	55	67
			1939		
Population (in thousands) ...	*41,215*	*5,010*	*1,295*	*47,520*	*2,934*
Births	620,257	86,899	25,254	732,410	56,097
Birth rates	15·0	17·4	19·5	15·4	19·1
Deaths	499,192	64,413	17,549	581,154	41,730
Death rates	12·1	12·9	13·6	12·2	14·2
Marriages	437,406	46,257	8,837	492,500	15,192
Marriage rates	10·5	9·2	6·8	10·4	5·2
Deaths under 1 year	30,927	5,955	1,777	38,659	3,643
Infant Mortality rates ...	50	69	70	53	65

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The Overseas Trade of the United Kingdom 1930–39

By Henry W. Macrosty

[Read before the Royal Statistical Society, May 21st, 1940, the President, Professor A. L. Bowley, C.B.E., in the Chair.]

Eight years ago, on May 24th, 1932, I read a paper before this Society on "The Overseas Trade of the United Kingdom, 1924–31," and it now appears to me that it would not be inappropriate to continue that review up to the outbreak of the present war and to put on record the position which we had reached. In my previous paper I showed how the recovery in British export trade which had begun in 1923 continued, except for a setback caused by the industrial warfare of 1926, till 1929, when the American *débâcle* started a depression which rapidly spread and brought down British exports by about 18 per cent. in volume in 1930.

The years 1924–31 were far from normal on account of the hidden weaknesses left by the First German War, of reparation troubles, of the accumulation of gold in France and the United States, and of a desire in many countries to increase home production and reduce imports. I summed up the situation as follows :—

> " Want of confidence in the economic situation has spread through Europe like a plague, and the sight of the United States flung out of her artificial, paper-valued, instalment-bought prosperity into a trough of the deepest depression has made all countries try, at whatever hazards, to conserve their funds. The fear to buy is universal, and everywhere the old doctrine that if you will not buy you cannot sell is abandoned. Germany, France, the Netherlands, Switzerland, Czechoslovakia, and several other countries have established quotas for fixing the amounts of a variety of goods that can be imported into their countries. Further, a number of countries in Europe and South America have placed restrictions on exchange so

that exporters who have sold goods to them cannot obtain payment. And everywhere since the war tariff walls have been raised higher. If we are to move ever farther away from the principle of the international division of labour, if countries will only sell and not buy (except raw materials), international trade must diminish and such goods as are sold must be dumped at unremunerative prices. The world level of costs must then rise and consumers be relatively worse off. This may or may not concern us, but what does concern us is that we are a great exporting nation and that, if we cannot expand our markets, our industry and population must dwindle, unless in the alternative we can achieve a vast reconstruction and live mainly on our home trade."

I. THE GENERAL ECONOMIC STATE OF 1930–39

The ten years 1930–39 have been no less abnormal. The great depression widened and deepened and the collapse of finance and industry continued in the United States long after the clouds began to lift in this country. The immediate effects in the financial sphere were disastrous, for there was a tremendous quantity of foreign capital in this country on short term, and, as at the same time we had "borrowed short and loaned long," when the "bad money" was recalled our financial system was threatened with complete overthrow. After some futile attempts to maintain the exchange value of the pound, the unbelievable happened and the United Kingdom went off the gold standard in the autumn of 1931. Now the pound was left to find its own level, but later the establishment of the Exchange Equalization Fund enabled some stability to be imported into the exchanges, and the pound was linked to gold in the form of the dollar in the United States and to the franc in France, though most of the other countries also rapidly devalued their currencies. The increasingly more violent drive in Germany and Italy to the attainment of complete self-sufficiency and the continuously greater use of quotas, of import duties, of various forms of exchange control, and of barter agreements for the purpose of restricting imports and fostering exports produced a disorganization of international trade equal to that which existed in international finance. Since the end of the war disorderly marketing at home of a great variety of products was destroying the profitability of industry in Great Britain, and that country stood almost alone in having no means of exerting pressure on unethical competition from abroad.

In 1932 the second unbelievable happened and the United Kingdom abandoned free trade. The function of making recom-

mendations as to appropriate import duties was handed over to an independent Commission, though the responsibility for the actual adoption of duties lay with the Government and with Parliament. The duties imposed were in the main moderate, and at least in the case of the steel industry the temporary imposition of a penal duty forced the Continental Steel Cartel to agree to an equitable division of world markets. The principles of British trade were laid down at the Ottawa Conference in the same year—that British producers were to have the first claim on British markets according to their ability, that next a preference was to be given to Empire products, and that then the field was open to foreigners. At home a policy was adopted involving the revival of agriculture and the organization of its more important branches either as self-governing units or as subject to a degree of State control in return for some form of monetary assistance. The different manufacturing industries were also encouraged to organize themselves as units, either voluntarily or with the help of State compulsion, and were given to understand that the object of import duties on their products was to improve efficiency. The general purpose was to abolish destructive competition at home and restore reasonable profits, thus allowing, among other things, special prices to be made for export goods.

Against this background it was sought to widen international trade by bilateral agreements whereby concessions in imports were made in return for undertakings to buy larger quantities of British goods. Those arrangements did not always work well, as, for example, the agreements with Scandinavian countries to take more coal, which benefited Scotland and the North-East Coast but shifted Polish competition to the detriment of South Wales. They were, moreover, a poor substitute for the full system of multilateral trade, and there was little prospect of a really substantial loosening of the fetters that impeded commerce until Mr. Cordell Hull started his policy of commercial agreements and lower tariffs. Beginning with Latin America, he has now over a score of such treaties to his credit, by far the most important being the Anglo-American Trade Treaty, which, unlike bilateral agreements, embodied the "most favoured nation clause." Subsequent events, however, have prevented that treaty from producing its beneficent effects. It might have begun a new era in international trade and put an end to the diversion of world trade from its natural use as an instrument for the exchange of necessary goods to its misuse as a weapon for the attainment of political objectives.

The advent of the Nazi party to power in Germany has been followed, as everyone now knows, by an increased expenditure on

armaments everywhere, though our own country was not seriously involved till the latter part of the period under review. Such expenditure, besides impoverishing the country indulging in it, seriously alters the normal course of trade; if guns are to be preferred to butter, then the importation of materials for arms and ammunition must be encouraged and the importation of food must be relegated to a subordinate place. Besides this running sore, the Sino-Japanese "incident," if we must not call it a war, caused our exports to China and Japan to fall from 3·77 per cent. of our total exports in 1929 to 1·30 per cent. in 1938, and the imposition of "sanctions" on Italy on account of her aggression in Abyssinia practically put an end to our exports of coal to that country in 1936.

Against such a background of economic history it would seem almost impossible to look for clear improvement or definite trends, but I think that a review of the past ten years shows that, but for the impending and ever more definite threat of almost universal war, the situation was not void of hope and that our great exporting industries still retained a core of strength and were supported by new industries of vigour.

II. THE TRADE POSITION AS A WHOLE

The base year from which the Board of Trade frames its calculations of changes in the volume and value of trade has been shifted from 1924 to 1930, and again from 1930 to 1935. The periodical calculations published in *The Board of Trade Journal* give particulars from 1930 to 1936 on the 1930 basis and from 1936 to 1940 on the 1935 basis, a connecting link being provided by the calculation of 1930 on the 1935 basis. To provide a continuous series, I have adjusted all the relatives from 1930 onwards to the 1935 basis, and though the simple arithmetical process involved may have brought about some slight lapses from strict accuracy of measurement, there can be no substantial error, for the period under review does not extend more than five years on each side of the base.

British trade to-day, as it has been since statistics were collected, is characterized by the importation of foodstuffs and raw materials and the exportation of manufactured goods. Our only exported raw material of importance, but one of very great importance, is coal, and our exports of food, etc., are mainly of manufactured foods. Imports of "articles wholly or mainly manufactured," on the other hand, contain large quantities of semi-manufactured metals, of leather, and of some yarns and other commodities, all of which are used as raw materials for British industries. How little the relations of the great categories of our trade to one another have altered is

shown by the following table, giving the different classes as percentages of the total declared values :

TABLE I

British Imports and Exports

Classes	1929	1937	1938
Retained Imports.			
I. Food, Drink, and Tobacco	45·8	43·9	48·8
II. Raw Materials, etc.	25·7	29·2	25·3
III. Articles mainly Manufactured	27·5	26·3	25·1
IV, V. Animals and Parcel Post	1·0	0·6	0·8
Total	100·0	100·0	100·0
Exports of British Produce and Manufactures.			
I. Food, Drink, and Tobacco	7·5	7·4	7·6
II. Raw Materials, etc.	10·8	12·4	12·1
III. Articles mainly Manufactured	78·8	77·6	77·6
IV, V. Animals and Parcel Post	2·9	2·6	2·7
Total	100·0	100·0	100·0

In spite of political juggling with economic factors, the massive structure of world trade changes but slowly. The *Statistical Year-Book of the League of Nations* gives estimates of world's trade "in old U.S.A. gold dollars" which indicate a fall of 54·0 per cent. between 1929 and 1937, but the relative position of the countries was little altered. The United States was still the chief exporting country, though its share had fallen from 15·6 to 12·7 per cent., and the United Kingdom was still second with 10 per cent. instead of 10·7. The United Kingdom's position as an importer was greater, the percentage having risen from 15·2 to 17·3 per cent., while the share of the United States declined somewhat, from 12·2 to 11·0 per cent. Germany and Italy, with their policy of autarky, naturally lost ground in both exports and imports, while Canada and Japan gained as exporters. France, too, lost ground as an exporter (from 6·0 to 3·7 per cent.) and yielded the fourth place to Canada, with 4·3 per cent.

Export Trade and Home Trade

The following table gives some material for estimating the comparative growth of home and export trade. The category, "Materials" is taken as Class II of the Import figures, and does not include large quantities of semi-manufactured metals, yarns, chemicals, leather, etc., classed as "Articles Wholly or Mainly Manufactured," which are really important materials for British industries. The Production figures are from the Board of Trade Index of Production.

TABLE II

Production and Exports of Manufactures
(Index Numbers of Quantity)

Volume of	1930	1931	1932	1933	1934	1935	1936	1937	1938	1939 Qr. 1	1939 Qr. 2
Materials: Retained Imports ...	90·9	85·2	87·0	95·4	102·4	100·0	111·1	120·9	106·9	103·9	116·3
Manufactures: Exports ...	113·8	84·8	87·2	90·1	97·2	100·0	102·1	112·3	98·7	104·3	101·8
Production of Manufactures *including*	100·0	96·7	97·2	103·9	108·5	117·1	129·1	138·0	128·9	135·8	139·6
Building Materials and Building ...	100·0	—	—	—	133·4	147·0	157·1	153·2	145·4	126·7	137·6
Food, Drink, and Tobacco	100·0	103·7	97·6	99·2	101·8	106·8	114·5	119·9	121·4	119·5	129·3
Gas and Electricity ...	100·0	142·4	147·0	156·2	121·8	132·6	147·4	161·2	168·0	206·8	161·6

It should be noted that in 1935 the Index of Production was revised, the weights of the several items being recalculated on the basis of their importance in the Census of Production for 1930; a new item—construction of new buildings—was then introduced. The production indices shown in the above table for 1931, 1932, and 1933 are calculated on the base 1924 = 100; the indices for 1934 were calculated on both bases, those on the 1930 bases are given in the table and the corresponding indices on the 1924 basis were: Manufactures 110·7; Food, Drink, and Tobacco, 99·9; and Gas and Electricity, 122·4—all relatively small differences.

The table, though imperfect, would seem to show that retained imports of raw materials have increased more rapidly than exports of British manufactures, and that British manufacturing production has also grown more rapidly than the export trade. Exports of manufactures have grown steadily at a moderate annual rate of increase since 1931, except for a setback in 1938, from which, however, there was a recovery in the first half of 1939. The prospects for our export trade, though not extravagant, would have been satisfactory had war not come. The details of its growth will be investigated later. The development of manufactures for consumption at home is also a matter for congratulation, for it can only have been made possible by an increase in the purchasing capacity of the people, either individually or through the State as a whole. How far the increase shown in the later years of the period is due to production of armaments and other warlike equipment cannot be precisely determined, but in so far as it existed it must be set down as an economic loss though productive, one hopes, of security. The table also shows that a great part of the improvement in the decade was due to the clearing of slums and the rehousing of the people, purposes which brought with them an increased output not only of all sorts of materials used in building, but also of all kinds of household equipment. As schemes approached completion, activity in building began to slacken in 1937, and since the outbreak of the war has been reduced to a low figure. This stoppage will in its

TABLE III

Volume and Value of Imports and Exports

Particulars.	1930	1931	1932	1933	1934	1935	1936	1937	1938	1939 Qr. 1	1939 Qr. 2
RETAINED IMPORTS											
I. Food, Drink, and Tobacco:											
Volume ...	99·6	107·7	103·7	101·7	102·2	100	102·8	103·1	106·3	105·1	104·0
Value ...	132·2	107·5	100·6	94·0	95·3	342·6*	105·4	118·3	115·0	108·5	107·8
II. Raw Materials and Articles Mainly Unmanufactured:											
Volume ...	90·9	85·2	87·0	95·4	102·4	100·0	111·1	120·9	106·9	103·9	116·3
Value ...	127·6	94·6	88·6	88·9	97·2	183·1*	105·8	125·7	111·0	104·6	106·7
III. Articles Wholly or Mainly Manufactured:											
Volume ...	133·9	136·0	87·3	90·0	104·3	100·0	111·6	127·0	115·5	117·8	130·6
Value ...	123·4	104·8	97·5	90·6	89·8	171·2*	103·3	115·2	109·0	108·8	109·3
Total:											
Volume ...	106·3	109·3	96·0	97·5	103·0	100·0	107·2	113·8	108·7	108·0	113·8
Value ...	128·4	104·0	96·7	91·5	94·1	700·7*	105·0	119·5	112·7	107·9	108·1
BRITISH EXPORTS											
I. Food, Drink, and Tobacco:											
Volume ...	118·7	98·6	94·8	86·1	97·3	100·0	110·6	117·2	111·1	107·7	109·5
Value ...	126·3	111·9	106·0	102·3	99·1	31·6*	101·8	104·7	102·4	102·9	104·1
II. Raw Materials and Articles Mainly Unmanufactured:											
Volume ...	110·2	90·4	85·3	92·9	93·9	100·0	92·4	101·2	89·9	89·3	93·9
Value ...	109·6	98·6	96·9	93·7	97·3	52·8*	105·2	120·8	119·9	116·5	118·4
III. Articles Wholly or Mainly Manufactured:											
Volume ...	113·8	84·8	87·2	90·1	97·2	100·0	102·1	112·3	98·7	104·3	101·8
Value ...	117·8	104·8	96·4	95·1	95·4	328·9*	101·5	109·6	112·6	109·1	108·1
Total:											
Volume ...	115·0	88·0	88·3	90·5	97·1	100·0	101·5	111·1	98·6	102·5	101·4
Value ...	116·6	104·2	97·1	95·5	95·8	425·8*	101·9	110·2	112·1	109·1	108·6
EXPORTS OF IMPORTED MERCHANDISE											
Volume ...	123·0	117·3	105·7	96·1	89·7	100·0	99·1	103·2	98·0	92·8	92·4
Value ...	127·7	98·5	87·3	92·3	103·3	55·3*	110·2	131·7	113·6	111·4	108·2

* £ million ; value figures for other years are percentages of 1935.

turn, as during the last war, cause a shortage of houses, which will have to be made good when peace comes with a consequent increase of imports of timber, iron ore, and other materials.

Volume and Value of Imports and Exports

It will be convenient at this stage to introduce a table showing the changes in the volume and value of the three main classes of Imports and Exports; the totals include not only those classes, but also "Animals, not for Food," and "Parcel Post" (non-dutiable in the case of Imports). The changes in the re-export trade as a whole are also shown. In the "Value" lines the figures under "1935" are the declared values in £ million; for the other years the figures are the declared values as percentages of 1935.

III. THE TRADE IN FOOD, DRINK, AND TOBACCO

The following table deals with retained imports of Class I, Food, Drink, and Tobacco, and shows the declared values of the retained imports for 1935, and then indices, on that year as a base, registering the changes in volume of the several groups for 1930 and for 1933 to 1939 (first half). The corresponding figures for 1931 and 1932 are not included, as the tables in the *Board of Trade Journal* for those years combine the three groups "Fresh Fruit and Vegetables," "Beverages and Cocoa Preparations," and "Other Food."

TABLE IV

Retained Imports of Food, Drink, and Tobacco

(Changes in volume: 1935 = 100)

Commodities	1930	1933	1934	1936	1937	1938	1939 Jan.–June	Declared Value 1935 (£ mill.)
Grain and Flour	95·2	105·6	104·0	104·8	100·5	99·0	103·6	56·0
Feeding Stuffs for Animals	56·7	76·2	97·8	108·2	112·1	116·9	126·9	7·7
Animals, Living, *for Food*	167·9	114·0	95·2	117·5	102·7	102·4	93·5	5·6
Meat	108·7	110·0	105·9	99·1	104·7	104·5	109·5	76·9
Dairy Produce	90·4	96·7	102·9	104·9	103·8	107·0	106·9	57·6
Fresh Fruit and Vegetables	94·9	101·2	95·9	95·8	98·6	98·8	108·7	37·9
Beverages and Cocoa Preparations	107·5	88·4	103·1	105·5	102·9	107·1	89·2	36·5
Other Food	99·6	100·4	104·4	102·5	104·8	110·4	110·7	47·7
Tobacco	95·1	83·9	94·1	108·6	110·7	137·6	77·2	16·7
Total	99·6	101·7	102·2	102·8	103·1	106·3	104·6	342·6

Imports of cereal by-products and oil-cake for cattle food had long been increasing with the development of meat and milk production, and grew rapidly during the decade. Despite heavy taxation retained imports of unmanufactured tobacco also increased from an average of 192 million lb. in 1930–32 to an average of 285 million lb. in 1936–38. The year 1938 "was a record for imports of

unmanufactured tobacco, the total reaching 345·8 million lb., an increase of 29 per cent. over 1937 and 28 per cent. more than the previous highest total (in 1936) . . . while entries for home consumption (190·0 million lb.) exceeded those in 1937 by 4 per cent. and constituted a new high record" (*Board of Trade Journal*, January 19th, 1939).

As regards imports of foods and beverages for human consumption, their growth in volume in 1930–38 was about 3·5 per cent., an increase nearly equal to that of the population of the United Kingdom. This apparent stagnation is due to the Government policy of the stimulation of home agriculture and the corresponding regulation of the imports of many foods. The imports of Living Animals for Food are almost entirely imports of cattle, sheep, and pigs from the Irish Free State. The following table gives triennial averages of the quantities of retained imports of certain important foods and beverages, which in recent years accounted for about three-quarters of the total value of this Class, excluding Feeding-stuffs for Animals and Tobacco. Particulars for the first half of 1939 are also included.

TABLE V

Retained Imports of Certain Foods and Beverages

Commodity	Unit	Average 1930–32	Average 1933–35	Average 1936–38	Jan.–June 1939
Wheat	Mill. cwt.	109·0	104·8	98·5	56·2
Wheat Meal and Flour ...	,,	10·2	9·0	8·2	4·1
Maize	,,	44·4	55·5	65·4	25·9
Beef, chilled and frozen ...	,,	11·4	11·4	12·0	6·2
Mutton and Lamb, frozen	,,	6·8	6·6	6·6	3·6
Bacon and Hams	,,	11·1	8·4	7·4	4·0
Butter	,,	7·5	9·3	9·5	4·8
Cheese	,,	3·0	1·9	2·8	1·4
Eggs in shell	Th. Gt. Hunds.	24·2	19·0	25·7	14·6
Apples	Mill. cwt.	7·0	6·6	5·9	3·0
Bananas	Mill. bunches	15·6	17·3	21·1	10·1
Oranges	Mill. cwt.	9·5	10·4	10·5	7·4
Potatoes	,,	12·7	3·6	4·6	·2·8
Tomatoes	,,	2·8	2·9	2·8	1·5
Tea	Mill. lb.	462·0	423·4	430·0	144·8
Wine	Mill. galls.	13·2	14·6	16·6	7·2
Fruit, canned	Mill. cwt.	3·0	3·4	3·8	2·5
Lard	,,	2·5	2·4	1·5	0·9
Sugar, unrefined	,,	39·9	38·9	45·4	23·2
Fish, fresh or salted ...	,,	3·1	2·2	2·1	1·4
Fish, canned	,,	1·2	1·3	1·5	0·7

These figures confirm the general remarks made above. The passing of the Wheat Act in 1932, with its limited bounty intended to make good to farmers the difference between the ascertained home price and a standard sum per cwt., was followed at once by an

increase of 30 per cent. in the wheat acreage of the United Kingdom and by some variation afterwards. Comparing 1932 and 1937, and reckoning imported flour in terms of grain, there appears to have been in the latter year a reduction of about 5 million cwt. in the quantity of home-grown and imported wheat available for consumption in the United Kingdom. This may have been partly offset by a reduction in stocks of grain held in the country, but, as population increased in the period by something short of a million persons, we are entitled to conclude that there has been an improvement in the incomes of the people, leading to the substitution of other foods for bread, which is always the staple nutriment of the poorest classes. The recovery in wheat imports in the first half of 1939 may be related to the Government's policy of accumulating stocks as a provision in case war should come.

Imports of beef and mutton were stabilized, following the adoption of the Ottawa policy, as the table shows, and a stimulus was thus offered to the home producers and exporters from the Dominions. The restriction of imports from South America was more than made good by increased imports from Empire countries. Taking gross imports for the two years 1932 and 1938 we get for gross imports:

TABLE VI

Gross Imports of Beef and Mutton

(In thousand cwt.)

Kinds of Meat	1932			1938		
	British Countries	Foreign Countries	Total	British Countries	Foreign Countries	Total
Beef, chilled and frozen	1,569	9,792	11,361	3,366	8,899	12,265
Mutton and Lamb, frozen	5,060	1,867	6,927	5,585	1,305	6,890
Total ...	6,629	11,659	18,288	8,951	10,204	19,155

Disorderly marketing of home produce and glutted markets led to the collapse of prices paid to producers and importers (except where the stocks were in strong hands, as is the case with South American beef), and the Live Stock Act of 1937 was passed granting an annual subsidy for producers of beef in the United Kingdom. Commercially, the year 1932 is remarkable for the first attack on the South American monopoly of chilled beef in the form of experimental shipments from Australia under a new process; in 1938 533,000 cwt. of chilled beef were imported from Australia, 361,000 cwt. from New Zealand and 110,000 cwt. from other British

countries. Over the decade the substitution of chilled beef for frozen beef and of lamb for mutton went on, indicating both an improvement in the purchasing power of the people and also a greater appreciation of the less gross forms of meat.

One may be pardoned for declining the attempt to unravel the tangled efforts to assist the production of pigs and bacon in this country and their relation to our import policy. But it is pertinent to note that the Danish export trade in bacon to the United Kingdom had grown steadily owing to the scientific technique of the producers till it reached 6,118,000 cwt. in 1930, but in the following year Poland more than doubled her exports to Britain and Holland tried to develop a trade in bacon instead of fresh pork, which had been stopped, so Denmark apparently set out to kill competition by bringing up her exports to this country in 1932 to about a quarter more than in 1930. This madness went far towards ruining all exporters as well as British producers, and in 1933 the Board of Trade induced the leading exporting countries to agree to voluntary reductions of their exports, and a scheme for the encouragement of home production and the limitation of imports to the quantity which British producers could not supply was put forward. In this scheme the average annual consumption of 1926–30, 10,670,000 cwt., was taken as the normal requirement of bacon and hams at the then level of population. We can see the effects of this stabilization in the imports of bacon from Denmark, which were 3,826,000 cwt. in 1935, 3,373,000 cwt. in 1936, 3,429,000 cwt. in 1937, and 3,389,000 cwt. in 1938.

The years 1930 and 1931 were bad years for the British crop of potatoes, and an increase in imports was therefore to be expected over the three years 1930–32, but in fact the increase in 1931 was 188 per cent. and that in 1932 was 169 per cent. over 1930, and in 1932 the home crop returned to about its normal size. The enormously increased imports, especially from Germany, Belgium, and the Netherlands, collapsed in 1933. A scheme was then introduced based on the calculation that consumption was stable at about 4,000,000 tons a year, and importation was limited to the period when the British crop was not on the market. There was no real reduction in the consumption of potatoes over the decade as the import figures would seem to imply, and the abnormal position of potatoes in 1930–32 is one of the reasons why the whole group, " Fresh Fruit and Vegetables," does not show that increase in volume which one might have expected from the vigorous campaign to eat more fruit and vegetables. But variations in imports are in part governed by the size of home crops, and in part by weather conditions in the producing countries. The war in Spain, sanctions against Italy,

and disturbances in Palestine all in varying degrees affected the export trade of those countries in their standard fruits. Attention should also be drawn to the increasing importance of South Africa as an exporter of fruit, especially of the finest quality.

Of the other commodities in the table it is important to note the increase in the imports of raw sugar, for it shows that the fostering of the cultivation of sugar-beet in the United Kingdom has not reduced the import trade, and it further follows that British sugar industries are prosperous. In 1936 the British production of white sugar was limited to 560,000 tons of white sugar and the output of white sugar (equivalent) in the 1936–37 campaign was 522,000 tons, but in 1937–38 only 377,000 tons.

Exports of United Kingdom produce of Food, Drink, and Tobacco at declared values were 8·3 per cent. of total exports in 1930, 7·4 per cent. in 1935, and 7·6 per cent. in 1938. In volume, taking 1935 as 100, the figure for 1930 was 118·7, that for 1937 was 117·2, that for 1938 was 111·1, and that for the first half of 1939 was 108·6. The two groups, "Beverages and Cocoa Preparations" and "Other Food" account for about two-thirds of those exports. The volume of the first group was 101·5 in 1930, 100·0 in 1935, 145·8 in 1937, 137·0 in 1938, and 136·0 in Jan.–June 1939; for the second group it was 110·1 in 1930, 100·0 in 1935, 99·4 in 1937, 93·5 in 1938, and 85·7 in the first half of 1939.

The following table shows the movement of some leading exports for three triennial periods and for the first half of 1939.

TABLE VII

British Exports of Food, Drink, and Tobacco

Commodity	Unit	Average 1930–32	Average 1933–35	Average 1936–38	Jan.–June 193
Fish, fresh	Mill. cwt.	1·3	0·9	1·2	0·5
Herrings, cured or salted	,,	4·0	2·8	2·6	0·5
Beer and Ale	Th. Stand. bbls.	287	221	316	143
Spirits	Mill. pf. galls.	5·8	5·8	9·1	4·5
Sugar, refined	Mill. cwt.	4·6	6·8	6·9	4·0
Tobacco, manufactured ...	Mill. lb.	23·8	25·9	32·8	17·3

The chief comment to be made is that the heavy decline in the exports of fish continues a movement caused by the impoverished state of the Continent after the War of 1914–18 which brought down exports of all kinds of cured, salted, and canned fish (herrings and other) from 10,999 thousand cwt. in 1913 to 7,174 thousand cwt. in 1930; in 1938 the aggregate was 4,377 thousand cwt. British exports of Food, Drink, and Tobacco in 1938 were valued at £35,911,000; 48·3 per cent. went to British countries. Out of

tobacco valued at £4,915,000, exports to British countries accounted for 71·2 per cent. British countries also took 77·8 per cent. of the exports of beer and 33·1 per cent. of the exports of spirits. The United States in 1938 took 43·2 per cent. of British exports of spirits by quantity and 49·3 per cent. by value; in 1933 the quantity was 96 thousand gallons, valued at £146,000 and the surrender of prohibition raised the quantity to 4,375,000 gallons in 1937 and to 3,934,000 gallons in 1938, the values being respectively £6,562,000 and £5,598,000 in 1938.

IV. British Exports of Coal and other Raw Materials

This class is, of course, dominated by coal, which in 1935 accounted for £31,559,000 out of the total of £52,836,000 for the class; the next largest groups were "G. Wool, Raw and Waste, and Woollen Rags," £8,171,000, and "J. Seeds and Nuts for Oil, Oils, Fats, Resins, and Gums," £3,200,000. With 1935 as a basis, as before, volumes moved as follows :—

Table VIII

Volume of Exports of Coal and Raw Materials

(1935 = 100)

Groups	1930	1931	1932	1933	1934	1936	1937	1938	1939 Jan.–June
A. Coal	137·4	108·0	100·3	100·4	101·7	88·9	101·2	90·6	92·8
G. Wool, etc.	47·8	47·1	52·5	80·7	96·2	85·2	78·7	68·3	63·1
J. Oils, Fats, etc.	84·3	78·3	62·6	56·6	60·1	95·0	97·1	96·9	84·9
Class II Total ...	110·2	90·4	85·3	92·9	93·9	92·4	101·2	89·9	91·6

Germany and the United States were the principal importers of British wool, wool waste, and noils, and in 1935 took together rather more than half of the total value of the Group—Germany 33·5 per cent., United States 17·6 per cent. Exports to Germany were at first of moderate dimensions and after 1935 they fell away rapidly. Group J. consists of unrefined fats, oils, etc. Canada, the United

Table IX

Production and Exports of Coal

(In million tons)

Particulars	1930	1931	1932	1933	1934	1935	1936	1937	1938	1939 Jan.–June
Production	243·9	219·5	208·7	207·1	220·7	232·2	228·0	241·2	228·0	118·3
Exports	54·9	42·7	38·9	39·1	39·7	38·7	34·5	40·3	35·9	18·5

States, and Germany have been our chief customers. Exports declined heavily from 1930 to 1933, there was a moderate increase in value in 1934, then a very rapid rise in volume from 1935 to 1937, with a falling-off in 1938. Exports to Canada were £166,000 in 1934, £1,156,000 in 1937 and £968,000 in 1938.

In 1913 production amounted to* 287,430,000 tons and exports to 73,400,000 tons. It will be noted that if the export figures are reduced to percentages of 1935 a range of relatives will be obtained differing from the volume figures given in the preceding table; that is because "volume" figures take into account changes in the relative proportions of the different kinds of coal exported which are included in the gross total. Even in the relatively good year 1929 production (257·9 million tons) was 10·3 per cent. less than in 1913 and exports (60·3 million tons) were down by 17·8 per cent. It is not necessary to repeat here the oft-told tale of the reasons for this decline in British coal trade. The further fall to the low point of 1932 reflects the collapse in the world's coal-using industries brought about by the great depression, and then for 1932–35 we have a period of stagnation during which sundry importing countries, such as France, severely regulated their imports of coal by increasingly strict quotas. The recession of 1936 was mainly due to the stoppage of imports of British coal by Italy as a retort to the "sanctions" policy of the British Government; one section of our community thus suffered severely without any compensation. In 1937 there was a substantial improvement of about 17 per cent., France contributing an increase of 24 per cent. on the previous year's quantities, but the following year showed a reduction of about 11 per cent. on 1937 although Italy raised her imports to about 53·5 per cent. of the average for the three pre-sanction years; France's imports of British coal were less by 30·6 per cent.

In 1930 the Coal Mines Act was passed, which enforced a regional organization on the industry and enabled the various boards to control output and prices, prevent internecine competition, and foster export trade, but throughout it has been a struggle against unfavourable economic conditions. Trade with the Baltic had particularly suffered from the competition of Polish coal, and when by their trade agreements with the Scandinavian countries the Government helped to recover those northern markets, the competition shifted to markets previously held by South Wales.

Table X on p. 465 shows the great changes which have taken place over the last thirty years.

Exports to British Countries (excluding Eire) show some curious changes. In 1909–13 exports to Malta averaged 492,000 tons but in 1937 were only 71,000 tons, owing probably to a change in naval

TABLE X

Exports of Coal to Leading Markets

Countries	Average 1909–13	1930	1937	Average 1909–13	1930	1937
	In million tons			Percentages		
France	10·6	13·0	8·9	16·2	24·8	23·6
Italy	9·2	7·2	2·2	14·0	13·7	5·8
Denmark, Sweden, and Norway	9·0	4·9	8·2	13·7	9·4	21·7
Germany	9·0	4·9	3·3	13·7	9·4	8·7
Russia	4·0	0·5 *	2·0 *	6·1	1·0	5·3
Argentina	3·1	2·7	2·2	4·7	5·2	5·8
Egypt	2·9	1·8	1·4	4·4	3·4	3·7
Spain	2·2	1·7	0·8	3·4	3·2	2·1
Netherlands	2·2	2·9	1·1	3·4	5·5	2·9
Belgium	1·7	3·4	0·9	2·6	6·5	2·4
Brazil	1·6	1·2	0·6	2·5	2·3	1·6
Other Foreign Countries	7·9	5·8	3·6	12·1	11·0	9·5
British Countries ...	2·1	2·4 †	2·6 †	3·2	4·6	6·9
Total	65·5	52·4	37·8	100·0	100·0	100·0

* Succession States; exports to Finland in 1937 were 1·4 million tons.
† Excluding exports to Eire, 2·6 million tons in 1937.

requirements from coal to oil. But exports to Gibraltar went up from 307,000 tons to 461,000 tons, being presumably for commercial bunkers. A great new trade with Canada, almost entirely in anthracite coal, has grown up since the end of the last war, exports in 1937 being 1,062,000 tons against 48,000 tons average in 1909–13. The table also brings out very clearly the great importance of the Baltic markets to us and the continuing importance of the French market.

One final comparison may be made between 1937 and 1930:

TABLE XI

Exports of Coal by Kinds

(In million tons)

Kinds of Coal	1930	1937	Decrease per cent.
Anthracite	4·1	3·8	7·3
Steam	36·5	27·4	24·9
Gas	6·6	4·0	39·4
Household	2·0	1·6	20·0
Other Sorts	5·7	3·5	38·6
Total	54·9	40·3	26·6
Bunker Coal	15·6	11·7	25·0

In 1937 Canada took 26·2 per cent. of our exports of anthracite and France 32·6 per cent.; Eire took over 80 per cent. of the exports of household coal. The exports of steam, gas, and other (or undescribed) coal were distributed mainly among the European industrial countries. About 22·7 per cent. of the exported coal in 1937 was shipped unscreened (through and through) against 26·5 per cent. in 1930, so that some advance has been made in preparing coal for export.

V. TRADE IN MANUFACTURES

The following short table summarizes the broad movements of change in imports of raw materials retained in the United Kingdom to be worked up either for home consumption or for export, in retained imports of goods described as "wholly or mainly unmanufactured" (many of which, as already explained, are semi-manufactured materials for British manufactures), and in exports of British goods "wholly or mainly manufactured" (some of which are semi-manufactured materials for overseas manufactures).

TABLE XII

Raw Materials and Manufactured Goods

(Changes in Volume: 1935 = 100)

Particulars	1930	1931	1932	1933	1934	1936	1937	1938	1939 Jan.–June
Raw Materials: Retained Imports	90·9	85·2	87·0	95·4	102·4	111·1	120·9	106·9	109·8
Manufactures:									
Retained Imports... ...	133·9	136·0	87·3	90·0	104·3	111·6	127·0	115·5	124·2
British Exports	113·8	84·8	87·2	90·1	97·2	102·1	112·3	98·7	103·1

Retained imports of articles wholly or mainly manufactured decreased in volume much more rapidly than British exports of the same class between 1930 and 1932, but they then recovered more quickly than British exports did. Their fall in 1938 by about 9 per cent., against about 12 per cent. for British exports, and their recovery in the first six months of 1939 were also more rapid. It has already been suggested that the condition of the home market accounts, in part at least, for this difference.

The five groups "Iron and Steel and Manufactures thereof," "Machinery," "Cotton Yarns and Manufactures," "Woollen and Worsted Yarns and Manufactures," and "Vehicles, including Locomotives, Ships, and Aircraft," accounted for over 62 per cent. of the total declared value of exports of British manufactures in 1930 and for about 60 per cent. in 1937. Attention may, therefore, be given to them first.

TABLE XIII

Trade in Iron and Steel and Manufactures thereof

Particulars	1930	1931	1932	1933	1934	1936	1937	1938	1939 Jan.–June	Declared Value 1935 (£mill.)
			(Changes in volume : 1935 = 100)							
Iron Ore and Scrap : Retained Imports...	82·3	41·5	37·0	51·8	92·8	155·3	168·9	124·4	111·0	5·0
Manufactures :										
Retained Imports...	247·2	234·3	122·4	84·0	117·9	123·8	157·1	105·7	115·5	8·6
British Exports ...	127·6	80·8	78·9	82·4	94·7	97·6	113·5	87·9	82·4	36·0
					(1950 = 100)					
Production of Iron and Steel in United Kingdom, Index of Production... ...	88·8*	65·9*	66·2*	82·2*	115·7†	150·1	166·6	131·5	‡	—

* On basis 1924 = 100; on that basis 1934 was 115·6.
† 1935 = 125·6.
‡ First quarter 149·1; second quarter 176·5.

Recovery in the iron and steel industry did not manifest itself conspicuously till 1933, after the Import Duties Act had been passed in 1932 and the industry had begun its internal regulation. The Import Duties Act forced the Continental Steel Cartel to agree to a fair division of neutral markets. The Index of Production, when compared with the index of export trade, clearly indicates that the relative importance of the home market was increasing, especially in the later years under the added stimulus of armaments.

The group contains a great mass of commodities of different degrees of finish—from pig iron to hollow-ware—and the following analysis of imports and exports into three great sub-groups will be found instructive. The first comprises pig iron, ferro-alloys and crude steel; the second includes bars and sections of all kinds, girders, hoops, strip, and uncoated plates; in the third are found galvanized sheets, tinplates, pipes and tubes, railway material, wire and its products, nails, screws, bolts, stoves, baths, hollow-ware, etc.

Broadly speaking, we import crude iron and steel and semi-manufactured goods as materials for our own industries, while our exports of those categories are relatively small, and imports of more finished products are comparatively unimportant in view of our exports. The effects of the Import Duties Act, 1932, and the arrangements with the Continental Steel Cartel on our imports are quite manifest, and to meet our inability to cover the demands for steel for armaments that Act was relaxed and imports of pig iron were raised to 638,000 tons in 1937, to fall to 395,000 tons in 1938, as high prices restricted home demand. In Great Britain the output of pig-iron was 6,192,000 tons in 1930 and fell to 3,574,000 tons in

TABLE XIV

Retained Imports and British Exports of Iron and Steel Manufactures

(In million tons)

Commodities	Average 1930–32	Average 1933–35	Average 1936–38
Retained Imports :			
Crude Iron and Steel	0·8	0·4	0·9
Rolling-mill Products	1·4	0·7	0·5
More Finished Products	0·2	0·1	0·2
Total	2·4	1·1	1·6
British Exports :			
Crude Iron and Steel	0·2	0·1	0·1
Rolling-mill Products	0·5	0·6	0·6
More Finished Products	1·6	1·5	1·5
Total	2·3	2·2	2·2

1932, but rose to 8,496,000 tons in 1937, receding to 6,768,000 tons in the following year, and again steadily improving to reach 716,000 tons in June, 1939, with a total of 3,638,000 tons for the first half of that year. The production of crude steel was 7,326,000 tons in 1930 and 5,261,000 tons in 1932, but then rose to 12,984,000 tons in 1937 with a recession to 10,392,000 tons in 1938 and a recovery to 6,406,000 in the first six months of 1939. Armaments, it is true, were responsible for a large part of this growth, but more was due, at least in the middle years of the period, to the development of home industries—for example the motor and canning trades. Two of our leading exports—galvanized sheets and tinned plates—had a very chequered course, and neither reached again the export figures of 1930 (432,000 tons for galvanized sheets and 508,000 tons for tinned plates). Exports of galvanized sheets averaged 332,000 tons in 1930–32, 265,000 tons in 1933–35, and 199,000 tons in 1936–38; the annual figures for tinned plates were 457,000 tons in 1930–32, 380,000 tons in 1933–35, and 370,000 tons in 1936–38; galvanized sheets showed a moderate rise and tinned plates a further fall in the first half of 1939 compared with the corresponding period of 1938. Decline in exports to India accounted for a large part of the falling-off in galvanized sheets, but the losses in exports of tinned plate were widely spread. Wrought tubes showed a progressive export trade from 171,000 tons average in 1930–32 to 231,000 tons in 1936–38.

Trade in Machinery and Vehicles

The machinery and vehicles (including locomotives, ships, and aircraft) trades may logically be taken next, for they are large users of steel and contributed respectively 15·8 and 12·2 per cent. of our

exports of manufactured goods in 1938. The following table gives particulars of volume of imports and exports.

TABLE XV

Trade in Machinery and Vehicles

(Changes in volume : 1935 = 100)

Particulars	1930	1931	1932	1933	1934	1936	1937	1938	1939 Jan.–June	Declared Value 1935 (£mill.)
Retained Imports :										
Machinery	162·9	146·1	82·3	70·7	99·5	148·7	194·1	163·7	176·2	12·4
Vehicles	120·7	68·2	54·2	38·1	75·3	101·3	163·6	97·5	109·5	4·4
British Exports :										
Machinery	126·6	86·6	81·7	73·4	89·5	100·9	115·7	123·4	115·9	49·8
Vehicles	161·7	98·2	82·3	96·8	107·0	115·3	134·6	140·5	166·0	47·7

Quantities of machinery are reckoned by tons in the Trade Accounts, and that is not a very useful means of comparison. The slump caught both the import and export trade very severely, and recovery did not come till 1934. By declared values retained imports were £16,529,000 in 1930, £12,373,000 in 1935, £23,274,000 in 1937, and £20,891,000 in 1938, while British exports were £48,274,000 in 1930, £39,352,000 in 1935, £49,741,000 in 1937 and £57,907,000 in 1938. In 1930 imports consisted chiefly of agricultural machinery, machine tools, prime movers, printing machinery, and textile machinery. Imports of agricultural machinery first declined and then recovered to about the level of the early part of the period, and those of tractors, which were small in 1930, have now very largely increased. There has also been a very great increase in imports of machine tools, especially from the United States, although we have also obtained substantial contributions from Germany. The other classes named above have shown great increases after the slump (especially in marine internal combustion engines), except in the case of textile machinery; imports of typewriters and sewing machines are also far from having regained their former eminence.

Our exports of machinery cover an immense variety of goods, and not only, as already said, is comparison by weights deceptive, but the summarizing of details into broad categories conceals very varied movements among the included items. However, here are particulars of the ten leading groups of our exports at their declared values in 1930, 1935, and 1938, set against the declared values of the whole group, those values on the basis of 1935 values, and the movement of average values (*i.e.*, the price movement) of the whole group. It must not, however, be supposed that the declared values of the listed items would, if reduced to the basis of 1935 show the same movement as the whole group.

TABLE XVI

British Exports of Machinery

Particulars	1930	1935	1938
Index of Average Values (1935 = 100) ...	96·9	100·0	119·3
		(£ million)	
Value of Group on 1935 basis	49·8	39·4	48·6
Declared Value of Whole Group	48·3	39·4	57·9
Agricultural Machinery and Tractors ...	1·6	1·1	1·3
Boilers and Boilerhouse Plant	3·3	2·2	3·9
Cranes, Hoists, etc.	2·3	1·2	2·3
Electrical Machinery	6·3	4·1	7·9
Machine Tools	1·9	2·2	4·5
Mining Machinery	—	2·3	2·3
Prime Movers	4·1	3·6	4·2
Pumps	1·2	0·9	1·6
Sewing Machines and parts	2·2	1·4	1·3
Textile Machinery	8·6	6·9	8·4

The most briskly developing items are boilers, etc. (especially water-tube boilers), electrical machinery, and machine tools. Prime movers are dominated as to 60 or 70 per cent. of the total value by the oil internal combustion engine, and it should be noted that engines for aeroplanes, motor-cars, and cycles are excluded here.

The Vehicles Group may be divided into three main sections—railway vehicles, motor-cars, and ships—with cycles as an old and aeroplanes as a new industry, which are also of importance. A table similar to that given for Machinery follows :—

TABLE XVII

British Exports of Vehicles

Particulars	1930	1935	1938
Index of Average Values (1935 = 100) ...	108·9	100·0	107·7
		(£ million.)	
Value of Group on 1935 basis	47·7	29·5	41·4
Declared Value of Whole Group	51·9	29·5	44·6
Aircraft and parts	2·0	2·7	5·4
Railway Vehicles and parts...	10·0 *	3·9	7·5
Motor Cars and parts †	8·9	12·2	15·1
Motor Cycles †	2·9	1·0	1·1
Other Cycles †	2·2	3·0	3·3
Rubber Tubes and Tyres	4·5	3·2	3·3
Ships : Steam Vessels	9·8	1·5	4·1
Motor Vessels	9·3	0·7	2·0
War Vessels	0·7	0·9	2·0

* Add £1 million for wheels, tyres and axles, then included with "Iron and Steel."

† Except tyres separately consigned.

Exports of railway vehicles are obviously dependent on capital investment in the receiving countries, and that has notoriously been lacking during the last ten years. The collapse of our exports of ships is also the reflection of the economic depression, the growth of "self-sufficiency" or economic nationalism, the superfluity of shipping tonnage, and the poverty of shipowners. Competition in the Vehicles Group is practically limited to motor-cars, retained imports of which in 1930 were valued at £5·1 million—five-sixths of the whole group; in 1935 motor-cars and vehicles contributed £3·8 million out of £4·4 million and in 1938 £3·0 million out of £4·1 million. Some perturbation was expressed in some newspapers at the time on account of the large increase of cheap cars from Germany, but they only amounted to 8,582 cars in the two years 1937–38, valued at £736,000, out of a total of 28,276 private cars imported and valued at £3,909,000, and part was re-exported. Since it first received protection under the "McKenna Duties," the British motor-car industry has established itself in an inexpugnable position at home and has won a strong position in Empire markets. Taking new private cars and commercial vehicles and chassis exported in 1938 to the value of £11,158,000, exports to British countries accounted for £8,305,000, or close on three-quarters of the whole. Comparisons are not always easy, since, for example, Australia in 1938 took 3,070 complete private cars and 4,053 chassis for its body-building industry, while New Zealand in the same year took 13,316 cars and only 1,254 chassis.

Other Metal Trades

Non-ferrous ores and semi-manufactured metals are not only raw materials for their own industries, but also for machinery, electrical goods, watchmaking, etc., and the cutlery, hardware and other trades are dependent on steel.

Semi-manufactured non-ferrous metals are imported instead of ores for economy of transport, but copper ore from Canada and tin ore from Nigeria and Bolivia are imported, smelted and in part exported in crude forms. British exports of raw materials are chiefly wastes and residues. Generally speaking, our non-ferrous exports are in more finished products than our imports. The effect of armament demands in recent years is obvious, but the supply to Germany has been exaggerated in the popular mind; *e.g.*, about an eighth of the crude nickel exported went to that country in 1936–38. Except in scientific instruments, cinematograph films, clocks and watches, and (to some extent) needles, our exports in the Group of Cutlery, Hardware, Tools, etc., far exceed our imports. In the Group of Electrical Goods and Apparatus we are beyond competition in

TABLE XVIII

Other Metal Trades

(Changes in volume: 1935 = 100)

Particulars	1930	1931	1932	1933	1934	1936	1937	1938	1939 Jan.–June	Declared Value 1935 (£mill.)
Retained Imports:										
Non-Ferrous Ores, etc.	134·5	110·7	87·3	78·8	99·1	112·0	135·8	126·8	120·8	11·2
Non-Ferrous Manufactures	80·3	81·4	61·3	65·8	95·8	99·6	119·6	115·7	120·0	24·0
Cutlery, Hardware, etc.	112·5	119·8	65·7	71·4	95·2	112·8	128·6	114·8	105·6	5·5
Electrical Goods ...	205·3	198·3	86·4	84·6	104·9	114·3	122·0	94·2	90·0	3·0
British Exports:										
Non-Ferrous Ores, etc.	27·5	27·9	30·7	63·2	64·1	113·3	122·6	122·0	128·3	1·8
Non-Ferrous Manufactures	84·1	56·3	56·9	86·4	93·0	83·7	95·9	84·7	101·5	14·1
Cutlery, Hardware, etc.	68·8	52·4	61·4	70·2	90·3	98·1	116·0	103·8	105·0	8·1
Electrical Goods ...	119·7	76·4	61·8	72·1	86·7	102·4	119·0	127·7	123·4	9·5

electric wires and cables and telegraphic and telephonic apparatus, which made up more than half our exports in 1937; in electric lighting bulbs, etc., we have a superiority, but in several other classes of appliances there is keen competition.

The Textile Industries

Table XIX on p. 473 summarizes the course of trade in textiles. The immediate effects of the slump and the imposition of import duties is very clearly seen in the diminution of imports of manufactured goods.

The most comprehensive comparison of the course of our textile industries over the past 30 years is afforded by Table XX showing the retained imports of raw materials.

The textile industries recovered with some speed from the depression. Retained imports of raw cotton, indeed, in 1937 reached 16·0 million centals, thus exceeding the 1929 level, but they fell to 11·6 million centals in 1938 and fell still further in 1939. The imports of silk, hemp, and jute also receded in some degree, but wool imports continued to rise, largely through the accumulation of stocks for army purposes. Flax rose a little in 1938, and textile pulp for artificial silk also increased; textile pulp comes mainly from Canada and Norway.

In the mass, retained imports of textile manufactures form no great menace to British industry, for in 1937 they were in value only 13·3 per cent. of British exports of textiles and 7·2 per cent. was composed of special yarns for British weavers; also 27·4 per cent. of the gross imports came from British Countries, chiefly from India.

The immediate effect of the imposition of import duties is seen, for example, in the diminution of the imports of "Other Woollen Tissues"; these were 31·5 million square yards in 1930 and 44·5 million square yards in 1931 but fell to 7·4 million in 1934 and to 4·4 million in 1935, and averaged 8·4 million in 1936–38.

TABLE XIX

Trade in Textiles

(Changes in volume: 1935 = 100)

Particulars	1930	1931	1932	1933	1934	1936	1937	1938	1939 Jan.–June	Declared Value 1935 (£mill.)
RETAINED IMPORTS										
Raw Materials:										
Raw Cotton and Cotton Waste ...	93·6	87·0	99·0	113·0	100·6	122·2	131·0	96·5	84·0	35·0
Wool, Raw, Waste, and Rags ...	85·7	101·9	101·0	101·6	90·4	107·3	96·6	105·7	165·2	25·2
Silk, Raw, etc. ...	39·4	48·3	62·1	74·2	94·1	105·0	118·5	123·5	92·9	1·5
Other Textile Materials	77·6	81·6	81·6	88·5	106·2	113·4	110·0	108·8	139·2	9·2
Manufactures:										
Cotton Yarns and Manufactures ...	312·1	312·7	58·4	79·6	81·5	121·8	143·5	138·3	141·3	2·2
Woollen and Worsted Yarns and Manufactures ...	556·7	662·5	89·6	95·2	95·2	133·4	178·1	183·1	221·9	2·0
Silk Yarns and Manufactures ...	276·0	253·6	125·0	106·8	118·7	84·2	87·4	77·1	106·3	2·7
Other Textiles ...	174·9	207·3	134·7	119·5	111·2	117·0	128·6	126·1	166·4	6·2
BRITISH EXPORTS										
Manufactures:										
Cotton Yarns and Manufactures ...	110·0	87·3	106·2	100·4	99·8	99·8	101·4	74·8	76·6	60·2
Woollen and Worsted Yarns and Manufactures ...	91·4	72·5	75·9	86·9	91·8	101·8	99·8	77·3	83·9	30·4
Silk Yarns and Manufactures ...	91·9	77·1	85·6	74·0	108·4	99·7	112·0	97·4	119·9	1·1
Other Textiles ...	94·7	79·2	87·5	94·0	104·9	111·8	125·1	91·5	95·6	15·7

TABLE XX

Retained Imports of Textile Raw Materials

Materials	Unit	1913	1929	Average 1930–32	Average 1933–35	Average 1936–38	1939 Jan.–June
Cotton, Raw	Mill. centals	19·2	15·3	12·0	12·5	14·2	4·9
Wool, Raw	Mill. lb.	500	486	562	578	612	498
Silk, Raw, Waste, etc.	,,	—	4·6	4·0	6·3	7·1	2·7
Flax (excl. Tow) ...	Th. tons	—	35·4	30·7	33·2	41·9	31·6
Hemp. (excl. Tow) ...	,,	—	67·1	61·7	75·0	84·2	43·4
Jute	,,	—	202·4	131·7	161·0	165·3	103·4
Pulp	,,	*	*	31·6 †	38·3	68·3	21·0

* Not available. † 1932 only.

The following table summarizes the leading British exports of textiles:

TABLE XXI

British Exports of Textile Manufactures

Particulars	Unit	Average 1930–32	Average 1933–35	Average 1936–38	1939 Jan.–June
Cotton Yarns	Mill. lb.	137	136	144	67
Cotton Piece Goods	Mill. sq. yds.	2107	1991	1741	702
Cotton Thread	Mill. lb.	15·1	16·6	15·2	7·2
Wool Tops	"	32·9	47·8	41·6	21·4
Yarns of Wool, Alpaca, etc....	"	47·5	52·1	40·9	18·6
Tissues of Wool, Alpaca, etc.	Mill. sq. yds.	99·0	108·5	116·7	49·0
Silk Tissues, pure and mixed	"	4·8	2·7	3·0	1·7
Artificial Silk Tissues, pure and mixed	"	55·5*	57·1	67·8	34·4
Linen Piece Goods	"	64·2	77·2	74·5	36·6
Jute Piece Goods	"	89·5	113·1	114·4	52·2
Cordage, Ropes, etc. ...	Th. cwt.	346	412	428	268

1932 only.

In most cases exports increased up to 1937, receded in 1938, and started a recovery in the first half of 1939, but the recession began in 1937 in the case of wool tops and yarns and linen piece-goods. Exports of cotton piece-goods had recovered in 1932 to within 8 per cent. of the 1930 level, but then steadily declined; a slight recovery in 1937 was followed by a slump of 28 per cent. in 1938, and there was no sign of recovery in the first half of 1939. Further losses in our Indian trade are shown below:

TABLE XXII

British Exports of Cotton Piece Goods

(In million square yards)

Kinds of Piece Goods	1930 To India	1930 To all Countries	1937 To India	1937 To all Countries
Unbleached	281	581	37	317
Bleached	327	876	197	573
Printed	84	349	47	413
Dyed in the Piece	77	507	73	506
Coloured Cottons	9	94	2	112
Total	778	2407	356	1921

Our Chinese and Japanese markets have been almost destroyed by the war in China. For a further analysis of our markets, it is

useful to consult the special articles prepared annually by the Board of Trade (e.g. *Board of Trade Journal*, July 20th, 1939). The improvement in the wool industry is the more remarkable in that exports of woollen and worsted tissues to China, which in 1933–35 averaged 3,170,000 square yards, averaged only 891,000 square yards in 1936–38, and those to Japan, which averaged 5,120,000 square yards in 1930–32, fell to 2,730,000 square yards in 1934, rose again to 6,229,000 square yards in 1936 and, at 262,000 square yards almost disappeared in 1938. Excluding flannels, mohair tissues, damasks, etc., exports of " other " woollen and worsted tissues rose between 1930–32 and 1936–38 by 17·6 per cent., but those to Canada, United States, and Argentina (comprising nearly 31 per cent. of all markets in 1936–38) rose by 27·6, by 49·3, and by 73·9 per cent., respectively.

Trade in Chemicals, Drugs, Dyes and Colours

The declared value of retained imports in 1935 was £11,201,000 and of British exports £21,329,000.

TABLE XXIII

Volume of Imports and Exports of Chemicals, etc.

(1935 = 100)

Particulars	1930	1931	1932	1933	1934	1936	1937	1938	1939 Jan.–June
Retained Imports	102·0	103·9	70·3	78·0	95·6	108·4	115·9	110·3	135·0
British Exports ...	96·8	81·3	89·1	87·6	93·7	100·6	115·5	96·5	103·7

To a very great extent imports and exports are non-competitive, the former including items like borax, calcium carbide, kainite and other potassium salts, nitrate of soda, tanning and dyeing extracts, etc., which are most conveniently obtained (partly for reasons of carriage) from the neighbourhood of the source of the raw materials, while imported fertilizers like kainite, superphosphates, guano, are only partly competitive with British-made fertilizers like sulphate of ammonia. Imported dyes, again, are by special legislation restricted to those not easily obtainable in this country. Great Britain has nothing to fear in the production and export of heavy chemicals, such as sulphate of ammonia, bleaching powder, copper sulphate, disinfectants, sodium compounds, basic slag, but has to face competition in fine chemicals, drugs and medicines, proprietary medicines, dyes, paints, coal-tar products. The export trade in agricultural chemicals is naturally dependent on the prosperity of the agricultural countries. Thus we find :

TABLE XXIV

British Exports of Agricultural Chemicals

(In thousand tons)

British Exports	Average 1930–32	Average 1933–35	Average 1936–38
Ammonium Sulphate	460	288	280
Copper Sulphate	43	39	29
Disinfectants, Insecticides, etc.	363	378	382

Two other heavy chemicals show progress in the three-yearly periods, viz. Sodium Carbonate and Caustic Soda, exports (in thousand tons) of the former being 90, 197, and 214, and of the latter 89, 102, and 111. In 1936–38 the value of retained imports of paints and colours was 3·2 per cent. less than in 1930–32, and British exports increased by 45 per cent.; imports are, morever, to some extent used in making mixed paints in paste or oil.

Miscellaneous Industries

The industries discussed in the preceding pages accounted for about 71 per cent. of our retained imports and 83 per cent. of our exports in 1937. Space and time will not allow the remainder to be treated in other than a somewhat perfunctory manner.

Taking those industries in the order of British exports, we find a declining trade in coke and briquettes; it is important to note for the present that our chief markets have been in the Scandinavian countries. Pottery imports are mainly bricks, tiles, and china, and these all declined, with some recovery after 1934. Our exports (mainly sanitary ware and earthenware) were more than double the value of imports in 1930–32 and treble in 1936–38, but in quantity were 15 per cent. less in 1937 than in 1930. Foreign countries (especially Belgium) maintain their superiority in cheap sheet glass, while British makers have improved their position as exporters of plate glass. Imports of cheap domestic glass have declined by a third, while our exports of dearer ware have increased heavily. Our exports of cement in 1936–38 averaged about 70 per cent. of those of 1930, but the imports of cheaper cement also declined. Imports of wood and timber reflect the prosperity or adversity of the building, mining, and furniture trades; manufactured imports are chiefly builders' woodwork and plywood from the timber countries and British exports are chiefly furniture.

Retained imports of apparel were declared at a total value of £20,252,000 in 1930, equivalent to £14,782,000 at 1935 prices, while in 1936–38 they averaged £10,393,000, also at 1935 prices; the

Table XXV

Miscellaneous Industries

(Changes in volume : 1935 = 100)

Particulars	1930	1931	1932	1933	1934	1936	1937	1938	1939 Jan.–June	Declared Value 1935 (£mill.)
					RETAINED IMPORTS					
Raw Materials :										
Wood and Timber...	89·0	73·1	71·5	86·0	104·7	117·2	126·5	93·9	73·9	35·2
Oil Seeds, Nuts, Oils, etc.	91·0	95·4	89·6	88·7	97·6	103·0	111·6	118·7	136·4	24·5
Hides and Skins, undressed ...	73·8	50·8	89·1	128·3	124·0	156·8	155·2	118·2	116·5	6·4
Paper-making Materials	76·6	71·5	84·7	86·2	99·7	111·4	126·5	110·5	116·8	10·7
Rubber	97·1	68·7	39·3	58·1	124·1	—	74·1	105·9	55·5	7·3
Miscellaneous ...	94·6	99·2	105·5	91·6	94·4	108·2	117·0	110·4	86·9	8·3
Articles Mainly Manufactured :										
Pottery, Glass, etc.	145·6	140·1	72·1	86·8	103·2	101·8	113·5	100·0	89·1	7·3
Wood Manufactures	99·4	102·4	91·4	90·1	85·2	117·1	119·7	91·9	88·7	6·0
Apparel ... ... } Footwear ... ... }	186·1	206·8	92·1	104·6	116·7	114·6	124·0	121·6	122·6	7·9
							181·9	216·6	279·0	
Oils, Fats, etc. ...	86·1	82·6	82·8	91·8	98·8	105·6	110·3	116·0	122·5	30·9
Leather and Manufactures	142·1	147·9	94·8	104·4	98·5	108·4	94·9	71·7	92·0	7·1
Paper, etc.	98·6	99·5	84·0	89·0	101·1	109·2	120·2	97·9	105·2	13·4
Rubber Manufactures	168·4	158·0	114·2	129·7	95·8	109·6	151·5	178·5	210·3	0·5
Miscellaneous ...	133·3	154·4	94·5	98·9	108·5	113·0	122·2	121·1	131·2	15·8
					BRITISH EXPORTS OF MANUFACTURES					
Coke, etc.	109·7	100·0	94·8	97·3	92·4	89·9	98·9	74·1	88·6	3·0
Pottery, Glass, etc.	121·5	91·9	81·2	82·3	93·8	103·2	116·2	112·9	109·7	8·3
Wood Manufactures	169·8	130·2	121·4	104·8	104·6	114·3	112·1	100·3	115·9	1·1
Apparel ... ... } Footwear ... ... }	133·9	109·9	104·7	100·4	100·8	105·0	106·7	89·7	93·7	11·1
							99·2	94·5	78·0	
Oils, Fats, etc. ...	92·8	79·3	80·3	83·4	95·5	92·1	103·8	99·9	106·4	5·2
Leather and Manufactures	87·2	62·8	62·0	87·8	77·3	101·3	113·2	87·2	96·6	3·8
Paper, etc.	105·2	83·2	92·5	89·7	92·8	103·6	122·5	96·2	101·3	6·6
Rubber Manufactures	78·3	71·3	74·7	85·5	99·0	103·3	102·5	103·9	105·0	1·5
Miscellaneous ...	109·8	90·0	89·5	91·9	94·1	109·7	119·2	115·7	124·5	23·0

declared value of the imports of 1932 was only £7,832,000 or £10,106,000 at 1930 prices. Where comparisons by quantities are available, the import position of 1930 was maintained at the end of the period in gloves but not in hats or hosiery; in women's leather shoes imports recovered to the old level in 1937–39. In 1936–38 the export trade in waterproofed garments was held at about the 1930 level, but there was some falling-off in hats and caps, and there were losses (by quantity) in woollen stockings of 26 per cent., and of over 50 per cent. in artificial silk stockings. Exports of boots and shoes wholly or mainly of leather are far below the 1930 level; reckoned by dozen pairs the average exports of 1936–38 were down by about 49 per cent. in men's boots and shoes, about 68 per cent. in women's, and about 54 per cent. in children's. Eire and South Africa took half our exports in 1930, but have since developed local

industries and cut down their share in 1937 to about a sixth, and in 1938 to about 16 per cent. of their 1930 takings. Comparisons of apparel trade in different years are difficult owing to changes in classification and quantity units, and to the number of entries by value only.

The Oils Group might well be divided into two—Petroleum and its Products and Other Oils, Fats, etc., and Manufactures thereof—for in 1937 the former was one-seventh of the "Raw Materials" oils, etc. Group, and about 92 per cent. of the "Manufactures" Group. Imports of crude petroleum were 461 million galls. in 1930 and averaged 536 million galls. in 1936–38. In 1930 retained imports of refined petroleum were 1,870 million galls., nearly one-half (909 million galls.) being motor spirit; for 1936–38 the average was 2,483 million galls., motor spirit 1,337 million galls. Exports of British refined petroleum run in the neighbourhood of 130,000 to 145,000 galls. For want of a standard of oil values, measurement by weight of oil-seeds and nuts and crude oils is imperfect, but there was an increase of about 30 per cent. in the tonnage imported in 1937 over 1930. Very significant is the rapid growth of the imports of crude whale oil (except for a severe setback in 1932) from 82,500 tons in 1930 to 225,600 tons in 1938 and 214,200 tons in the first half of 1939—stocking up against the threat of war. Refined vegetable oils are foodstuffs and do not enter largely into our import or export trade. Imports of soap were never very important, but our export trade, which is of more consequence, fell off by a third in quantity between 1930–32 and 1936–38.

Table XXIV sufficiently indicates the volume movement of the imports of hides and skins, and it may be left to the expert to analyse them further according to their uses; cattle hides and sheepskins are from one point of view by-products of the meat industry, and their imports in any one year are not necessarily indicative of the activity of the leather trade in that year. Comparing 1937 with 1930, retained imports of undressed leather were down by about a sixth, and those of dressed leather by a third. British exports of undressed leather were about the same in both years, and exports of dressed leather rose by about a half. The volume of imports of undressed leather in 1937 was nearly four times that of British exports. The real market of the British leather trade is the British boot and shoe trade, and in 1937 the Board of Trade index of production for leather and boots and shoes together was 111·9 against 100 for 1930.

The important thing about our imports of paper-making materials is that in 1938 seven-eighths by value consisted of wood-pulp, practically all from Baltic countries. Imports of both wood-pulp

and esparto have increased considerably, paper and paper-using trades being prosperous. Paper imports are chiefly newsprint, packing paper, and boards; newsprint was six times more in 1937 than in 1930 and two-thirds of it came from Canada and Newfoundland. Imports of packing paper and board seem to have followed the course of merchant business; the former exceeding 1930 quantities from 1933 onwards, and the latter from 1935. British exports of newsprint and other printing papers go mainly to British countries and have barely maintained themselves. There is a substantial export of good-quality writing-paper and of stationery, but this too has hardly kept up. Exports of packing papers and boards are relatively small, but have increased.

London has a large entrepot trade in rubber, and so the quantities retained in any one year are no indication of the condition of the rubber trades; in 1936 re-exports exceeded imports. Rubber tyres and tubes are regarded as parts of vehicles and are dealt with in Table XVI; they form the main section of the rubber manufacturing industry but both in home and overseas markets they are dependent on activity in the production of cars and cycles.

Conclusion

Space and time have prevented any serious treatment of shipping movement, of the distribution of trade, and of prices, each of which might well occupy a separate paper. But the following percentage table of distribution is instructive and brings out the increasing importance of the non-self-governing Crown Colonies, protectorates, and mandated territories which together form no mean colonial empire.

TABLE XXVI

Percentage of Total Trade

Imports from	1930	1938	Exports to	1930	1938
Dominions ...	18·67	25·82	Dominions ...	24·59	29·87
India	4·89	6·08	India	9·28	7·74
Colonies, etc. ...	5·56	8·49	Colonies, etc. ...	9·64	12·26
Total	29·12	40·39	Total	43·51	49·87

With regard to values, the general average of retained imports fell from 128·4 in 1930 to 100·0 in 1935 and rose to 119·5 in 1937, falling to 112·7 in 1938, while British export values fell from 116·6 to 100·0 and rose to 110·2 in 1937 and to 112·1 in 1938. Import food values were 132·2 in 1930, 118·3 in 1937 and 115·0 in 1938, and those

of raw materials were 127·6 in 1930, 125·7 in 1937 and 111·0 in 1938. The average value of exports of British manufactures (Class III) was much less violent in movement than raw materials, 117·8 in 1930, 109·6 in 1937 and 112·6 in 1938.

Though not so good a year as 1929, 1930 was still a fair year for export trade, and that there was a reduction of only 3·4 per cent. in the volume of British exports as a whole and of 1·3 per cent. in exports of goods wholly or mainly manufactured is very satisfactory, considering that in cotton yarns and manufactures the fall was 7·8 per cent. and in coal 26·4 per cent. As the volume of British exports in 1931 was 23·5 per cent. below that of 1930 and the improvement in 1933 was only 2·8 per cent. on 1931, the recovery afterwards was rapid. The falling-off of 11·3 per cent. in 1938 compared with 1937 was partly due to high prices of materials, partly to the coming shadow of war, partly to increasing concentration on armaments, and partly to depression in cotton exports. There were signs of recovery in 1939, and it is not being unduly optimistic to say that had the world remained at peace British exporters would have expanded their markets, even if the rate of growth would not have been sensational.

Discussion on Mr. Macrosty's Paper

The President thanked Mr. Macrosty for acceding so readily to his request to fill a gap which had occurred at that meeting. It was fortunate that there was this gap, because otherwise the very interesting analyses which the author had put before them would not have been forthcoming. The paper showed also what an enormous amount of important material was contained in the Board of Trade reports, and how very little of it was worked out and shown to an interested public. Mr. Macrosty had given the Society a number of examples from this very important and interesting material. He had suggested lines of analysis, but he had by no means exhausted the material even within the sphere which he had selected.

Dr. E. C. Snow said that he was very glad to propose a vote of thanks to his colleague Mr. Macrosty, for two reasons: the first because of the excellence of the paper itself, and the second because it afforded him the opportunity of expressing his gratitude to Mr. Macrosty and to Dr. Isserlis, for bearing so large a part of the burden of the Society in these days. For some months the Society had not been able to hold their meetings, but a good deal of administrative work had to be done, and this had entailed a greater burden, upon Mr. Macrosty especially, than ever before.

The paper contained a great deal of material for discussion. Nowadays the question of export trade had to be considered almost

as a weapon of war, and he hoped that his friend Sir Cecil Weir of the Export Council of the Board of Trade would contribute to the discussion from the point of view of the importance of export trade in war-time. He himself would deal with it, however, from the statistical point of view and as a general peace-time proposition.

He would turn first to the table (p. 457) giving the volume and value of imports and exports for the period 1930–39—which exhibited what Mr. Macrosty had referred to as a major surgical operation. In spite of the operation, he thought some highly important facts were made perfectly evident. The period Mr. Macrosty had taken, from 1930 onwards, was, of course, a highly important period in the history of this country, but the year 1930 was very exceptional, and he was not sure that in comparing 1930 with 1938, the last full pre-war year, they were comparing two corresponding points in the trade cycle. He wished to suggest another form of comparison over a rather longer period. It so happened that the earlier volume of the *Board of Trade Journal* dealing with these exports and imports gave certain particulars which enabled one to take these figures back to 1913, which happened to be just a generation ago, and the last full year before the previous war. Thus a comparison of the figures of 1913 with those of 1938, the last full year before the present war, was likely to be of some value.

The table given in the paper was divided into two main parts—retained imports and British exports—and the last figures in each set related to the totals of the subdivisions food, raw materials, and manufactures.

The table showed that, taking the year 1935 as 100, the volume of retained imports in 1930 was 106·3 and in 1938 108·7. These figures did not appear to indicate a very great change over that period, but if the year 1913 were taken, the corresponding index of volume was actually about 90. Obviously the figures over such a long period could only be approximate; nevertheless it was the fact that the volume of imports in the period between 1913 and 1938—just one generation of 25 years—had advanced from about 90 to 108.

The next line gave the corresponding index of value in £ mill. Again taking 1935 as 100, the figure for 1930 was 128·4, and for 1938 112·7. If, again, that figure was taken back to 1913, the corresponding index would be found to be 120. In other words, while the volume index went up from 90 to 108·7, which was an increase of 20 per cent., the value index went down from 120 to 112·7, a decrease of 6 per cent. That was the position with regard to imports.

Turning to exports, the volume of 115 in 1930 went down to 98·6 in 1938. If the corresponding figure be taken in 1913, it would be found to be approximately 180—that is to say, the volume of exports fell during those 25 years from 180 to 98·6, a decrease of 45 per cent. With regard to value, the index of average price, which in 1930 was 116·6 and in 1938 112·1, in 1913 was only 85. Thus there had been a rise in price of exported material to the extent of 35 per cent., accompanied by a fall in volume of 45 per cent.

That was the general picture of the import and export trade, the imports increasing in volume, not greatly changing in value; the exports diminishing considerably in volume and increasing considerably in price.

He was hoping that Professor Jones might be able to draw certain conclusions from those figures on the question of the price level, the importance of labour in respect of exports, and quite a number of other factors. He had merely desired to direct attention to the statistical aspect of the problem, which was extremely striking.

Another point which was well brought out in the paper was the result of the import duties which were placed on imported articles in this country in 1931. In this connection he drew attention to the tables showing the changes in volume of raw materials and manufactured goods and in trade in machinery and vehicles. In 1930 retained imports of manufactures had reached a figure of 133·9 (again taking 1935 as 100) and in 1931 the figure was 136. Then came the duties, which immediately reduced it to 87·3, and in 1933 it was 90. But the point he wanted to make concerned what had happened in the subsequent years. There had been a rapid improvement, and, on the whole, the import duties after a few years did not seem to have had a very great effect on the volume of the manufactured goods which were imported into this country. The effect was purely temporary, spread over two or three years, and afterwards the imports increased substantially. This was brought out particularly in regard to machinery and vehicles. In the case of machinery the retained imports in 1930 were 162·9 and in 1931 146·1. They immediately dropped to 82·3 in 1932 and to 70·7 in 1933. Then the tide turned; in 1934 they were up to 99·5 and by 1937 they had actually reached the figure of 194·1, in spite of the duties on machinery, which he suspected were not less than 20 per cent., and in many cases probably more. Thus imports recovered their volume in the course of a few years.

Mr. Macrosty, in referring to the trade in textiles, mentioned particularly the falling off in the exports of cotton yarns and manufactures, particularly the exports to India. Those of them who two years ago took part in negotiations with India in the attempt to get a trade agreement, must have formed the opinion that any such attempt to increase textile trade with India on a permanent basis must encounter very severe obstacles. Indeed, it seemed quite a hopeless proposition. The loss in export trade of cotton yarns and cotton piece goods was shown in the table giving British exports of textile manufactures. In cotton piece goods the trade was down from 2,107 million square yards in 1930–32 to 1,741 in 1936–38, and was probably less in the following year, but that was to some extent compensated by a substantial export of textile machinery from this country to India. Whether it was a compensation to export the machinery with which the goods were made rather than to export the goods themselves was perhaps a matter of controversy, but it was a fact that the loss of the export trade in the article which was consumed was frequently accompanied by an increase in the export trade in the machinery which made the article.

There was another extremely good illustration of that in the figures given for the exports of footwear. Before the last war the exports of footwear amounted to more than 20 million pairs a year, and recently the exports had been 4 or 5 million pairs a year. Even if the trade with Ireland were described as exports, the fact remained that these exports were only about one-fifth of what they used to be. This was largely brought about by the fact that within the last generation the making of boots had come to be much more of a machine matter than a handicraft, as it was in the younger days of some of those present. There was a very substantial industry in footwear machinery; actually it was almost the world monopoly of one firm, and that firm was continually expanding its operations in other countries. This was a tendency which quite definitely had come to stay; he saw no hope of reversing it, although a certain amount of export trade in footwear might be gained through the impetus which Sir Cecil Weir's organization was giving to export trade generally. At the same time he thought that people everywhere had a habit of wanting to make the goods which they themselves wore.

There was just one other matter to which he wished to refer in connection with export trade—namely, the relative value of the figures for exports of the various articles. All the figures given, of course, were gross. They talked about the exports of £20 million worth of coal and £20 million worth of cotton goods, but, of course, the two things were not of equal value to this country. The coal was entirely produced here, but the cotton goods first required an import of raw material which had to be paid for. In a paper which he read to the Society about nine years ago he attempted to indicate a way by which they might convert those gross values of exports into a kind of net output of exports, and he desired to repeat an example taken from an industry about which he happened to know something.

He took in the first place £1 million worth of exports of leather and came to the conclusion that out of this, £200,000 represented native raw materials, £380,000 raw materials which had to be imported, £210,000 wages and salaries, and £210,000 remuneration of capital. In other words, of the £1 million worth of leather exported, only £620,000 represented the net output, owing to the heavy importation of raw materials. He had then gone on to deal with the corresponding figures in the case of footwear, a more highly manufactured article. If, again, £1 million worth of exports were taken, only £60,000 represented native raw material, £280,000 imported raw materials, £370,000 wages and salaries, and £290,000 remuneration of capital. In other words, there was £720,000 of income to the country in exporting £1 million worth of boots, as against only £620,000 in exporting leather. The obvious conclusion was that it was better to export boots than leather. He had rather hoped that somebody would have developed that suggestion and set out other figures, because it seemed to him to be of considerable importance, from the point of view, for instance, of the Export Council, to know which were really the most valuable exports. Coal was

probably at the top of the list; he did not know which was at the bottom, but his own article of leather could not be far from the bottom, because so much raw material had to be imported for its manufacture.

Sir Cecil Weir said that he was interested in the export trade both as a peace-time study and also in his capacity as an executive member of the Export Council, but at the present time he was most particularly interested in it as an important weapon of war. In the few remarks he had to make—and he had not prepared anything to say to the meeting, having come merely to listen to the paper—he would deal first with the general question of exports in ordinary times when the nation was not at war. He had felt for a good many years that this matter was being allowed to drift, and as Chairman of the Special Export Committee of the Association of British Chambers of Commerce he had the opportunity of presiding over a well-informed body of men who met in the latter part of 1938 and who, in January 1939, produced a report which was submitted to the Government, making a number of proposals designed to improve what seemed to them at that time to be the somewhat unsatisfactorily organized and ill-regulated condition of the British export industry. They had been hopeful that effect would be given to many of those recommendations, and this, he thought, would have been done had the war not broken out. The proposals were of a character which would have stimulated the export trade. He was strongly in agreement with Dr. Snow that the whole situation of this country as an exporting country in normal times had very greatly altered since the year, 1913, which he had taken as his starting point. There had been, for example, the change to which Dr. Snow had alluded whereby, instead of exporting cotton piece goods, textile machinery was in some cases being exported instead. That showed even in pre-war times the way in which the wind was blowing, and he thought it gave a good deal of support to the view of quite a number of people that if secondary industries were going to continue to be developed in other countries as they were already being developed, there would have to be a much greater migration of population in order to provide permanent work for people who hitherto had chosen to reside in countries like Great Britain which relied for their prosperity on continuing to be great industrial countries.

Dr. Snow had asked him to refer to the work which the Export Council was now doing. He desired in the first place to pay a tribute, as the President had done, to the statistical information that could be obtained from the Board of Trade. He had been working at the Board of Trade for the past three months, and had been amazed at the wonderful supply of information that there was in the Statistical Department of that office, and he was hoping that under the new organization of industry which they were trying to bring about for export purposes in war-time that statistical information would be made readily and easily available to industries in general. He did not know how it would have been possible for

the Board of Trade to have given or individual firms to have obtained much of that information under normal conditions, but the organization of industries in Export Groups had provided bodies which could ask for and obtain all the relevant statistics which they required for any attack any of them might desire to make upon a particular market. Up to the present it had been his experience that there was practically nothing which they had asked for, however detailed its character, which had not been produced by that Department at a very few hours' notice. The export groups felt that this was going to be very helpful to them in the task they had undertaken.

The importance of export trade in war-time had been referred to by Dr. Snow, and in that meeting it was quite unnecessary for him to emphasize it. There was a gap of £60 to £70 million between the import and export figures, and, as many of those present knew perfectly well, they had no longer got the investments abroad and the other assets which they could use to pay for those imports over an indefinite period. It was therefore necessary to export larger quantities of goods if they were to continue to bring in as much as they would like to do.

But he did not desire to suggest that they must at the present time necessarily pay for all their imports by exports. When fighting a war of a *Blitzkrieg* nature, the important thing was to get the goods which were wanted at once, and he thought it would be wise policy for the Government to spend a larger proportion of money in the first six months of the year on essential imports, instead of spreading it throughout the year, and thus to make sure that such raw materials as were wanted were in the country and were made available for the conduct of the war.

Sooner or later, however—and the sooner the better—they must try to make up for these enormous increases in their import figures by increasing the total value of their exports. In his position on the Export Council he would have their sympathy when they looked round Europe and realized that within the last three months the Scandinavian markets had been lost, and now Holland and Belgium, all of them extremely valuable markets for British industry. It was not perhaps entirely a loss from the material point of view, for these countries were themselves fairly substantial exporters, and, retaining—as he hoped Britain would always continue to do—the command of the seas, it should be possible for Great Britain to take over, for the time being at any rate, a good deal of the export trade done by European countries which would now come under the blockade imposed by the British Navy.

The system which the Export Council was adopting was now, he thought, pretty well known. They were trying to bring all the industries of the country into Export Groups, founded on their existing trade associations, but representative ultimately of all the companies and firms who were manufacturers and merchants in each of the industries, whether members of their trade associations or not. It had been a comparatively easy job to bring about this organization, because in no single case had any objection been taken by any firm which was asked to join an export group. When one

remembered that in ordinary times under peace conditions there was a great deal of bitter competition between firms, and that there were rival claims by trade associations to represent the interests of single industries, it was a remarkable tribute to the patriotism of industry that it should have been possible in a short time to have formed 105 Export Groups, and he believed that by the end of the present month the figure would be nearer 200.

One important question had been raised by Dr. Snow in his analysis of the figures given in the paper. He had referred to the distinction in value between one form of export and another. That was one of the matters to which particular attention was being paid. Obviously when they were obliged to cut down as far as they could their expenditure on imported raw materials, they wanted to export the most highly fabricated articles rather than those which were merely of a semi-manufactured character. They were taking the opportunity of ascertaining from industries what was the value of an article sold in comparison with the value of raw material which it contained, and it was extraordinary what importance certain classes of exports had assumed in their minds and in the minds of the officials of the Board of Trade who were working with them. They discovered one type of article, for example, where the expenditure of perhaps £200 on a raw material such as steel would give an export running into a value of many thousands of pounds. Take another thing like needles, these were very valuable exports because the raw materials content was comparatively small in proportion to the value of the finished articles. That was an illustration of how a small amount of raw material might greatly enhance value.

He did not entirely agree with Dr. Snow—partly perhaps because he was himself engaged in the leather industry—that leather was not a good example of valuable export! Perhaps it was not as good as some others, but it had a fairly considerable value. Anything which had a value of two to one was a good export from the country's point of view, though obviously it could not compare from the home content standpoint with coal, which was of 100 per cent. value as an export. There were, however, other considerations in the case of coal which could not be excluded. He thought that Mr. Macrosty would be pleased to know that while in the figures he had had at his disposal it was rather difficult to tell just which classes of machinery were making progress and which were not, under the Export Group system it was possible to obtain the most minute details as to what was being sold in certain markets, and that was going to be an extremely valuable thing for export manufacturers and for the Government in the future.

Although he was now talking about a war problem, the paper itself was concerned with export trade in peace years without any particular reference to the effect which the war must necessarily have had upon it. Nevertheless the figures which Mr. Macrosty had presented must be very valuable in any consideration of the post-war situation as it would have to be studied in due course. The tendency in British industry had always been intensely in-

dividualistic. This had been its strength and its weakness. It had been its strength when Britain was the greatest industrial nation in the world, and its weakness in the period between the last and the present war. One good thing would come out of the involvement of this country in the war—namely, that under war conditions it had been possible to get industry to adopt a system of organization which it would have been almost impossible to have brought in only a short time ago. Having once had the experience of working together for this national purpose, those concerned would find that so many benefits were derived from it, not only for the industry as a whole, but even for the individual members of the industry, that when the war was over those who had come together to work as partners in the interests of the nation to increase its export trade would continue to work together as a matter of enlightened self-interest. There would inevitably be a greater co-operation between individual firms in the future than there had been in the past.

DR. ISSERLIS said that Mr. Macrosty had mentioned at the beginning of his remarks that by the bye-laws of the Society it was the duty of the Secretary to act as a stop-gap and to read a paper on some specific subject as and when directed. On this occasion he had contributed an analysis of statistics similar to the analyses he had made on previous occasions, in which the valuable material published by the Board of Trade was presented in a properly digested form and became available as a record for comparison in future years. It was his duty to second the vote of thanks to Mr. Macrosty, but Dr. Snow had already expressed those thanks in words better than he, the present speaker, could have commanded, and he had proceeded to make comments on the contents of the paper and to add much that was of value to the information by taking the comparison back to the year 1913. He thought that on that point a little additional analysis might be added. As far back as 1925 it was obvious that the great difficulty in the maintenance of exports was the fact that we were imitating the Dutch of the old rhyme—

> In matters of commerce the fault of the Dutch
> Is offering too little and asking too much.

We were keeping our import prices at a very low level, and, perhaps because of the conditions under which industry was carried on in this country, export prices at a high level. If the index of value were divided by the index of volume, it would be found on comparing 1935 with 1938 import "prices" as a whole went up in the latter year to 103·7 as compared with 100 in 1935, while export "prices" as a whole went up to 113·7. Those were the figures for imports and exports as a whole, but it was well worth analysing them further, as Mr. Macrosty had done in the paper. The story was exactly the same when 1938 was compared with 1930, and on going back to 1913, Dr. Snow had shown that the difference was still more marked. He was therefore a little worried over the author's concluding sentence: "There were signs of recovery in 1939, and it

is not being unduly optimistic to say that had the world remained at peace British exporters would have expanded their markets, even if the rate of growth would not have been sensational." He thought that the kind of organization which Sir Cecil Weir had described would probably become very necessary after the war if the discrepancy between import and export prices was to be reduced.

There were one or two detailed points on which he might touch. One was with reference to the imports of raw sugar. Mr. Macrosty had shown that in spite of the fostering of the cultivation of sugar beet in this country, the import trade had not been reduced. We imported something of the order of 2 million tons of sugar a year, but whereas in 1913 that sugar came on an average a distance of 2,000 miles, it now came on an average a distance of 6,000 miles or so. We might be importing a rather larger quantity of sugar, but with it we were importing a much larger quantity of service in kind. They had been told how valuable were the exports of leather goods because two-thirds represented home production. This applied equally to the exports of shipping services, although it was true that when the service consisted of carrying from China to Peru, then very nearly two-thirds of the freight was left behind in payment for services there. He hoped that the activities on which Sir Cecil Weir was engaged would be crowned with great and rapid success.

The vote of thanks was put to the meeting at this point and carried unanimously. The President being obliged to leave, the chair was taken during the rest of the meeting by Dr. Snow.

MR. W. J. WILLIAMSON, who, after twenty years' membership of the Society, was, he remarked, making his maiden speech, said that from the point of view of industrial organization they were not so much concerned to-day with figures and comparisons as with present-day facts. In a letter which he wrote to the *Glasgow Herald* of April 11, he had pointed out that in January and February of this year there was an adverse trade balance of over £116 million, and for March one of £63,500,000, and for April £57,250,000, making a total of £236,750,000. He asked what was in the intelligent minds of exporters with long-range vision and practical experience of overseas trade. He thought they would want to know, in view of all this organization of the industrial groups concerned, what could be done to organize foreign buyers to buy British goods—how to entice or compel them to purchase the goods. Twenty-five years ago he was concerned in an exhibition under the auspices of the Manchester Chamber of Commerce showing what goods the then enemy countries were producing which could be produced here. During the six days that the exhibition was open it was visited by 4,700 manufacturers and merchants, and it proved to be the foundation work of the newly established British Overseas Club. It was necessary to bear in mind that the potential foreign buyer could neither be induced nor compelled to buy British exported goods if the price was wrong; he must seek the lowest market. It was

not so much figures that counted, it was what could be sold and what could be done in this state of emergency to help the country.

MR. G. L. SCHWARTZ said it might sound ungracious, but he was rather disturbed to find any practical importance being attached now to a particular point raised by Dr. Snow—namely, that in the case of some British exports a larger proportion, and in the case of other British exports a smaller proportion, was attributable to home effort. It was very plausible, of course, to talk about £100 export of British coal in comparison with £100 export of British cotton goods, and to point out that practically the whole value of the coal was attributable to British effort, but that only about £30 of the cotton export was similarly attributable, and therefore to say that the coal export was the more important. But he thought the comparison was completely fallacious. The real comparison was between £30 earned net by a cotton export and £30 earned net by a coal export, and £1 net export obtained in the one way was just as good as £1 net obtained in the other. Suppose that £1 million worth of diamonds were imported, and that the workman here by diamond cutting added a value of £1,000, so that the value when exported was £1,001,000, would it be said in such a case that the value of the British effort was quite insignificant and that it would have been much better had the diamond-cutters been transferred to the coal industry, when the whole of the value of the export would be attributable to British labour? That would be a tremendous mistake. Men would be transferred from diamond-cutting, in which they were very proficient, to coal-cutting, in which they were not proficient at all, and to work out a figure on those lines as indicative of British effort really took one nowhere.

MR. MACROSTY in reply said that Dr. Snow's comparison between 1913 and 1938 dealing with retained imports as a whole and British exports as wholes, was quite an interesting little calculation, but, he would submit, only of limited value. On analysing the totals into at least their component groups it would be found that the increase in the volume of imports was very largely due to imports of food and that the falling off in exports was largely due to coal and cotton.

On the other point to which Mr. Schwartz had just alluded, he did not know that it was quite a fair thing to compare exports, say, of coal entirely raised by British labour and exports of cotton goods partially produced by British labour. Of course, in the first place, it must be remembered that in exporting coal one was destroying a permanent piece of capital. Secondly, when considering that the raw cotton had been imported one should also take into account the export of British goods to pay for that imported cotton.

He had been particularly interested in what Sir Cecil Weir had said, which was extremely valuable. It was ten years ago, almost to the day, since he, the present speaker, left the Board of Trade as an official for the last time, and now he was gratified to find that Sir Cecil Weir and his colleagues had discovered that in the Board of

Trade there were statistics of such enormous value and that, moreover, there were men in that Department who possessed an expert knowledge of those statistics which it would be very difficult to equal anywhere else. Was it not a pity that it had taken a war—two wars—to make the merchants and manufacturers of this country learn the value of one great public Department? He hoped that when this war had been won and they settled down to reconstruction those gentlemen would be good enough to remember what they were now learning and would be a little bit more forthcoming in assistance in peace time than they were in the years when he was at the Board of Trade. He was glad that Sir Cecil Weir had indicated that there was good hope that that would be the case.

He thanked the members for their kind reception of what was certainly an important but, in his handling of it, rather a dull subject.

As a result of the ballot taken during the Meeting the candidates named below were elected Fellows of the Society:—

George Abrahamson, D.Econ.
Math Nath Bhargova.
Charles T. MacGarvey.
Ernest John Partridge.

Corporate Representatives.

Arthur James Turner, M.A., D.Sc. *representing* The Linen Industry Research Association.
James Urmston *representing* Callender's Cable and Construction Co.

Some Constituents of the National Income

The Valedictory Address of the President, Professor A. L. Bowley, C.B.E., Sc.D.

[Given before the Royal Statistical Society, June 18th, 1940]

Preliminary

The no doubt laudable custom that a second address shall be exacted from the President on his termination of office has not been abrogated by the existing disturbances to detached and academic investigations. Thanks to the active co-operation of the Secretaries, of those members of the Council still in London, and especially of the office staff, who have carried on the usual routine of the Society in difficult circumstances, the issue of the *Journal* has not been interrupted, nor, I think, have its contents deteriorated. Possibly even the discussions on the Papers, carried on by correspondence, have been more mature than the impromptu speeches to which we are accustomed. The membership of the Society has hardly declined and the financial position gives no special cause for anxiety. With the new President and in general the same personnel, the Society may confidently hope that any further difficulties will be met with energy and efficiency.

In choosing my subject for to-day I naturally take that which has occupied a great part of my time for nearly two years, and which in fact is closely related to the first Paper I had the privilege of reading to the Society—that is, in 1895. It must not be assumed that I have thought of, or worked at, nothing but National Income during the intervening 45 years. Though all that I put forward to-day is on my own responsibility, and mainly without consultation with my colleagues, it is in fact largely the result of co-operative work. In 1938 the National Institute of Economic and Social Research delegated to a Committee of the London School of Economics the task of reporting on the National Income in any detail that should be found possible. During the twelve months or more prior to the outbreak of war, systematic research was carried on under my direction, with Mr. H. S. Booker as secretary and principal investigator. Since September I have worked through and extended the material, in consultation with Mr. H. Campion on several important problems. Only part of the subject has been completely explored, but I have the permission of the National

Institute to offer some results in a preliminary way, on the understanding that the Institute is in no way committed, and that revision and amendment may be made in any subsequent publication. As an Addendum to this paper, Mr. Campion has prepared a study of Salaries, on the same understanding as to the Institute's rights.

The central date for the investigation is 1931. For at that date we have the Censuses of Population and of Production, and the Ministry of Labour's Report on Earnings. Having established estimates for that year, we work back to 1924 to key on to the account that Lord Stamp and I gave for that year, and, on the other hand, carry it forward to 1938 or 1939. Here I do not attempt complete estimates, but only examine the course of the major constituents over the period. Also, having expressed my views on the measurement of Real Income at Manchester last November, I deal only with money income. Questions of definition do not arise till the relevant items have been measured. In fact I propose rather to examine the material and to test its sufficiency than to present any finished product. It may appear that I have spent my effort in straining at a gnat and swallowing a camel. But the gnats are interesting, if vexatious, creatures, and we cannot decide that they are unimportant till we have examined them. As to the camels, my gullet is less capacious than that of some writers on Income, but I have done my best to de-hump them and put them in suitable compounds.

It is only possible within the limitations of a Paper to describe the sources and methods and present the results. The whole *Journal* itself would not contain all the data and the tedious computations involved in their analysis. For example, there is an enormous mass of arithmetic behind the statement that the successive Returns of Earnings confirm the estimates of the course of wages derived from other sources.

The Number of "Persons Gainfully Occupied."

The Population Censuses of Great Britain show in considerable detail the numbers of persons classed as gainfully occupied at the Census dates. For intermediate years it is necessary to estimate them by interpolation and hypothesis.

The procedure adopted is to take the Registrar-Generals' estimates of numbers of all persons and their age and sex distribution each year, and apply to each grade the percentage of occupied to all recorded for the year 1931. In Table I it is seen that there have been significant changes in some of these percentages in recent decades, and it is interesting to speculate on their causes. For boys

under 18 years, and especially under 16 years, the percentage has diminished, it may be hoped for the reason that more continue their education or training after 16 years. On the other hand, for girls and young women in the grades up to 25 years the percentages occupied have increased, presumably a transfer from "helping mother" to paid work. In other age-groups below 65 years for males and 55 years for females there has been little change. In the higher age-groups for females the reduction (at least between 1911 and 1921) is due to lower employment of married women and widows. For males the reduction is only since 1921; this is possibly due to old-age pensions, possibly to greater reluctance to employ elderly men, possibly to changes in age-grouping within the grades. These changes apply to the relatively less important age-groups, while in the central grades which include the bulk of the occupied population the changes have been small, and smaller for the numerically more important males than for females.

The age-grouping in respect of occupation is not the same under 25 years as that in the Registrars' Annual Estimates, and adjusted percentages have been used as shown in the lower part of Table I.

The Census of North Ireland has not been taken at the same dates or in the same detail as that of Great Britain, but some estimate is available every year. The numbers computed as occupied in Great Britain have been raised year by year by a fraction falling from 1·0288 in 1924 to 1·0277 in 1931 and rising a little after 1931.

TABLE I

Occupied Persons as Percentage of all Persons

Great Britain

<table>
<tr><th rowspan="2">Age</th><th colspan="4">Males</th><th colspan="4">Females</th></tr>
<tr><th>1911</th><th>1921</th><th>1921</th><th>1931</th><th>1911</th><th>1921</th><th>1921</th><th>1931</th></tr>
<tr><td>14– ... ...</td><td>72·8</td><td>64·8</td><td>64·8</td><td>63·2</td><td>47·8</td><td>44·7</td><td>44·7</td><td>50·6</td></tr>
<tr><td>16– ... ...</td><td>92·0</td><td>91·5</td><td>91·5</td><td>89·8</td><td>69·4</td><td>71·2</td><td>71·2</td><td>75·6</td></tr>
<tr><td>18– ... ...</td><td>96·8</td><td>96·8</td><td>96·8</td><td>96·6</td><td>65·4</td><td>66·5</td><td>66·5</td><td>70·9</td></tr>
<tr><td>25–</td><td rowspan="2">98·5</td><td rowspan="2">97·9</td><td>97·9</td><td>98·4</td><td rowspan="2">29·3</td><td rowspan="2">28·4</td><td>33·6</td><td>36·5</td></tr>
<tr><td>35– ... ...</td><td>97·8</td><td>98·2</td><td>22·8</td><td>24·4</td></tr>
<tr><td>45–</td><td rowspan="2">94·1</td><td rowspan="2">94·9</td><td>96·8</td><td>96·7</td><td rowspan="2">21·6</td><td rowspan="2">20·1</td><td>20·8</td><td>21·0</td></tr>
<tr><td>55– ... ...</td><td>90·8</td><td>91·2</td><td>19·2</td><td>17·7</td></tr>
<tr><td>65–</td><td rowspan="2">56·9</td><td rowspan="2">59·0</td><td>79·8</td><td>65·4</td><td rowspan="2">11·5</td><td rowspan="2">10·0</td><td>15·2</td><td>12·2</td></tr>
<tr><td>70– ... ...</td><td>40·2</td><td>33·4</td><td>6·5</td><td>5·5</td></tr>
<tr><td>Aggregate 14– ...</td><td>92·7</td><td>91·8</td><td>91·8</td><td>90·5</td><td>35·4</td><td>33·7</td><td>33·7</td><td>34·2</td></tr>
</table>

Computed from the 22nd *Abs. of Lab. Stat.*, pp. 2–3.

Percentages Applied to Annual Estimates of Population to Approximate to Number Occupied

Age-group	Males	Females
14–	51·6	40·0
15–	92·3	70·6
25–	98·4	36·5
35–	98·2	24·4
45–	96·7	21·0
55–	91·2	17·7
65–	47·9	8·2

TABLE II

Estimated numbers of Occupied Persons — Great Britain and Northern Ireland

Estimated numbers of Wage-Earners — Great Britain and Northern Ireland

Year 1921	Males 1401	Females 583	Total (0000's omitted) 1984	Males 1068	Females 401	Total 1469
1924	1413	627	2040	1060	429	1489
1925	1426	630	2056	1068	431	1499
1926	1438	632	2070	1075	431	1506
1927	1456	635	2091	1086	434	1520
1928	1467	637	2104	1092	434	1526
1929	1481	641	2122	1100	437	1537
1930	1495	642	2137	1109	437	1546
1931	1519	643	2162	1124	437	1561
1932	1537	642	2179	1135	436	1571
1933	1543	638	2181	1137	432	1569
1934	1549	633	2182	1139	429	1568
1935	1569	639	2208	1152	433	1585
1936	1587	645	2232	1163	436	1599
1937	1595	647	2242	1166	437	1603
1938	1613	653	2266	1177	441	1618

Estimated Distribution by Age of Occupied Persons

Great Britain and Northern Ireland

Age-group	Males			Females		
	1924	1931	1938	1924	1931	1938
14–21	164	151	141	299	286	274
21–55	683	681	679	626	626	626
55–65	112	122	127	54	63	71
65–	41	46	53	21	25	29
	1000	1000	1000	1000	1000	1000

The number of occupied persons in the United Kingdom exhibited in Table II have been computed by the methods described in the preceding paragraphs.

Change in Age Distribution.

The well-known changes in the age and sex distribution of the population are naturally reflected in the ages of the occupied. The result is summarized in the lower part of Table II.

For both sexes the proportions under 21 years have diminished, those for the central group 21 to 55 years have changed little, and those in the higher age grades have increased.

Since gainful occupation diminishes rapidly among women after the age 25, the effect of this change is shown in a slow rate of increase in the total occupied in the whole period 1924 to 1938, and an actual fall from 1932 to 1935, when the low birth-rates of 1916–29 were effective.

Number of Wage-Earners. Earning Strength.

It is important to emphasize that for purposes of estimating the National Income by the method of aggregating individual incomes it does not matter much whether a person is counted as a wage-earner or as salaried and in the intermediate group so long as the number concerned is small, since the income in marginal cases is much the same in the two groups. But for purposes of estimating the wage-bill we must have some hypothesis on which to base the change in the number of wage-earners, and obtain factors to apply to a better-founded estimate at one date, in this case at 1931.

In my *Wages and Income Since 1860,* Appendix E, I classify the occupational groups in the Censuses as Wage-Earning and Others, and obtain

Percentage of Wage-Earners to All Occupied.

	Males	Females
1921	75·5	68·7
1931	74·0	68·0

Here shop-assistants are excluded from wage-earners.

I have assumed that the fall in the percentages here shown has continued uniformly from 1924 to 1938; *e.g.*, the percentage for males is taken as 75·05 in 1924 and 72·96 in 1938. These percentages are applied in Table II to the numbers occupied to obtain estimates of the numbers of wage-earners in each year.

Allowance is now to be made for the numbers of unemployed wage-earners. For this purpose the usual percentages of insured persons unemployed are averaged annually for males and females separately, with some adjustment in 1937 and 1938 to eliminate the effects of change in the method of enumeration and the inclusion of additional occupations (Table III).

The numbers of wage-earners thus discounted are shown as "Employed Wage-Earners" in Table III. The numbers for the year 1926 were placed in brackets and are subject to modification in the sequel, for the percentage unemployed that year, excluding as it does the men actually involved in the coal-stoppage, does not measure the whole reduction in employment; nor is it probable that it allows sufficiently for reduction of work in other industries.

TABLE III

Year	Unemployed Per cent.		Employed Wage-earners 0000's			Index-numbers						Aggregate annual Wages (exc. shop-assistants) £Mn.
						Earning Strength	Wage-rates		Wage Bill			
	M	F	M	F	Total		A_1	B_1	A_2	B_2	D	
1924	11·0	8·6	943	392	1335	1000	1000	1000	1000	1000	1000	**1495**
1925	12·2	8·6	938	394	1332	999	1014	1016	1013	1015	1015	**1520**
1926	(13·5)	(9·9)	(930)	(388)	(1318)	(990)	1016	1012	(1006)	(1002)	993	**1485**
1927	11·0	6·2	967	407	1374	1034	1015	1006	1049	1040	1040	**1555**
1928	12·3	7·0	958	404	1362	1026	1007	991	1033	1016	1016	**1520**
1929	11·6	7·3	972	405	1377	1039	1001	982	1040	1020	1020	**1525**
1930	16·5	14·8	926	376	1302	983	992	984	975	968	968	**1450**
1931	22·6	17·9	870	359	1229	929	979	965	910	897	897	**1340**
1932	25·2	13·7	849	377	1226	928	962	947	893	879	879	**1315**
1933	23·1	11·4	874	383	1257	953	949	940	904	896	896	**1340**
1934	19·3	10·0	917	386	1303	989	949	944	939	934	950	**1420**
1935	17·6	9·8	949	391	1340	1019	960	955	978	973	1000	**1495**
1936	14·8	8·6	991	398	1389	1058	985	982	1042	1039	1060	**1585**
1937	12·15†	7·7†	1024	403	1427	1088	1020	1019	1110	1109	1120	**1675**
1938	13·85†	11·6†	1014	390	1404	1072	1056	1059	1132	1135	1130	**1690**

† Adjusted from the published figures to allow for the change in the method of enumeration and for the inclusion of additional occupations.

It is noticeable that with the increase in female unemployment since 1936, the number at work is lower in 1938 than in 1924.

The index-numbers of wage-rates used as one factor in estimating the National Wage Bill make no allowance for change in distribution of the labour force by sex or age or between industries.

From details given on p. 497 below it appears that the average wage of males under 21 years is about 45 per cent. of adult wages. It may be taken for granted that the average earnings of men over 65 is lower than that of men aged 21 to 65, but it does not seem possible to judge how much lower. In the following Table the effect on average wages of the change in age distribution of males over the whole period 1924–38 is shown on various hypothesis.* By adult wage-rates is meant the average for men aged 21 to 65; by "elderly" is meant over 65 years, by "juveniles" under 21.

* For example, the central entry is computed from the lower part of Table II, thus:

$$\frac{141 \times 0{\cdot}45 + 806 \times 1 + 53 \times 0{\cdot}80}{164 \times 0{\cdot}45 + 795 \times 1 + 41 \times 0{\cdot}80} = 1{\cdot}0113.$$

Effect of Changes of Age Distribution of Males on Average Wages (1924 to 1938)

Per cent. elderly to adult wages	Per cent. of juvenile to adult wages		
	40	45	50
50	1·009	1·007	1·006
75	1·012	1·011	1·009
80	1·013	1·011	1·010
85	1·013	1·012	1·012
100	1·015	1·014	1·012

The effect of the relative diminution of the young is therefore about 1·1 per cent. spread over 14 years.

For females the average wage over 21 years is about 64 per cent. of average adult wages. If the average wage over 55 years was also 64 per cent. of average adult wages, no correction would be needed between 1924 and 1938. If we assume that the average under 21 is 64 per cent. of the average 21 to 65, while over 65 the average is half that for 21 to 65, the correcting factor is 1·0057, that is about half that for males. The correction is in any case so small that it is unnecessary to aim at greater precision.

The effect of the increase in the number of male earners relative to female is of more importance.

Write kw for the average wage of females, where w is that of males.

Write I for the index-number of wages, allowing for no change in age or sex distribution.

Use the figures for employed wage-earners in 1924 and 1938.

Then a little consideration will show that when changes in sex and age distribution are taken into account, we should have for the change of the National Wage Bill from 1924 to 1938 :

$$\frac{1014 \times 1{\cdot}011 + 390 \times 1{\cdot}006k}{943 + 392k} I$$

instead of

$$\frac{1014 + 390}{943 + 392} I$$

k is known to be in the neighbourhood of 0·5, and the ratio of these two fractions hardly changes as k varies from 0·45 to 0·55.

The first fraction is nearly 2 per cent. greater than the first, and this is the whole increase to be applied over the period to allow for increased earning strength.

To obtain an index of earning strength we have therefore to increase the total numbers of employed wage-earners (Table III)

progressively about 0·14 per cent. per annum, and it is not worth while to try to adjust this regular progression. This having been done the column is transferred to the base 1000 in 1924 to give the index of earning strength in Table III.*

The Index of Wage-Rates.

In Table III are given two index-numbers of wage-rates: A_1 that computed monthly for the London and Cambridge Economic Service and explained in detail in Memorandum 28 of that Service; B_1 that published by the Ministry of Labour, and discussed by Mr. Ramsbottom in the paper to the Royal Statistical Society (1935, Part IV) and brought up to later dates in the Journal 1938, p. 202, and 1939, p. 289. The structure of these numbers is well known. B_1 depends on a wider range of industries than does A_1 and it is adopted in the sequel. It is remarkable that they march closely together and are nearly identical in 1937 (compared with 1924).

Both index-numbers are on the basis of unchanged numbers and age and sex distribution and should therefore be applied to the index of earning strength to obtain an index of the National Wage Bill, not to the unadjusted numbers of earners.

They also neglect any change in the relative importance of industries. I have examined this effect in my *Wages and Income*, pp. 107–110. From 1924 to 1931 it is negligible, and from 1931 to 1935 the computed effect is an increase of under 1 per cent. in the industries covered by the Census of Production. But when mining and agriculture are included there may be some modification. I have combined this change with the more important adjustment that I proceed to discuss.

It is clear that index-numbers that depend primarily on changes of time- and piece-rates do not necessarily give an exact measurement of earnings. For apart from complete unemployment, for which allowance has already been made, there are facilities for increased earnings on piece and for overtime and less broken time, when industry is prosperous. Further it is well known that an arranged change in piece-rates is not necessarily proportional to the corresponding change in earnings. These relationships can be tested by the general returns of earnings published by the Ministry of Labour for 1924, 1928 and 1931, supplemented by information regarding coal-mines, railways and agriculture. On the basis of these data I estimated average earnings for males and females separately in my *Wages and Income*, p. 51. It remains to combine these by the help of the numbers of occupied wage-earners.

* For example: the figure for 1938 is $1000 \times 1404 \times 1{\cdot}019 \div 1335 = 1072$.

Average Earnings

	Males		Females		All Wage-earners			
	Numbers 000's	Average	Numbers 000's	Average	Average		Index-numbers	
							B_1	B_3
1924	943	54·4*s*.	392	27·5*s*.	46·5*s*.=	100·0	100·0	100·0
1931	870	53·7*s*.	359	26·9*s*.	45·9*s*.	98·7	96·5	97·0
1935	949	54·7*s*.	391	27·3*s*.	46·7*s*.	100·5	95·5	96·3

Here the index-numbers first given are proportional to average earnings; B_1 is that from Table III, and B_3 is B_1 adjusted for the greater change in " earning strength " than in numbers occupied, a change which has already had its effect on the estimates of average earnings.

We find then that earnings of employed wage-earners had decreased rather less than rates between 1924 and 1931, and increased between 1931 and 1935, when rates dropped.

We can make a rough test of the relation between wage-rates and earnings by comparing the details of Mr. Ramsbottom's index with records of earnings in eight industries, viz.: Cotton, Wool, Boots, Bricks, Pottery, Coal, Railways and Iron and Steel Production. The first five depend on the statistics of members employed and earnings, given monthly in the Ministry of Labour Gazette. For this purpose the average earnings shown each November, as compared with those a year before, have been computed for comparison with Mr. Ramsbottom's December figures. For coal the average earnings per shift in the fourth quarter of each year are taken; for Railways the annual report on earnings for March or

Table IV

Index-numbers of Approximate Earnings (E) *Compared with Constituents of Wage-Rates Numbers B_1 for December*

	Cotton		Wool		Boots		Bricks		Pottery		Coal		Railways		Iron and Steel		General Average	
	E	B	E	B	E	B	E	B	E	B	E	B	E	B	E	B	E	B
1924	100	100	(103)	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
1926	90	100	100	100	101	103	98	101	110	100	—	99	103	100	—	95	100	100
1928	99	100	102	100	93	98	95	101	103	100	87	87	102	96	96	90	97	96½
1929	94	94	103	99	100	98	96	101	103	100	87	87	102	96	98	90	98	96
1930	85	94	95	91	98	98	96	101	96	91	88	85	100	98	92	90	94	93½
1932	86	86	92	84	94	93	85	100	89	91	87	84	95	94	87	86	89	90
1934	86	86	95	82	95	93	94	100	95	91	87	84	95	95	98	89	93	90
1935	89	84	99	82	98	91	96	101	96	95	88	84	97	95	111	92	97	90½
1936	91	87	100	89	97	92	99	104	99	98	95	91	99	96	108	96	98½	94
1937	98	92	95	89	97	96	103	104	99	100	103	97	102	99	118	110	102	98½
1938	96	92	103	89	101	99	100	104	97	100	106	97	105	99	120	116	103	99½

April each year; for Iron and Steel Production the annual earnings as computed by the Iron and Steel Institute. There are many minor and some more important difficulties in making the comparison, and it does not seem to be worth while to do more than give rough averages.

The series of general averages agree so closely till 1932 that it seems best not to modify the original index-numbers in Table III prior to 1934; but in the years of improving trade adjustment seems to be justified. The industries selected probably contain on the whole a larger element of piece-rates or greater opportunities of overtime (in the case of railways) than do industries in general, and a smaller increase than that suggested by the figures here is probably justified.* The following paragraphs throw some light on the problem.

The Wage Bill and Income Tax, Schedule D.

There is a close relationship between the computed index-numbers of the Wage Bill and "Actual Income" under the Income Tax, Schedule D. The data for the years 1924 to 1935 are given in Table V and in the accompanying diagram. 1926 is omitted from the computation, since the average number of occupied wage-earners in that year is doubtful.

By the usual method of partial correlation, in which the effect of the time-element is separated we obtain the equation

$$Y' = 434 + 0{\cdot}57X + 4{\cdot}6t, \pm 12$$

where Y' is the wage-bill index, £X Mn is the actual income in Schedule D † and t is the number of years measured from 1930.

The mean square deviation of Y' as computed from Y the given index is approximately 12, or 1·2 per cent.

The fit of the computed line (Y') to the given line is very close from 1929 to 1934.

Y' is computed from values of Y raised in 1934 and 1935 on *à priori* evidence that earnings increased more than weights.

If, however, we used the original values of B_2 in these years we obtain a modified equation

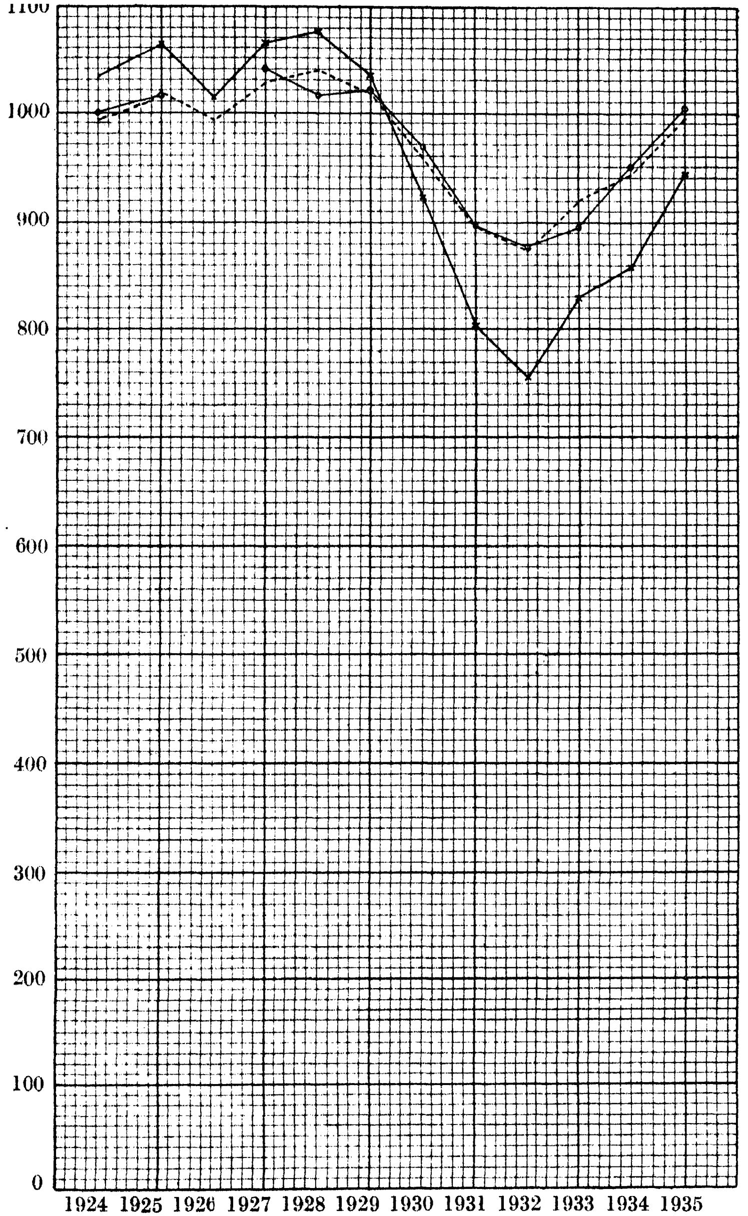
$$Y_1' = 490 + 0{\cdot}505X + 1{\cdot}29t.$$

Neither equation should be regarded as representing a permanent relationship between the wage-bill and Schedule D. Different

* In Table IV it is noticeable that from 1935 to 1936 in the general averages earnings increased less than rates. It appears from more detailed records that rates were raised at a later stage in the period of improving trade than the increase of earnings, as indeed we might expect.

† The assessment of each year, *e.g.*, 1925–26, is compared with the wage bill of the previous year (*e.g.*, 1924).

Wage-Bill Index and Schedule D



X. Schedule D. Actual Income £ mn. ×——× 1925–6 placed under 1924, etc.
Y. Index of National Wage Bill. 1924 = 1,000. o——o.
$Y' = 436 + \cdot 57X + 4\cdot 6t$ where t is number of years after 1930. – – – – –

TABLE V

Schedule D		Wages-Bill Index Estimates Computed from Formula			
Year	Actual Income £ X mn.	Year	Y	Y'	Y_1'
1925–26	1033	1924	1000	995	1004
1926–27	1064	1925	1015	1017	1021
1927–28	1013	1926	—	993	996
1928–29	1065	1927	1040	1027	1024
1929–30	1078	1928	1016	1039	1032
1930–31	1032	1929	1020	1018	1010
1931–32	922	1930	968	959	956
1932–33	802	1931	897	896	896
1933–34	756	1932	879	874	874
1934–35	829	1933	896	920	912
1935–36	858	1934	949	941	928
1936–37	944	1935	1005	995	973

periods of years yield different coefficients. But a close relationship between the movements is demonstrated, and explanations may be sought in years where the divergence is at all considerable.

On the evidence of these figures I propose to adopt the number 993 as the wage-bill index for 1926.

For the years 1934 to 1938 the evidence is somewhat conflicting. Using provisional estimates for Schedule D for the years 1937–38 to 1939–40, we have alternative estimates as follow :

Estimates of Wage-Bill Index

	A_2	B_2	Y'	Y_1'	Adopted
1934	939	934 × 1·016 = 949	941	928	950
1935	978	973 × 1·030 = 1002	995	973	1000
1936	1042	1039 × 1·024 = 1064	1048	1019	1060
1937	1110	1109 × 1·016 = 1127	1090	1054	1120
1938	1132	1135 × 1·016 = 1153	1075	1038	1130

Here the multiplier applied to B_2 is obtained by assuming that the general excess of the movement of earnings over rates is half that indicated by Table IV. For 1935 the excess is in the same proportion as that indicated by the Wage Census (p. 499 above).

During the time of expanding employment, 1935–37, the wage index-numbers show a stronger movement than does Schedule D, and are I think to be preferred.

In 1938 there was a reaction in industry and employment, and it would be anticipated that the factor to be applied to B would be less than unity. But the tendency to increased wage-rates was continued, while the actual amount of Schedule D is not known to me at the time of writing.

From these considerations I adopt the figures shown in the last column with considerable hesitation.

The results are shown under D in Table III.

For completeness at this stage the Wage-Bill Index is applied to the estimate of earnings in 1931 discussed below and the results are shown in the last column of Table III. So far the earnings of shop-assistants are excluded.

It is interesting to compare the estimate for 1924, here £1,496 Mn, with that made in *The National Income, 1924* (Bowley and Stamp), which (excluding shop-assistants) was £1,480 mn. The present estimate is based primarily on reports on earnings in 1931, worked back by index-numbers of wage-rates and earning-strength. The earlier estimate was based on the Wage Census of 1906, worked forward by estimates of the occupied and employed population, and wage-indices computed partly from the Wage Returns of 1924, but largely from other sources. A possible error of ± 5 per cent. was suggested for 1924. It is very remarkable that the two estimates differ by only 1 per cent.

The whole series from 1924 to 1931 is about 1 per cent. higher than that given in my *Wages and Income*, p. 77. From 1931 to 1935 it is 1½ per cent. higher. The revision of the wage and earnings index and of the number occupied for 1936 makes the new estimate 1 per cent. lower than the old in that year.

Note on Wages of Juveniles.

There is need for a systematic study of the rates of wages of boys, girls and young persons up to the age at which full adult rates are received, and indeed of the progression of earnings also for younger adults when the maximum is not reached till after some years of experience. I commend such a study to Fellows of the Society or to competitors for the Frances Wood Memorial Prize. Meanwhile I have put together the data relating to the subject in "Standard Time Rates and Hours of Labour, 1929," a publication which was kindly annotated for me at the Ministry of Labour in connection with another investigation, so that most of the figures used apply to the year 1937.

The Table and diagram given should be regarded as the results of only a preliminary and rough investigation, the main purpose of which was to estimate the effect on "earning strength" of the relative falling-off of young earners. On any reasonable hypothesis that effect was found (p. 497 above) to be trifling.

The averages in Table VI are simple averages based on 262 entries for males (including 49 for agriculture) and 128 for females.

The entries are not distributed in accordance with the importance of industries, but nothing appeared to be gained by any system of weighting; one which I applied did not affect the result.

TABLE VI

Juveniles' Weekly Wage-Rates

(shillings)

Age	14	15	16	17	18	19	20	21
			Males					
Bricks, chemicals	13·1	15·8	19·1	23·2	28·9	34·1	39·3	
Metal, engineering	11·2	13·4	15·9	19·4	25·3	29·8	34·3	
Textiles	11·3	13·2	16·7	20·9	26·3	31·1	36·2	
Clothing	11·4	14·5	19·5	24·1	29·0	32·9	35·8	
Boots (minimum)	12·0	16·0	20·0	25·0	30·0	36·0	44·0	
Food	12·6	14·7	17·6	21·5	27·5	33·9	42·0	
Wood, furniture	—	—	15·5	17·1	25·6	34·1	41·5	
Paper, printing	11·3	15·0	19·2	24·4	30·2	36·4	44·5	
Building, artisans	9·8	11·3	14·2	17·2	21·5	28·5	40·8	
,, labourers ...	14·4	14·4	21·5	21·5	28·7	28·7	43·1	
Transport	16·0	16·0	20·0	25·0	30·0	35·0	—	
Distribution	12·1	14·6	18·4	23·0	28·7	36·1	42·5	
Agriculture *	11·2	13·9	16·8	20·4	23·9	26·6	29·0	
Miscellaneous	—	15·2	18·0	22·6	28·6	34·2	39·8	
General average	12·1	14·3	18·0	21·8	27·4	32·7	39·0	
			Females					
Chemicals	11·8	13·8	16·1	19·4	22·2	24·2	26·0	28·1
Metal, engineering	10·5	12·3	14·6	17·2	22·0	24·6	25·2	25·3
Textiles	10·8	13·4	16·4	20·1	24·0	26·3	27·9	28·5
Clothing	10·1	13·0	17·0	21·1	25·7	28·3	30·6	31·6
Boots...	12·0	16·0	20·0	25·0	30·0	30·0	30·0	30·0
Food	11·8	14·7	16·7	20·9	24·5	27·3	29·5	34·1
Furniture	10·7	12·8	16·6	21·9	29·0	33·8	38·6	38·6
Paper, printing	8·9	11·4	15·9	22·3	25·6	27·3	29·0	31·6
Distribution	10·2	12·4	15·1	18·2	23·1	26·9	30·2	32·7
Miscellaneous	—	11·8	14·4	18·1	22·4	25·5	29·0	30·9
General average	10·6	13·2	16·3	20·4	24·9	27·4	29·6	31·1

* In the year 1933.

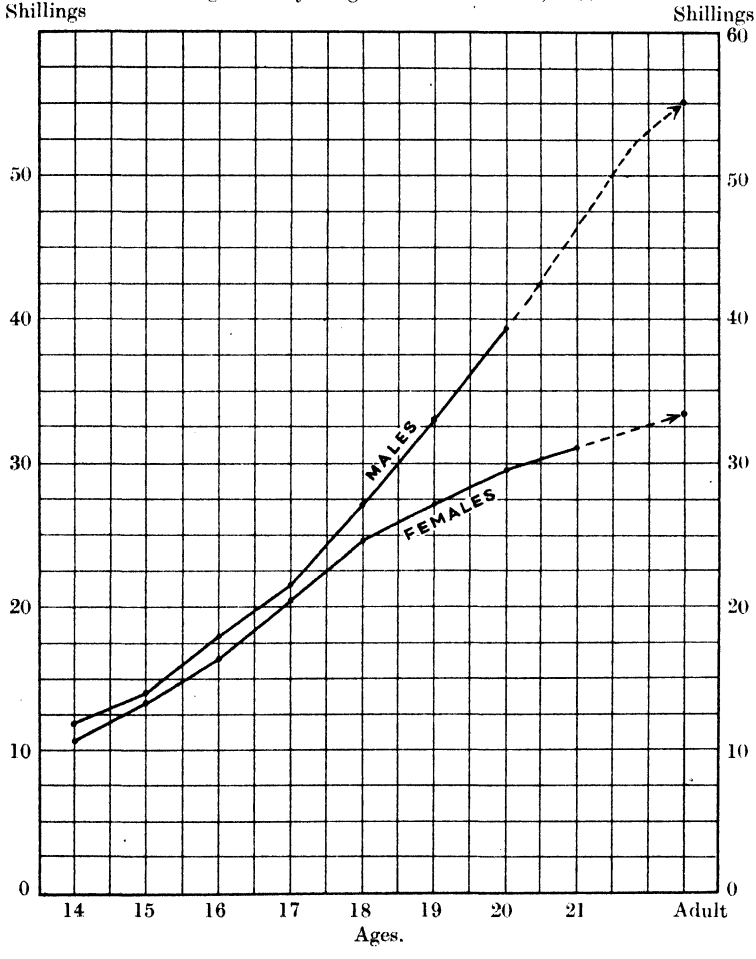
There is considerable variety in the upward steps in wages year by year in different occupations, and there is often a distinction between apprentices and others. On the whole the statements confirm the general impression that in unskilled male occupations wages start higher, increase at first more rapidly and reach their maximum at an earlier age, than in skilled occupations. In the latter the wage at 20 years is considerably below the standard rate for adults, and it appears that there is a sudden jump at the age 21.

From the Table and diagram it is seen that girls start at nearly the same rates as boys, but make slower progress especially after

17 years of age, and by the time they are 21 are usually receiving adult wages. The actual age at which adult rates are payable varies from trade to trade.

The dotted lines in the diagram are intended to indicate progress towards an average adult wage of 55*s*. to 60*s*. for men and 33*s*. to 36*s*. for women.

Average Weekly Wage-rates of Juveniles, 1937



The general averages for females agree closely with those recorded in the *New Survey of London Life and Labour*, Vol. VI, pp. 82–4. Their earnings in factories, workshops, etc., average 13*s*. 6*d*. at 14 to 16 years, 18*s*. 6*d*. at 16 to 18, 25*s*. 6*d*. at 18 to 20, 30*s*. 6*d*. at 20 to 25, and 33*s*. 6*d*. for all ages 20 to 65. It might have been

expected that London wages would be higher than provincial, but the figures here assembled do not support this view.

Total wages in 1931.

The Population Censuses of England and Wales and of Scotland of 1931 and that of Northern Ireland in 1926, together with some subsidiary information, enable us to classify gainfully occupied persons as follows :

United Kingdom, 1931

(000's)

	Males	Females	Totals
Wage-earners	9,035	3,793	12,828
Shop-assistants, etc.	606	494	1,100
Salaried and employers in agriculture ...	215	20	235
Others	2,548	1,211	3,759
Workers on own account	965	361	1,326
Total	13,369	5,879	19,248
Unemployed	2,014	575	2,589
Total	15,383	6,454	21,837

In this total is included an estimate of the number of fishermen and seamen absent on Census night.

In the present section shop-assistants are distinguished from other wage-earners and subsequently from the rest of the salaried group, since the various sources of information about their earnings do not allow any class distinction in their case between manual operatives and salaried or administrative.

For weekly earnings we depend primarily on the returns for "Average Earnings in the Principal Industries in Great Britain and Northern Ireland" in the week October 18th to 29th, 1931, summarized in the *Ministry of Labour Gazette*, 1933 (pp. 8, 45, 82). These, when supplemented by the periodic returns of coal-miners' earnings, cover the Industrial Groups III–XIV in Table VII. Other sources are used for agriculture and railway earnings. For domestic servants a small special investigation was carried out, which gave sufficient information to allow of a fairly reliable estimate. For other occupations we depended on known wage-rates for particular occupations or for the class of labour employed. Allowance was made for the value of board, lodging and clothing in the cases where it was relevant.

In the main group covered by the Ministry of Labour's returns there were in some cases difficulties in judging the relative import-

ance of firms employing less than ten persons and of larger firms, for which the returns were separately given. In all cases there was a little doubt whether the returns, which were on a voluntary basis, were closely typical of the whole industries.

The whole of the data were treated independently by Mr. Booker and myself. His first estimate of the total was $3\frac{1}{2}$ per cent. below mine; of this difference (£47 mn.) £15 mn. was due to coal-miners' earnings, where I found evidence of a higher average number of shifts, £5 mn. to his use of minimum rates (instead of earnings) in agriculture, £13 mn. to an allowance for deficiency in the returns from small building employers, where the earnings were at lower rates than in larger firms, and the remaining £14 mn. came from a number of industries, and principally from the assumption that small firms were insufficiently represented. I have not had the opportunity of collating these estimates with him completely, but I think that we would agree to at least half of my additions, so that we might write the total wage bill at £1,330 $\pm$ 10 mn., so far as this variation of estimate is concerned. In this paper, however, I have taken the higher quantity, £1,340 mn., as the principal estimate.

Apart from this difference of interpretation of the data there is the possibility of small errors in every factor and estimate involved. These are independent of each other and tend to cancel out, and would in that case give a margin of some 1 per cent. or less in the total.

Another consideration is that the earnings returns are for one week in October, and may not be applicable as the average of the year 1931. Actually changes in wage-rates were very small during the year, and the fluctuations in employment were not such as to lead us to expect any important difference in the relation between earnings and rates, when October is compared with the whole year.

There is, however, the further consideration whether the number recorded in the Census as employed in April was equal to the average number employed during the year. Among insured persons the percentage unemployed was 20·4 in April and 21·3 as the average for 1931 (21·9 in October), so that the percentage not unemployed was 79·6 in April and 78·7 for the year. But the insured population increased by 3 per cent. during the year, and the number of insured persons employed was very nearly the same in April as the average for 1931. In the occupations outside insurance the relationship may have been different, and on the whole it seems best not to adjust the main estimate, but to realize that here is a further error of less than 1 per cent. It may be added that various systems of

TABLE VII

Estimate of Earnings and of Salaries under £125 p.a. in 1931

United Kingdom

Industrial Group	Earners: Males: Numbers	Earners: Males: Weekly Earnings	Earners: Females: Numbers	Earners: Females: Weekly Earnings	Earners: All Annual Earnings	Employers, Managers, and Non-manual Operatives: Males: Numbers All	Males: Under £125: Per cent.	Males: Under £125: Average Salary	Females: Numbers All	Females: Under £125: Per cent.	Females: Under £125: Average Salary	All Total Salaries under £125
	000's	Shillings	000's	Shillings	£ Mn.	000's		£	000's		£	£ Mn.
I. Fishing	38	44·0	1	22·0	4	2·5	20	70	0·2	80	80	0·1
II. Agriculture	778	34·0	52	25·0	65	6·1	20	70	2·0	80	80	0·2
III. Coal Mines	920	49·0	4	24·0	108	20·2	16	70	1·7	75	86	0·3
Other Mines, etc.	91	51·3	2	26·5	11	6·3	16	70	0·7	75	86	0·1
Mineral Products	44	65·2	2	29·2	7	5·2	16	70	1·2	75	86	0·1
IV. Bricks, Pottery, Glass ...	134	51·7	47	22·4	19	11·6	20	62	4·1	85	77	0·4
V. Chemicals, etc.	110	58·8	36	27·5	18	46·5	13	80	17·8	65	80	1·4
VI. Iron and Steel production...	129	54·7	1	22·4	17	14·1	21	63	2·8	80	72	0·3
Engineering	326	52·4	19	26·8	42	65·3	20	65	19·5	80	75	2·0
Electrical Engineering ...	139	50·6	45	27·4	20	37·2	12	70	17·7	80	77	1·4
Vehicles	256	57·3	23	28·5	37	39·4	13	70	14·5	80	77	1·3
Ships	145	51·8	2	26·9	18	13·1	24	48	2·5	91	68	0·3
Other Metal Industries ...	400	52·3	91	22·8	55	60·1	16	66	20·8	75	74	1·8
Precious Metals, Watches ...	20	57·8	11	26·2	4	5·6	20	70	3·4	80	75	0·3
VII. Cotton	151	45·3	269	27·2	34	16·0	22	74	4·7	88	72	0·6
Wool	85	49·3	121	27·6	18	10·9	22	74	3·1	88	72	0·4
Other Textiles	107	49·3	252	26·5	29	25·2	22	74	16·1	88	72	1·4
Dyeing, Bleaching	72	49·3	21	24·3	10	8·6	22	74	3·2	88	72	0·3

VIII.	Leather	42	52·7	21	29·9	7	9·4	25	70	3·8	70	85	0·4
IX.	Clothing	203	53·6	435	27·6	54	57·9	14	80	37·6	80	73	2·8
X.	Food	193	58·2	126	26·3	35	87·3	12	88	57·7	65	80	3·9
	Drink	73	56·9	17	23·8	11	24·2	24	80	3·4	64	90	0·7
	Tobacco	12	58·8	29	36·7	4	6·9	20	75	3·3	70	80	0·3
XI.	Wood, Furniture	184	52·0	25	26·8	24	33·0	20	75	6·6	80	75	0·9
XII.	Paper	38	57·2	12	26·3	6	4·4	15	65	1·9	60	90	0·1
	Stationery	22	56·1	48	26·8	6	8·7	25	60	5·6	80	70	0·4
	Printing	161	77·4	72	29·2	35	61·7	22	63	21·4	82	71	2·1
XIII.	Building, Contracting ...	790	58·2	4	27·0	111	94·7	14	85	9·6	90	85	1·9
XIV.	Rubber	26	57·3	16	27·0	5	8·2	30	60	4·0	66	90	0·4
	Other manufactures ...	69	52·9	40	28·0	11	24·5	30	60	12·2	66	90	1·2
XV.	Gas, Water, Electricity ...	187	62·3	3	26·5	28	33·1	15	75	4·8	75	77	0·7
XVI.	Railway Transport	458	66·4	6	30·0	74	67·7	10	80	9·7	50	90	1·0
	Road Transport	359	62·2	6	32·8	54	46·1	30	70	11·8	90	70	1·7
	Other Transport	288	55·0	6	27·0	38	53·8	20	70	7·5	80	75	1·2
XVII.	Distribution	505	50·0	116	27·0	68	425·4	28	70	140·0	77	66	15·4
	Banking, etc.	42	50·0	19	20·0	6	261·3	15	80	71·9	55	93	6·8
XVIII.	Defence	336	45·7	3	28·0	37	39·3	8	74	3·3	55	82	0·4
	Central Government ...	32	58·6	10	39·5	5	235·8	20½	74	79·4	55	82	6·8
	Local Government	408	52·6	68	26·0	56	83·9	15	76	27·8	41	87	2·0
	Police	—	—	—	—	—	67·0	0	0	—	—	—	0·0
XIX.	Teachers	—	—	—	—	—	66·8	1	111	164·0	3½	106	0·7
	Nurses	—	—	—	—	—	—	—	—	151·6	76	80	9·2
	Professions	69	50·0	83	23·0	13	65·0	18	72	51·0	75	77	3·9
	Others	—	—	—	—	—	148·2	0	0	36·8	20	80	0·6
XX.	Entertainments, Sport ...	55	52·0	16	24·0	8	44·7	20	75	30·8	70	85	2·5
XXI.	Personal Service	507	36·0	1,604	22·0	124	86·2	25	75	107·3	85	70	8·0
XXII.	Other	32	36·0	8	25·0	4	8·9	20	70	7·8	75	78	0·6
	Total	9,035	51·6	3,793	24·2	1,340	2,548·0	17½	73	1,209·0	62½	77	90·0

weighting the separate returns within the total have been tested, and that they give very nearly the same results.

The remaining difficulty is in estimating the number of weeks to be applied to the October earnings to obtain annual earnings. More study and more information are needed here. Provisionally absence from work owing to holidays (usually without pay at that date) and from sickness is taken as averaging 4 weeks per annum, so that the weekly earnings in Table VII are multiplied by 48 to obtain annual earnings.

After consideration of all these elements it seems reasonable to regard the estimate of total annual earnings as subject to a margin of error of less than 5 per cent., or, since the possible errors suggest an over- rather than an under-estimate, as between £1,280 mn. and £1,370 mn. This is exclusive of shop-assistants.

Salaries below the Income Tax Exemption Limit.

As in the years 1911 and in 1924, a direct investigation was made into salaries by questionnaires issued to employers. We were successful in obtaining a great deal of information, and at least for the purpose of Table VII, which is to obtain the total of earned incomes other than wages below the exemption limit of £125, the returns were sufficient.

In the computation which leads to the total (£90 mn.) there are a very large number of independent estimates of percentages and averages, and so far as these are concerned the majority of the entries are unlikely to be more than 10 per cent. in error and the total should be correct within 5 per cent. But there is some doubt about the numbers to which these percentages should be applied. The columns headed "Numbers. All" include the entries in the Population Census under Employers and Managers (except in Agriculture), as well as estimates of the numbers of non-manual operatives, and it is doubtful how far the returns to the questionnaires included the superior group. Also in some cases there were included rather arbitrary estimates for quite small employers. Taking all these uncertainties into consideration, we may with some confidence write the total as between £80 mn. and £100 mn., keeping £90 mn. as the most probable estimate.

Of course we have not in this Table the material for computing average salaries of all employed, but only the average of the lower 17 per cent. of the men and the lower 62 per cent. of the women. Since the men include beginners preponderantly, while the women include many who have reached their maximum, it is not surprising that the average in these sections is greater for women than for men.

The numbers of those with small salaries are not stated in the Table, to save space. The last line may be written:

Under £125 per annum

	Numbers	Average Salary	Aggregate of Salaries
	000's	£	£ Mn.
Males	448	73	33
Females	746	77	57
Total	1,194	75	90

Shop-assistants.

It will be seen in the sequel that when the aggregate number of incomes is to be estimated, it is necessary to know what employees are classed as "weekly wage-earners, employed by way of Manual Labour" and are assessed on a half-yearly basis. Unfortunately neither the instructions nor the practice are clear as regards employees in shops. Shop-managers, shop-walkers, window-dressers, are given as examples of occupations "not regarded as manual," while working-dressmakers are "regarded as manual." Counter-hands are not named. The covering definition is: "'Manual Labour' includes all occupations which depend mainly on the exercise of physical exertion, even though a considerable amount of dexterity and training may be involved."

We have found on enquiry that some distributors include all shop-assistants in their general return to the Income-Tax authorities, merging them with clerks, etc., while others include them in the special return relating to manual labour.

As a compromise it has been decided for the present purpose to assume that the number of shop-assistants receiving less than £125 per annum—approximately 50*s.* weekly—is equal to the number returned as manual labourers, and that the remainder are included among the number assessed on an annual basis for income tax.

We have received reports that cover 125,000 shop-assistants—that is, about 12 per cent. of the number so classed in the Census of 1931—and the numbers and average salaries of those receiving less than £125 per annum in 1938.

Salaries of Shop-assistants in 1938

Percentage under £125 p.a.	Males 39	Females 89
Average salary:		
Under £125	£61·4	£78·3
£125 or more	£189	£162
All...	£139	£87

It is quite possible that the returns, which came from large distributors, do not contain a sufficient proportion of quite low rates

for young girls, and that the average (about 35*s*. weekly) for all females is too high. I have reduced the average under £125 somewhat arbitrarily in the estimates that follow.

Our returns indicate that salary-rates were nearly stationary from 1924 to 1936, that they rose 2½ per cent. in 1938 and a further 1 per cent. in 1939.*

From this information we have to deduce the proportions and average salaries under £125 in 1931 and subsequent years, and under £150 from 1924 to 1930.

As regards the numbers concerned we have from the Population Census:

Salesmen and Shop-assistants

(000's)

	Males	Females
Great Britain, 1921	352	411
,, ,, 1931	511	484
Northern Ireland, 1926	11	9

The statistics of Insured Persons under the heading "Distribution" have a wider content, especially of males, many of whom are presumably drivers and porters.

Insured Persons : Distribution

(000's)

Great Britain and Northern Ireland	Males	Females
1924	808	544
1931	1,137	738
1939	1,282	814

The increase here is much more rapid than that shown by the Census. In the sequel I have taken the Census figures with a uniform rate of growth from 1924 and 1931, and assumed an annual increase of 1 per cent. from 1931 to 1938 and no change in 1939, since the Insurance statistics indicate stationariness.

The change in the income-tax exemption limit, the movement in rates of salary and the correction for the presumed over-estimate of women's salaries affect the proportion and average rates below and above the exemption limit. After trying various hypotheses I have provisionally adopted the estimates in the following Table. These may be modified in the light of further information.

* See Mr. Campion's paper, p. 531 below.

TABLE VIII A

Shop-assistants

Estimates of factors involved

Year	Limit £150 1924	1930	1931	Limit £125 1936	1937	1938	1939
Males							
Total number (000's) ...	409	505	521	546	551	558	538
Below limit :							
Percentage	50	50	42	42	39	39	39
Number (000's)	204	252	219	229	215	218	218
Av. salary (£)	80	80	62	62	61·4	61·4	61·4
Females							
Total number (000's) ...	443	485	493	498	503	508	508
Below limit :							
Percentage	95	95	91	91	89	89	89
Number (000's)	421	462	449	453	447	452	452
Av. salary (£)	75	75	72	72	70·7	70·7	70·7

Between 1924 and 1930 and again between 1931 and 1936 uniform progression in numbers is assumed, and we obtain :

TABLE VIII B

Shop-assistants, Numbers and Aggregate Salaries

Male and Female

Year	Number (000's) Under £150	£150 and over	Total	Aggregate salaries, £ mn. Under £150	£150 and over	Total	Add roundsmen wages to small salaries £ mn.
1924	625	227	852	48	43	91	*57*
1925	640	235	875	50	44	94	*59*
1926	655	243	898	51	45	96	*60*
1927	670	251	921	52	47	99	*61*
1928	684	260	944	53	48	101	*62*
1929	699	268	967	54	50	104	*63*
1930	714	276	990	55	52	107	*64*
	Under £125			Under £125	£125 and over		
1931	668	346	1,014	46	64	110	*55*
1932	671	349	1,020	46	65	111	*55*
1933	674	352	1,026	46	66	112	*55*
1934	677	355	1,032	46	67	113	*55*
1935	680	358	1,038	47	67	114	*56*
1936	682	362	1,044	47	67	114	*56*
1937	662	392	1,054	45	73	118	*54*
1938	670	396	1,066	45	75	120	*54*
1939	670	396	1,066	45	76	121	*54*

These estimates should be regarded as subject to margin of error

of $\pm$ 10 per cent. throughout.* In particular some modification is probably needed to allow for the changing stress of unemployment.

In the Table on p. 513 the number classified as shop-assistants is 1,100,000; the excess of 86,000 over the number 1,014,000 in Table VIII A and B is due to the inclusion of "Roundsmen and Van Salesmen," whose annual earnings are about £9 mn. This sum is added to obtain the last column in VIII B.

Income in the year 1931

In consequence of the reduction of the income-tax exemption limit for earned income from £162 to £125 in the year 1931–32, a considerable number of incomes, estimated at 1,100,000, were assessed as taxable, though many were not taxed owing to additional allowances. The greater part of this number was presumably drawn from the "Intermediate Class," that is recipients of earned incomes less than £162 arising from non-manual work. The number in this class was estimated at 1,990,000 in 1924 (when the exemption limit was £150) with an aggregate income of £190 mn.

United Kingdom, 1931

	Total	Occupied Persons		Aggregate Income under £125 or Manual
		Under £125 or Manual	£125 and over Non-Manual	
	00,000's	00,000's	00,000's	£ Mn.
Employers				
Managers	40	12	28	90
Salaried				
On own account	13	6 *18*	7	50 *140*
Shop-assistants	11	7	4	55
Wage-earners	128	128	0	1,340
Total	192	153	39	1,535
Unemployed at Census date	26	—	—	—
Total	218	—	—	—

This Table shows 1,800,000 persons in the Intermediate Class (if as in 1924 shop-assistants are counted with wage-earners) with an aggregate income of £140 mn. The average income (under £125) at £78 is to be compared with the £95 (under £150) estimated in 1924.

The number of individuals assessed to tax in 1931–32 was estimated by the Income-tax Commissioners at about 4,900,000; the exact number is not known, and is less than the number of separate

* In *Wages and Income*, p. 77, the estimate for shop-assistants was £120 mn. in 1924, £100 mn. in 1931, and £120 mn. in 1936. These figures include some distributors who were not shop-assistants, and assumed (in the absence of evidence to the contrary) that the movement in salaries and unemployment was parallel to that in wage-rates.

assessments of income, since many persons are assessed under more than one schedule or more than once under one schedule. Among the individuals assessed is a considerable number who are not occupied and derive their income solely from property. A current estimate of this number is 250,000. Thus there are presumed to be about 4,650,000 individuals with incomes £125 or more, partly or wholly derived from gainful occupation. This is 750,000 in excess of the number of occupied persons (as shown in the Table above), whose income from occupation is £125 or over. The gap is serious, both as regards number and aggregate income, and is still under investigation.

One possible way of filling the gap can thus be found:

(*a*) Among the wage-earners may be included some, especially in Industrial Orders XVII to XX who are not regarded as manual and assessed half-yearly (including here some of the shop-assistants with less than £125 per annum).

(*b*) Among the salaries some will have additional income from property or supplementary work which bring their incomes from under to over £125.

(*c*) Among the salaried there are many married women, whose incomes are merged with their husbands' in assessment, and then the couple counts only as an individual.

(*d*) There is at present no basis for deciding how many of those working on their own account are assessed to tax.

On the assumptions that these classes account for 750,000 persons, and that their aggregate earned income is £60 mn. (*i.e.* £80 per head, men or women), the gap is closed. Part of this £80 mn. is included in the wage-earners aggregate, part in salaried. The Intermediate Class in the Income Total is replaced by a "residual class" that includes those incomes which are not assessed to tax or included under wages. The amount to be included is £80 mn., viz., the £140 mn. in the table less £60 mn.

Income from property that brings individual incomes above £125 is already included in the Income-tax total. Any earned income from occasional or supplementary sources should be added, but the amount is not known and can hardly be great enough to affect the total perceptibly.

The estimate for the total of the constituents of national income so far considered in 1931–32 is then:

Assessed to Income-tax ("actual income") ...	£2,065 mn.
Shop-assistants under £125	55 ± 5
Wage-earners	1,340 ± 40
Residual class	80 ± 20
Total of these constituents	£3,540 ± 50

The Income-tax assessment for Schedules A, B, C is that for the year 1931–32, that for D and E for 1932–33, which is assessed of the previous year's earnings. Income from wages is excluded from E.

The margins of uncertainty indicated are in part complementary, so that less than their sum is suggested for the total. It is possible, however, that on further investigation the £80 mn. for the residual class may need to be modified.

Total National Income, 1924–38.

There is still a number of relatively small constituents to be brought into account in any statement of Total Income. In Table IX these are included, whether they are definitely ascertained or are the subject of rough estimates, subject to considerable revision after further investigation.

The separate lines of the Table may be briefly explained :

1. The assessments for Schedules A, B, C for each Fiscal Year, such as 1925–26, are taken as applicable to the corresponding Calendar Year, 1925.

2. The assessments for Schedules D and E are antedated one year, so that the assessment for 1926–27 is taken as applicable to 1925.

The entries for 1938 are provisional.

3. Owing to the change in the method of assessment of profits from a three-year average to one year an addition is necessary in 1924.

4 and 5 have already been explained (pp. 498–514 above).

6. It has been assumed that the 1924 estimate is still valid till 1930, and then reduced at the time of the lowering of the exemption limit. Since 1931 some variation is assumed in parallelism with other entries.

7. Income from property accruing to persons whose total income is under the exemption limit is in some cases assessed under Gross Income and subtracted to arrive at Actual Income; in other cases there is no record. The whole sum was estimated from rough data at £77 mn. in 1924, and again at £45 mn. in 1931, with subsequent adjustments to 1938. In the absence of information the estimate for 1924 has been repeated till 1930, and then reduced as in line 6.

8. Besides the income of Charities, etc., assessed under Gross Income and subtracted to obtain Actual Income, it is known that a sum (estimated at £12 rising to £15 mn.) is unassessed. This estimate has been included.

9. It is assumed that interest at 1*s*. per annum is payable on outstanding certificates less the number purchased in the same year.

Preliminary Estimates of National Income

(£ mn.)

	1924	1925	1926	1927	1928	1929	1930	1931	1932	1933	1934	1935	1936	1937	1938
Actual Income:															
1. Schedules A, B, C ...	390	387	403	418	435	442	452	479	482	478	482	487	489	507	*515*
2. " D, E ...	1,686	1,738	1,713	1,775	1,800	1,776	1,726	1,585	1,523	1,616	1,682	1,801	1,938	2,045	*2,000*
3. Adjustment for date ...	118	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Total	2,194	2,125	2,116	2,193	2,235	2,218	2,178	2,064	2,005	2,094	2,164	2,288	2,427	2,552	*2,515*
Wages and earnings: *															
4. Shop-assistants ...	1,600 (4 and 5)	59	60	61	62	63	64	55	55	55	55	56	56	54	54
5. Wage-earners		1,518	1,486	1,554	1,520	1,525	1,448	1,342	1,315	1,340	1,421	1,496	1,586	1,676	1,691
6. Residual class	*190*	*190*	*190*	*190*	*190*	*190*	*190*	*80*	*80*	*80*	*90*	*100*	*100*	*100*	*85*
Unearned Income:															
7. Small property ...	*77*	*77*	*77*	*77*	*77*	*77*	*77*	*45*	*46*	*48*	*50*	*52*	*56*	*60*	*60*
8. Charities	*42*	*47*	*48*	*50*	*51*	*52*	*53*	*54*	*53*	*54*	*54*	*57*	*61*	*62*	*65*
9. Savings certificates ...	*16*	*16*	*17*	*17*	*16*	*16*	*16*	*16*	*16*	*17*	*18*	*18*	*18*	*19*	*20*
Sundries:															
10. Pensions	93	92	92	94	90	86	82	84	82	78	77	75	74	73	72
11. Employers' contributions	35	32	*30*	29	28	29	29	30	32	33	35	36	36	37	*37*
12. Others	*26*	*16*	*26*	*26*	*41*	*51*	*47*	*37*	*2*	*26*	*38*	*31*	*25*	*26*	*18*
A. Total	4,273	4,172	4,142	4,291	4,310	4,307	4,184	3,807	3,686	3,825	4,002	4,209	4,439	4,659	4,617
Transfers:															
13. Income due to Foreigners	*25*	*25*	*25*	*25*	*25*	*25*	*25*	*25*	*26*	*27*	*28*	*30*	*30*	*35*	*35*
14. National Debt Interest	*268*	*261*	*272*	*275*	*272*	*270*	*265*	*260*	*251*	*187*	*184*	*185*	*185*	*185*	*185*
B. Total: A less 10, 13, 14 ...	3,887	3,794	3,753	3,897	3,923	3,926	3,812	3,438	3,327	3,533	3,713	3,919	4,150	4,366	*4,325*
B. In round numbers ...	**3,900**	**3,800**	**3,750**	**3,900**	**3,925**	**3,925**	**3,800**	**3,450**	**3,325**	**3,550**	**3,700**	**3,900**	**4,150**	**4,375**	**4,325**

The items printed in italics are specially subject to revision.

* Under £150 or £125 p.a.

10. Pensions here include war pensions to men and dependents and medical treatment, non-contributory Old Age Pensions and the Government grant to Contributory Pensions.

11. Employers' contributions to the National Unemployment and Health Funds, which are not included in Actual Income under Schedules D and E, are here included, on the assumption that they are additive to the income of wage-earners which has been computed as including only those actually at work. It is arguable that the Government's contribution should be included. Or we might take the total amount of benefits less the workers' contributions, in which case we should substitute :

Benefits less Workers' Contributions (£ mn.)

(Health and Unemployment)

1924	39	1932	105
1925	41	1933	78
1926	49	1934	83
1927	44	1935	81
1928	49	1936	80
1929	53	1937	—
1930	97	1938	—
1931	115		

In Table IX, however, the procedure followed in 1924 has been adopted. If the new figures were used there would have to be an adjustment under " transfers."

12. This includes an allowance for evasion less over-assessment in 1924, which has been dropped in subsequent years; an adjustment for income from real property rising £1 mn. per annum between the years of re-assessment, when it is put at zero; a rough estimate of income accruing to Government from property or trading at home, and an addition (or subtraction) for excess of overseas Government receipts over payments.

13. This is largely conjectural.

14. Rough estimates are made of the interest payable to foreigners on the National Debt and are subtracted from the total interest paid.

The Total A corresponds to Aggregate Income in the 1924 study (Bowley and Stamp), while the Total B was termed Social Income. The figures here differ from the earlier owing to an adjustment in line 3 (using final instead of provisional assessments), additions in lines 8 and 9, and a modification and addition in line 12. Since probably there is some duplication of shop-assistants salaries in 1924 in lines 2 and 4, in the final line the total is reduced.

Proceedings of the Meeting

Professor Major Greenwood, in moving a vote of thanks to the President for his address, said that he had the qualification for doing so that he was a Past President of the Society and the further one that he had long admired the President's work, but he had no technical qualification for speaking on the subject-matter of the address or on the appropriateness of the particular methods which the President had used in arriving at his conclusions. Indeed, all that he felt justified in saying was that work of this kind had a certain resemblance to work to which, owing to the unwelcome leisure of recent months, he had paid a good deal of attention, namely, the work of the earlier writers. In the seventeenth and eighteenth centuries official or semi-official data of vital statistics were fragmentary. Enthusiasm, critical acumen, genius were required to bring out the really significant items and to bridge gulfs. It took a long time to create satisfactory official statistics of births and deaths—satisfactory statistics of illnesses were still to come. But, after all, the data of vital statistics were relatively simple. The data of economic statistics are more complex, with the result that, although both the general public and civil servants are much more interested in official statistics than their predecessors, the data are incomplete and require for their interpretation the high qualities of pioneers. He thought he could perceive these qualities in the President's work: enthusiasm and imagination held in check by a scientific caution which was not always manifest in the writings of pioneers of demography.

The President had pointed out that there were various subjects suggested as to which further research could be undertaken. For example, he had no doubt that some of their younger mathematical statisticians would think of many different ways of tackling the problem set out in the section of the address headed "The Wage Bill and Income Tax, Schedule D."

The President had also pointed out as of special interest the graph showing the average weekly wage rates of juveniles in 1937. It was clear, of course, that if one had males and females starting at the same point, and the males were in fact when they arrived at adult age going to reach a much higher level of wage rates than the females, at some point the gradient of the curve for females' earnings must fall below that for males' earnings. But why should there be, from age 14 to 18, a practical parallelism between the increments for males and females as shown by the two curves? Of course, directly one considered the possible implications, one approached that very thorny problem, from an economic point of view, the respective remunerations of male and female workers.

This was an example of one out of many occasions for research which suggested themselves, and he had no doubt that when they had all studied this paper carefully, with its various implications, even vital statisticians would find in it suggestions for research in their own special fields. He did not know—he was probably being

very amateurish—but even in these present anxious days he could not help feeling a kind of encouragement in the suggestion that a great economic storm had been survived, which enabled him to believe that at a graver crisis than that of 1931, we were not so far from recovery as some feared. Therefore he had great pleasure in moving that the Society accord a hearty vote of thanks to the President for his Valedictory Address.

Mr. H. W. Macrosty said that the annual meeting, concluding the work of the Session, was over, but on this one occasion in the year the Society had a peculiar convention of its own, and although the Session was finished, they were still in the last meeting of the Session, and it was therefore appropriate that the retiring Hon. Secretary should second the vote of thanks to the retiring President. It gave him great pleasure to do this, both on account of the Address itself and also because it afforded him the opportunity of voicing the opinion of all who had served during the last two years with Professor Bowley on the Council. He had been a first-class chairman of Council and of the Executive Committee, keeping them to the point when they were wandering and accelerating the business in the best possible way.

In considering the amount and the constitution of the national income there were two main methods of approach. One of these, which he might call the Census-of-Production method, was based upon a calculation of the amount of goods and their value when finally distributed. To this had to be added estimates for services and various other things. The other method, which, perhaps, he might call the Income-Tax method, proceeded from the ascertainment of the various classes of income, filling up the gaps, and aggregating the totals. The former method had been particularly associated with the name of Sir Alfred Flux, and in connexion with it he himself as a member of the Census of Production Department had done some work. The other was that followed by the President and Lord Stamp.

At times there had been some rather hot controversy as to which of these two methods was the better, but he did not think that in principle either had any pre-eminence over the other. In both cases a certain amount of definite information was available; in both a considerable amount of estimate and calculation had to be made. The important thing was that those two methods had been found, when calculations were made for 1924, not to be so widely divergent as to throw serious doubts on the results.

The present address had naturally been based upon the second method, and unless one had all the working papers before one and went through all the calculations which Professor Bowley and his colleagues had made, it was, of course, impossible to criticize the details of the paper. The most important thing to consider was whether the method of approach to the various problems that had presented themselves in the course of this labour had been an appropriate one or not. He did not claim to be an authority on wage and salary questions, but so far as an outsider who had had

to consider those problems at various times might reasonably offer an opinion, it was that the method of approach had been justified. But, after all, it meant a great deal of calculation, a great deal of estimating, a great deal of making of assumptions, and Professor Bowley in giving his address had indicated at various points that further research might lead to modification of the figures which he had deduced from the evidence before him.

That raised the question why it should be necessary to make those calculations, to make all those estimates, to construct all those formulæ. Why should they not have the correct information, not necessarily every year, but periodically, as to the amount of wages which was earned by the different classes of wage-earners? Why should they not have correct information as to the incomes of the people who were above the conventional limit which defined the wage-earner? A great deal of the information had been ascertained, and a great deal more of it could be ascertained quite easily. There was no reason why the Earnings and Hours Inquiry of 1906 should not be repeated and form a permanent part of the Governmental sources of information. There was no reason why the Inland Revenue, in collecting particulars of incomes assessable to taxation, should not put all that information together in a manner which would enable not only the total income to be shown, but also the totals at various grades of income. This was done in the case of the persons subject to super tax, why should it not be done for the whole of the incomes with which the Inland Revenue had to deal? It was a laborious piece of work, it would be an expensive piece of work, but the value of the information when obtained would be very great. After the troublous period we were passing through, it would be imperative to know, not only what the income of the country was, but what the stratification of that income was, in order that policies might be devised which would make for the greater happiness of the people. When more peaceable times returned, the Society, he trusted, would devote all its energies to pressing upon the Government of the day the importance and necessity of getting this definite information, and thereby, incidentally, of releasing all the talents of Professor Bowley and his colleagues for the interpretation of definite data, instead of compelling them to be applied to the estimating of probable data. He had great pleasure in seconding the vote of thanks.

The vote of thanks was put to the meeting by the proposer and carried unanimously.

PROFESSOR BOWLEY thanked the Fellows and visitors for their kind reception of his paper. When at the next meeting Mr. Macrosty read his Inaugural Address they would not be able to discuss it, but a different procedure obtained in the case of the Valedictory Address of the President, and therefore the address was open to discussion.

DR. C. OSWALD GEORGE said that having had no time to study the paper, he had had no intention of taking part in the discussion

until he saw the imperiously-beckoning finger of the President-elect. Yet he was glad to be able to acknowledge the incalculable debt he himself owed to Professor Bowley and his works, and he felt sure a similar acknowledgment would willingly be made by an army of students and others to one who had had greater influence on statistics and statisticians than any man of his time. This Society also owed him a great debt for his unremitting work on its behalf during a very troublous term of office. It was some time since he had vacated his academic Chair, and now that he was retiring from the Presidential chair, one might hope, yet could hardly expect, that his public appearances would be as frequent as in the past.

His valedictory paper was of such a character and importance as to demand careful and detailed study, without which comment was difficult and criticism almost impossible. Yet while it was being read two broad points seemed to stand out—its transparent honesty and its apparent simplicity, the outstanding characteristics of so much of Professor Bowley's work. The apparent simplicity was due not to the nature of the subject-matter, but to the author's capacity for analyzing a difficult subject and, while exposing all the essentials, making the process appear extremely simple.

The gnats and the camels had been put in their proper places. The effect of gnats might often be disturbing, yet, as microscopists knew, the study of gnats might be not merely interesting but profitable—provided the appropriate "objective" were chosen—and no one would dispute the President's choice of objective. As for the camels, some writers on National Income seemed scarcely to have realized there were such animals, and if the paper had done no more than emphasize the existence of these zoological curiosities, it would have been more than justified.

Although the author might claim to have completely "dehumped" them all, there might possibly remain one or two things to trouble one's œsophagus. One of these, possibly only a gnat, was the correlation method adopted for ascertaining the wage-bill index figure of 933 for 1926. Dr. George admitted he was always a little suspicious of such methods, maybe owing to prejudice, but he felt that correlation of time series resembled tragedy in that it was sometimes perilously near the comic. Professor Bowley had clearly avoided any such danger, for he had taken great care to point out the weaknesses, difficulties and dangers involved in the correlation computations and diagram. And the broad resemblances in the graphs of the computed wage-bill index (Y'), of the index of national wage-bill (Y), and of Schedule D (X) were, over what was after all a relatively short period, not perhaps very surprising. Yet one might doubt the validity of using as an index figure the crucial 1926 point—placed as it was on that half of the (Y') curve which noticeably meandered from the (Y) curve.*

Another interesting diagram was that relating to the wages of males and females, which might well encourage further research.

* The figure for 1924 in the Table for Y' did not correspond to the formula and has been corrected from 965 to 995 and the diagram amended accordingly. Thus corrected, the meandering of the diagram is much reduced.

Time correlation enthusiasts might be tempted to pursue the negative relation between average wage rates and average pitch of the voice; when boys' and girls' voices were nearer in pitch, their wage-graphs travelled together, but at or soon after the age when the deep bass voice of the male appeared, the male wage rate began to rise sharply. Others, of course, might consider the factors associated with the drop in the voice, such as increased physical powers or approach of that age at which marriage and its financial responsibilities might be undertaken. In such factors might perhaps be found at least a partial explanation of the general shape of the diagram.

Dr. George concluded with a final tribute to the paper and to the whole of Professor Bowley's work for the Society in particular and for statistics and statisticians in general, during the last three or four decades.

PROFESSOR BOWLEY subsequently wrote :—

Owing to pressure of time it was not possible to reply to Dr. George at the meeting, but I wish to express my appreciation of his kind speech. There can be no better reward for a long period of teaching and research than to find that the principles which I have tried to inculcate have found their place in scientific work, that the results of research have generally (after appropriate criticism) been accepted, and that there are numerous workers and writers, whom I am proud to have had the privilege of influencing, who are actively carrying on the tasks in which I have been seriously interested.

As a result of the ballot taken during the meeting the candidates named below were elected Fellows of the Society :—

Harold Edward Brightley.
The Rev. William Drury.
Cyril Cornelius O'Brien.

Changes in Salaries in Great Britain, 1924–1939

By Joan G. Marley and H. Campion *

This paper gives a summary of the preliminary results of two enquiries made by the Economics Research Section of the University of Manchester relating to changes in salaries in Great Britain from 1924 to 1939. The first was a sample enquiry of salaries paid in 1938 to 700,000 persons, and was undertaken in collaboration with the National Institute of Economic and Social Research. The second was the collection by the Economics Research Section of details of changes in salary rates made by private firms and public bodies between 1924 and 1939. The analysis of the results of the two enquiries has been suspended because of the war, but it is possible to give here details of salaries paid to different classes of salaried persons in 1938 and also index numbers of changes in salary rates from 1924 to 1939.

A. *Number of salaried persons*

For the purposes of both enquiries all persons employed in non-manual occupations in private industry and in public administration (excluding employers, persons "on their own account," the armed forces, and persons paid wholly or mainly by commission) were taken as salaried persons. This basis of classification was chosen to conform with the classifications of salaried persons used by the Board of Trade for the Census of Production, by the Board of Inland Revenue for assessments under Schedule E for income tax and by the Ministry of Labour for purposes of unemployment insurance. Thus our estimates of the number of salaried persons in Great Britain include (*a*) the technical, administrative and clerical staff—to use the description adopted in the Census of Production—employed in manufacturing industries, and (*b*) other persons not in manufacturing industries, who are assessed annually for income tax purposes under Schedule E (excluding weekly wage-earners in manual occupations and shop assistants) or are non-manual workers as defined for purposes of unemployment insurance.

No claim can, of course, be made to absolute accuracy in the estimates of the numbers of salaried persons used in this paper.

* Miss Marley has been solely responsible for the collection and analysis of the statistics given in this paper.—H. C.

The line of demarcation between manual and non-manual occupations has never been clearly drawn and is less distinct than it was. Other investigators might wish to exclude some persons whom we have included or to make additions to our totals. Their alternative estimates, if shop assistants are excluded, are not, however, likely to differ much from those given in this paper. Shop assistants have been excluded throughout from our figures of salaried persons, even though their work may be considered non-manual. Details of earnings of shop assistants and changes in their rates of earnings since 1924 are given separately in Section D of this paper.

Salaried persons have been classified into nine categories according to the branches of industry in which they were engaged. (In each branch of industry there may be persons engaged in similar occupations. More than half of the salaried class in 1931 were in clerical occupations.) The method of arriving at the estimates in Table I was first to separate persons employed in public administration from persons employed in private industry. Persons employed by the State and local authorities were then divided into salaried officials and wage-earners, and the salaried officials in turn sub-divided into the three categories (*a*) non-manual civil servants employed by the State, (*b*) administrative and clerical staff employed by local authorities and (*c*) teachers employed by local authorities. Salaried persons

TABLE I

Estimated number of salaried persons (excluding shop assistants) in Great Britain in 1931 *and* 1911

(Thousands)

Branch of Industry in which Employed	1931			1911		
	Total	Males	Females	Total	Males	Females
Total Salaried	2,900	1,775	1,125	1,532	954	578
A. *Private Industry*						
Manufacturing Industry and Agriculture	726	481	245	279	228	51
Distribution	488	281	207	178	117	61
Finance	324	252	72	57	50	7
Transport	159	130	29	38	36	2
Personal Service ...	122	50	72	49	18	31
Professions	394	213	181	442	224	218
B. *Public Administration*						
Central Government...	210	133	77	123	87	36
Local Government Administration and Industry	248	169	79	169	140	29
Teachers	229	66	163	197	54	143

employed in private industry had to be separated on the one hand from wage-earners and from employers and workers "on their own account" on the other. They were then sub-divided into the six categories of (*a*) technical, administrative and clerical staff employed in manufacturing industries and agriculture, (*b*) the salaried staff (excluding shop assistants) of firms engaged in wholesale and retail distribution, (*c*) officials of banks, insurance and finance companies, (*d*) the non-manual workers of railway, road and transport undertakings, (*e*) the salaried staff (excluding domestic servants) of individuals, clubs and businesses engaged in providing personal services and (*f*) the professional, clerical and administrative staff of all individuals and firms providing professional services.

In 1931 the number of salaried persons in Great Britain is estimated to have been 2·9 million and to have increased further to 3·2 million in 1938—one in seven of all occupied persons in the country. The number of salaried persons has been increasing since before the last war both in total and also relatively as a proportion of the total occupied population. The number increased from 1·5 to 2·9 million between 1911 and 1931, and this represented an increase from 8·3 per cent. of the total occupied population in 1911 to 13·8 per cent. in 1931. The increase in the number of females employed in non-manual occupations has been greater than for males. Females in salaried occupations were 18·0 per cent. of the total number of occupied females in 1931. The percentage of occupied males in salaried occupations was only 12·0.

Although the total of all persons occupied in public administration has increased by a greater rate than those occupied in private industry since 1911, the number of salaried persons employed in private industry has risen more than the number of salaried officials of the State and local authorities. There has been a marked expansion of non-manual workers in manufacturing industry, distribution, finance and transport. The number of salaried persons employed by businesses providing professional services has decreased mainly because many industrial concerns and public bodies now employ a larger number of officials with professional qualifications than they did before the last war. The slower rate of increase in salaried persons in public administration is, however, partly explained by the relatively slow growth in the number of teachers employed by local authorities. Civil servants, local government officials and school teachers form still the three largest homogeneous groups within the salaried class, but in 1931 they were less than one quarter of all salaried persons. 79 per cent. of all salaried males in 1931 and 72 per cent. of all salaried females were employed in private industry.

B. *Earnings of salaried persons in* 1938

The first enquiry undertaken by the Economic Research Section was to collect details of the average earnings of persons in each of the nine categories into which we had divided the salaried class in 1938. It was possible to obtain information directly covering the majority of civil servants and teachers, but for the remaining seven categories it was necessary to ask a large number of firms and institutions if they would give details of salaries paid to their technical, administrative and clerical staff in the calendar year 1938. They were asked to state :—

(*a*) The numbers of males 21 and over, males under 21, females 21 and over and females under 21, receiving salaries classified according to the amounts of salaries they were paid in the last financial year.

(*b*) The total salaries paid (without deductions for superannuation or for unemployment and health insurance contributions) in the last financial year to males and females in each salary group.

Excluding the information obtained for civil servants and teachers returns were received from firms and public bodies who employed 308,000 salaried persons and paid £80 millions in salaries in 1938. Most of the returns received related to the 12 months ending December 31, 1938. The returns covered approximately 11 per cent. of the total number of salaried persons (excluding civil servants and teachers) in Great Britain in that year.

TABLE II

Percentage distribution of numbers of salaried persons (excluding teachers and civil servants) in Great Britain in 1938 *according to salaries received*

(Percentages)

Range of Annual Salaries	Weighted			Unweighted		
	Total	Males	Females	Total	Males	Females
Total	100·0	100·0	100·0	100·0	100·0	100·0
Up to £40	2·2	1·3	4·7	1·5	1·0	3·0
£40–59	4·6	2·9	9·5	3·5	2·4	6·4
£60–89	9·3	5·7	19·1	8·4	5·8	15·8
£90–125	11·4	5·0	28·1	10·8	5·2	26·4
£126–162	10·0	6·6	18·3	9·6	6·4	18·8
£163–250	21·8	22·9	16·2	22·3	21·8	23·6
Over £250	40·8	55·5	4·1	43·9	57·4	6·1

The unweighted percentages given in Table II were calculated from the totals of all the returns received. Since, however, the number of persons covered by the returns was not the same proportion for each class of salaried persons, the percentages for each category were combined by weighting the number of persons in each category. The percentage distribution of salaries as weighted in this way is not very different from the unweighted distribution.

TABLE III

Percentage distribution of numbers of salaried persons classified by amount of salaries received

(Percentages)

Sex and Age of Salaried Persons	Range of Annual Salaries	Private Industry					Public Administration
		Manufacturing Industry	Distribution	Finance	Transport	Professions	Local Authorities
Males, 21 and over	Total	100·0	100·0	100·0	100·0	100·0	100·0
	Up to £40	—	—	—	—	—	0·3
	£40–59	0·1	0·1	—	0·1	—	0·3
	£60–89	0·4	0·8	0·5	0·2	1·2	0·8
	£90–125	3·2	8·5	2·4	1·3	7·5	3·1
	£126–162	9·0	20·0	3·7	5·5	18·4	8·9
	£163–250	33·1	30·5	11·7	29·5	34·5	34·4
	Over £250	54·2	40·2	81·7	63·6	38·5	52·3
Males, under 21	Total	100·0	100·0	100·0	100·0	100·0	100·0
	Up to £40	16·1	19·4	2·8	15·4	22·2	5·6
	£40–59	31·3	27·2	10·7	19·7	27·8	21·0
	£60–89	37·7	37·1	53·3	24·5	33·3	45·6
	£90–125	12·3	13·8	29·9	39·9	16·7	25·2
	£126–162	1·3	2·5	3·4	—	—	2·5
	£163–250	0·6	—	—	0·5	—	0·2
	Over £250	0·7	—	—	—	—	—
Females, 21 and over	Total	100·0	100·0	100·0	100·0	100·0	100·0
	Up to £40	0·1	2·0	—	—	1·3	0·7
	£40–59	0·6	1·8	—	0·3	—	1·0
	£60–89	9·4	9·0	2·3	1·4	5·3	3·8
	£90–125	43·5	32·6	18·6	20·3	21·7	21·4
	£126–162	29·3	27·2	23·8	22·0	29·0	25·4
	£163–250	14·5	20·0	45·0	54·2	32·2	35·8
	Over £250	2·7	7·5	10·3	1·8	10·5	11·9
Females, under 21	Total	100·0	100·0	100·0	100·0	100·0	100·0
	Up to £40	15·8	15·4	0·7	5·6	17·0	8·0
	£40–59	31·9	26·0	4·5	11·1	23·4	21·8
	£60–89	43·0	39·3	42·3	30·6	27·7	48·2
	£90–125	9·0	18·9	51·5	52·8	23·4	21·3
	£126–162	0·3	0·4	0·9	—	8·5	0·7
	£163–250	—	—	0·2	—	—	—
	Over £250	—	—	—	—	—	—

Table III does, however, show some noticeable differences in the ranges of salaries paid in different branches of industry.

The details of salaries paid to teachers and to civil servants were not received in a form which allowed them to be incorporated in Tables II or III. 85 per cent. of male school-teachers employed by local authorities received salaries in 1938 of more than £250 a year, 13½ per cent. received between £200 and £250, and 1½ per cent. less than £200. Corresponding percentages for female teachers were 54½ per cent. more than £250, 30 per cent. between £200 and £250, 15½ per cent. less than £200. The inclusion of both these classes in Table II would have raised substantially the number of males and females in the higher salary groups. Although detailed figures are not available, about 30 per cent. of salaried male civil servants received salaries of more than £250 a year in 1938, 60 per cent. between £125 and £250, and 10 per cent. less than £125. For females in non-manual occupations in Government employment the corresponding figures were about 8 per cent. more than £250, 53 per cent. between £125 and £250 and 42 per cent. less than £125.

C. *Changes in rates of salaries*, 1924–39

The second enquiry was undertaken to see how far rates of salaries had changed between 1924 and 1939 and to provide the information needed for compiling an index number of changes in salary rates. All the firms, institutions and local authorities covered by the first enquiry were again approached and most of them were kind enough to give information. In addition, other firms who had not been asked to assist in the first enquiry gave details of changes in their rates of salaries since 1924. Altogether information was received covering 550,000 persons or 17 per cent. of the salaried class in 1938. Details of changes in the salary rates of civil servants and of teachers were obtained mainly from official sources.*

Employers were asked to give details not of special increases or decreases made in salaries of individuals, but only of general changes in salary rates which affected all or sections of their salaried staffs. Since, however, many small firms employ only a few salaried persons and since even large firms may have no organized scale of salaries, any index number of changes in salary rates is necessarily less trustworthy than an index number of changes in wage rates.

For the returns received, the changes in salary rates for each firm were calculated from 1924 to 1939, expressed as percentages of salary rates paid at December 1924. The percentage changes in rates for different firms were then combined by weighting by the

* Reports of the Royal Commission on the Civil Service, 1929–31 (Cmd. 3989) and of the Committee on National Expenditure (Cmd. 3920).

salary bills of the firms in 1938. Separate index numbers were thus obtained of the changes in salary rates for each category of salaried persons and they are shown in Appendix I; the general index numbers were reached by weighting the percentages for each category by the estimated total salaries paid to all persons in each category in 1938.

D. *Earnings of shop assistants*

In 1931 the number of shop assistants in Great Britain was 975,000, of whom slightly more than half were males; the number is estimated to have increased to at least 1,050,000 in 1938. It was decided to collect figures of the earnings of this important group of operatives and of changes in their rates of earnings since 1924 at the same time as the two enquiries described above, since figures for shop assistants were not collected for the wage enquiries of the Ministry of Labour. The returns received covered 125,000 shop assistants who were paid £14¾ million in 1938.

TABLE IV

Percentage distribution of numbers of shop assistants in Great Britain according to earnings received in 1938

(Percentages)

Range of Earnings	Total	Males		Females	
		21 and over	Under 21	21 and over	Under 21
Total	100·0	100·0	100·0	100·0	100·0
Up to £40	10·4	0·1	31·4	0·3	21·9
£40–59	12·6	0·1	31·7	0·9	36·0
£60–89	17·9	0·3	24·1	29·5	37·4
£90–125	18·3	5·8	11·0	55·5	4·5
£126–162	12·2	23·7	1·6	10·2	0·1
£163–250	25·4	62·1	0·1	3·1	—
Over £250	3·3	8·0	—	0·6	—

E. *A general index of rates of wages and salaries*

The information given in this paper gives the opportunity for the first time of compiling a general index number of changes in rates of wages and salaries for Great Britain since 1924. Professor Bowley has prepared index numbers of wage rates from 1924, and it was possible to combine his figures with the index numbers of changes of salaries and of rates of wages of shop assistants given in this paper to obtain a general index. The three constituent series of the general index were combined by weighting according to the

estimated totals of wages and of salaries paid in 1938. The general index of rates of wages and salaries given in Table V thus covers all occupied persons, excluding employers, persons working "on their own account," commercial travellers or persons working mainly on commission and members of the armed forces.

TABLE V

Index numbers of changes in rates of wages and salaries in Great Britain

(Percentages)

		TOTAL	Wage Rates (excluding Shop Assistants)	Wage Rates of Shop Assistants	Salary Rates
1924	Dec.	100·0	100·0	100·0	100·0
1925	Dec.	100·4	100·6	100·0	100·0
1926	Dec.	100·7	101·2	100·0	99·7
1927	Dec.	100·3	100·5	100·0	99·8
1928	Dec.	99·5	99·4	100·0	99·6
1929	Dec.	99·1	98·8	100·0	99·7
1930	Dec.	98·5	98·2	100·0	99·0
1931	June	97·4	97·1	100·0	98·2
	Dec.	96·2	96·5	99·7	95·6
1932	June	95·4	95·4	99·7	95·4
	Dec.	94·9	94·6	99·7	95·5
1933	June	94·5	94·0	99·7	95·6
	Dec.	94·6	94·0	99·7	95·8
1934	June	94·7	94·0	99·7	96·0
	Dec.	95·1	94·3	99·7	96·8
1935	Dec.	96·3	95·7	99·7	97·6
1936	Dec.	98·1	98·1	100·3	98·1
1937	Dec.	101·7	103·1	102·8	98·7
1938	Dec.	102·9	104·7	102·8	98·9
1939	June	103·5	105·5	103·7	99·0

F. *Conclusions*

The preliminary results obtained from the two enquiries described in this paper may be conveniently summarized:—

(*a*) The salaried class has been growing during the last 30 years. In 1938 one out of every seven occupied persons in Great Britain was engaged in non-manual occupations.

(*b*) About 55½ per cent. of males and 4 per cent. of females in salaried occupations (excluding teachers and civil servants) received salaries more than £250 a year. 15 per cent. of the males and 61½ per cent. of the females in salaried occupations received less than £125 a year. Total salaries paid amounted to between one-sixth and one-fifth of the total personal incomes in Great Britain in 1938.

(*c*) Rates of salaries did not fall so much as rates of wages in the depression of 1929–33. Since salaries of important groups of non-manual workers changed between 1924 and 1939 according to the movements of the Ministry of Labour cost-of-living index number, there was a greater decline in the salary rates between 1924 and 1928 and a slower recovery in salary rates than in wage rates between 1934 and 1939.

APPENDIX I

Index numbers of changes in rates of salaries in Great Britain, 1924–39

			Private Industry						Public Administration		
		TOTAL	Manu-facturing Industry	Distribution	Finance	Transport	Personal Service	Professions	Civil Servants	Local Authorities	Teachers
1924	Dec.	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0
1925	Dec.	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0
1926	Dec.	99·7	99·9	100·0	99·9	100·0	100·0	100·0	98·1	98·9	100·0
1927	Dec.	99·8	99·9	100·0	100·5	100·0	100·0	100·0	98·1	99·0	100·0
1928	Dec.	99·6	99·8	100·1	100·4	100·0	100·0	100·0	96·1	98·3	100·0
1929	Dec.	99·7	99·8	100·2	100·4	100·1	100·0	100·0	98·1	98·4	100·0
1930	Dec.	99·0	99·5	100·2	97·6	100·1	100·0	100·0	96·1	97·7	100·0
1931	June	98·2	98·9	100·2	96·5	99·5	96·9	100·0	92·2	97·7	100·0
	Dec.	95·6	95·5	98·7	95·3	99·5	96·9	98·4	90·3	95·6	90·0
1932	June	95·4	95·1	98·7	95·2	99·1	96·9	98·4	90·3	95·8	90·0
	Dec.	95·5	95·0	98·7	95·0	99·1	100·0	98·4	90·3	95·1	90·0
1933	June	95·6	95·1	98·7	95·3	99·1	100·0	98·4	90·3	95·3	90·0
	Dec.	95·8	95·2	99·0	95·3	99·1	100·0	98·4	90·3	96·8	90·0
1934	June	96·0	96·0	99·0	95·3	99·5	100·0	98·4	90·3	97·3	90·0
	Dec.	96·8	96·9	99·1	95·3	99·6	100·0	98·4	91·2	97·5	95·0
1935	Dec.	97·6	97·9	99·1	95·3	99·7	100·0	100·0	92·2	98·0	100·0
1936	Dec.	98·1	98·6	99·9	95·4	99·9	100·0	100·0	92·2	98·1	100·0
1937	Dec.	98·7	99·9	101·1	95·6	100·2	100·0	100·0	92·2	98·8	100·0
1938	Dec.	98·9	100·1	101·6	95·8	100·2	100·0	100·0	92·2	98·9	100·0
1939	June	99·0	100·3	102·1	95·9	100·2	100·0	100·0	92·2	99·0	100·0

Note by the President

Since Mr. Campion's Paper and my Address are part of the same larger investigation and to some extent cover the same ground, it will be convenient to readers to have their relation to each other explained. While Mr. Campion is solely concerned with salaries, I was concerned on the one hand with the whole of Intermediate or "Residual" Income, but on the other only with that part of the salary aggregate which was below the exemption limit.

Consequently the statistics in my Table VII do not correspond very closely with any given by Mr. Campion and Miss Marley. The range of mine is somewhat greater, because some estimates were included for industrial groups for which there were no returns in the questionnaires, and in some cases (other than agriculture) there were small additions to the aggregate income under £125 for small employers. But the main difference is that Mr. Booker (whose detailed work I incorporated) took, as the totals to which the computed percentages were applied, the entries in the Population Census that included employers, while Mr. Campion's totals excluded them. Thus the total number of salaried males in April 1931 is given by him as 1,775,000 for Great Britain (p. 525), while in Table VII (which includes North Ireland) the total of Employers, managers and non-manual operatives for the same industrial groups is 2,430,000, of whom 1,578,000 are operatives and 852,000 employers or managers. For females, on the other hand, Mr. Campion's total is 1,125,000 for Great Britain, while in Table VII the corresponding total is 1,046,000 for the United Kingdom, including 136,000 employers and managers. These discrepancies are due to some differences of definition and to some doubt about the interpretation of the questionnaire on the part of those who filled it up.

In the industrial groups included in both estimates, Mr. Campion's total for salaries below £125 p.a. would be about £70 mn., while mine for all occupied non-manually (except persons working on their own account) is £77 mn., of which £2 mn. is attributable to North Ireland. So that the final discrepancy in the aggregate of these incomes is relatively small.

MISCELLANEA

Recent Advances in Mathematical Statistics

Bibliography of Mathematical Statistics (1939)

By H. O. Hartley, Ph.D.

The list of papers which follows is part of a series of bibliographies of papers on mathematical statistics which the Society is publishing at approximately annual intervals. The present list gives principally papers published during 1939. A few publications of an earlier date have come to our notice since the last bibliography was published and these have been included in the present list.

Because of difficulties of communications between various countries in war-time, a bibliography of this kind is, at present, of particular value to those engaged in research. At the same time its compilation has become a more difficult task. Firstly, certain journals are now practically unobtainable. (Fortunately the 1939 issues of most journals are complete, but the difficulty will be considerable when we have to deal with publications in 1940.) Moreover, the facilities offered by many scientific Libraries (particularly London Libraries) have had to be reduced considerably owing to emergency measures. But for the kind co-operation of Dr. J. O. Irwin, Mr. D. N. Lawley and Mr. D. J. Finney, it would have been difficult to aim at a completeness comparable with that of the two preceding bibliographies.

As far as scope and arrangement of this issue are concerned, the principles laid down previously have been retained.

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III. APPLICATIONS

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REPORT OF THE COUNCIL

For the FINANCIAL YEAR *ended December* 31*st*, 1939, *and for the* SESSIONAL YEAR *ending June* 18*th*, 1940, *presented at the* ONE HUNDRED AND SIXTH ANNUAL GENERAL MEETING *of the* ROYAL STATISTICAL SOCIETY, *held in the Hall of the Royal Society of Arts, John Street, Adelphi, W.C.*2, *on June* 18*th*, 1940.

THE Council have the honour to submit their One Hundred and Sixth Annual Report.

The roll of Fellows on December 31st, 1939, as compared with the average of the previous ten years, was as follows :—

Particulars.	1938.	Average of the previous Ten Years.
Number of Fellows at end of previous year ...	1083	1053
Number lost by death, withdrawal, or default ...	39	59
Fellows elected or restored to the list	64	60
Number of Fellows on December 31st	1108	1054

In addition, there were 12 Honorary Fellows.

The Council regret to report that during the sessional year ended June 18th, 1940, the Society lost by death the undermentioned Fellows :—

Fellows

	Date of Election.
*Barnes, J. Howard, F.I.A.	1887
*Higgs, Henry, C.B., LL.B.	1892
*d*Hobson, John A., M.A.	1904
Lewis, Sir Alfred E.	1916
Martin, Albert Victor	1936
*d*Pearsall, C. W., M.A.	1921
Poulton, Olive Elizabeth, B.Sc.	1937
Putnam, G. E.	1922
Sykes, Arthur A.	1917
Watson, William	1919
*d*Wickens, Charles Henry, C.S.O., F.I.A.	1918

Honorary Fellow

Colson, Clément Léon	1929

* Life Fellow. *d* Donor to the Library.

Among the ten Ordinary Fellows lost to the Society by death the most eminent in statistics was Mr. C. H. Wickens, Statistician to the Australian Commonwealth from 1922 to 1931, when he was forced to retire through ill health, largely the result of his strenuous labours. He remained a Fellow until his death last year. An obituary notice appeared in the *Journal*, Part IV, 1939. The Council also especially regrets the loss of two notable personalities—Mr. J. A. Hobson and Sir Alfred Lewis. Both had been associated with the Society for long periods, though they did not participate actively in its work. By the death of Mr. J. Howard Barnes the Society has lost one of its senior Fellows, while Miss Olive Poulton was one of the youngest recruits; her life was regrettably cut short by an accident. Mr. G. E. Putnam, of Messrs. Swift & Co., was an American well known to a number of Fellows when he lived in London, and his premature death is greatly regretted.

The death of Mr. Henry Higgs, some time of H.M. Treasury and well known as an economist, occurred after this Report was in print, and space can be given only for this brief announcement, with which must be expressed the deep regret of the Council, whose colleague he was in the past.

The Council has also to record the loss of an Honorary Fellow, Monsieur C. L. Colson, Vice-President of the French Conseil Supérieur de Statistique, and a member of the International Statistical Institute, who was elected to the Society in 1929. A biographical notice was published in the *Journal*, 1939, Part IV.

During the session 1939–40, the following 57 candidates have been elected Fellows of the Society :—

Abrahamson, George, Dr. Oec.
Barnes, Harry Ryde, B.Sc.
Bauer, George Albert, B.A.
Bhargova, Math Nath.
Bose, Mrs. Chameli.
Bowles, W. Reed.
Brightley, Harold Edward.
Calvert, Neil Geoffrey, A.I.A.
Caplen, Alfred Charles Edward.
Chevis, Annie Henrietta.
Coe, Leonard Dodworth.
Curphey, Noel.
D'Abreo, Alfred Rudolph.
Daniel, Goronwy Hopcyn, D.Ph.
Drury, Rev. William.
Edmonds, Stanley Harold George.
Evison, Gordon Quested, B.Com.
Foster, Wilfred John.
Gambrell, H. T. E., A.C.W.A.
Ganguli, Durgadas, B.Sc.
Goshawke, Walter John, A.I.A.
Hamilton, John Hyslop.
Hamilton-Russell, D.H.G.
Harris, Sydney Charles, B.Com.
Henry, Harry, B.Sc.
Heywood, Geoffrey, F.F.A.
Johnson, Arthur Hazel Lionel.
Khan, Ahmad, Ph.D.
King, Wilfred Ernest.
Kuipers, John Dennis, B.A.
Lander, Maxwell.
Liepmann, Kaethe K.
Loveday, W. H. N.
MacDougall, G. D. A.

MacGarvey, Charles J.
Mills, Vicente.
O'Brien, Cyril Cornelius.
Partridge, Ernest John.
Peat, Henry.
Poornapregna, V. N., M.A.
Ross, Norman William, F.I.A.
Rubin, I.
Rudd, Martin.
Schellenberg-Orloff, Professor M.
Scott, Jesse.
Sengupta, Dhrubajyoti, M.A.
Sinclair, Thomas Hilary.
Singh, Jaggit, M.A.
Silva, David Montague de, M.R.C.S., L.R.C.P., D.P.H.
Spitzer, Erika Anita, M.Sc.
Thompson, Clarence Reginald, A.I.M.T.A.
Wall, John Edward.

Corporate Representatives.

Ching, Joan.	*representing* Messrs. Buckmaster & Moore.
Lythgoe, James.	*representing* The Corporation of Manchester, in place of Mr. J. E. Bray, retired.
Neubert, John William.	*representing* The International Sugar Council.
Turner, Arthur James, M.A., D.Sc.	*representing* The Linen Industry Research Association.
Urmston, James,	*representing* Callender's Cable and Construction Company, Limited.

The number of Fellows is now 1,079, compared with 1,083 in June 1939.

The long-established tradition by which the Society holds a meeting in each month of the Session was broken in 1939–40 on account of the war. Darkness and the traffic difficulties of the early months, together with the scattering of Council members and other Fellows, many of whom were engaged on work for the Government, and the uncertainty of things in general, rendered it impracticable to make plans in advance. The Council, therefore, resolved to suspend the Ordinary Meetings for a few months and, in fact, the first was held on April 16th, 1940, when Professor J. Harry Jones opened a discussion on the Report of the Royal Commission on the Distribution of the Industrial Population. The meeting was well attended and the Council decided to hold the May and June meetings also, if practicable. On May 21st Mr. H. W. Macrosty, O.B.E., read a paper on "British Overseas Trade, 1930–39," and on June 18th, the President delivered his Valedictory Address, the subject of which was "Some Constituents of the National Income." It was decided to continue the publication of the *Journal* as in peace-time and to invite the submission of full-length papers, which, on acceptance according to the usual conditions, should be printed in the *Journal* in the same way as papers read before the Society, followed by a discussion in the form of written comments from selected persons. The list of these papers is as follows :—

BARTLETT, M. S., D.Sc. The Present Position of Mathematical Statistics.
MACDOUGALL, G. D. A. Inter-War Population Changes in Town and Country.
EDELBERG, VICTOR, Ph.D. Flexibility of the Yield of Taxation—Some Econometric Investigations.
ALLEN, R. G. D. The Unemployment Position at the Beginning of the War.
GREBENIK, EUGENE. Some Aspects of Population in Bristol.

The suspension of the Ordinary Meetings made it impossible, in view of the Bye Laws, to elect candidates for Fellowship until April. There were, however, a number of candidates for admission last autumn, and the Council, whose meetings were also suspended during the winter, authorized the Executive Committee, who were empowered by them to administer the affairs of the Society meanwhile, "to grant to all candidates of whom they approve, pending election at an Ordinary Meeting, all privileges of Fellowship other than those which, by the Bye-Laws, can be enjoyed only after formal election." This meant that the candidates elect were enabled to receive the *Journal* and use the Library although they had no right as yet to the use of the letters F.S.S.

The names of forty-eight candidates approved by the Executive Committee to the months October–March were submitted to the April meeting and all were duly elected.

The meetings of the Industrial and Agricultural Research Section also had to be suspended for an indefinite period, chiefly because most of the members were absent from London, many of them in Government departments. It is intended, however, to hold a meeting on June 20th, at which Mr. E. C. Fieller will read a paper on "The Bioligical Standardisation of Insulin." The Supplement is to be issued in two parts as before, but the publication is likely to be a good deal delayed.

The Study Group has not been able to hold any meetings. The war occupations of most members has absorbed their whole time, but the Group hopes to resume its activities in less strenuous days.

In the year ended May 31st, 1940, 1,315 works were added to the Library, compared with 1,750 the year before. These figures exclude periodicals regularly received and a number of Parliamentary Papers. During the same period 2,112 volumes were borrowed by 772 Fellows, against 1,717 by 786 Fellows the year before. As in 1914–18, the Society's Library has been extensively used by various Government Departments.

The Council have awarded a Guy Medal in Silver to Mr. H. Leak, for his paper "The Carrying Trade of British Shipping," read before

the Society in February 1939, and published in Part II of the *Journal* for 1939.

The Frances Wood Memorial Prize, value £30, offered for competition in 1939, was awarded in February 1940, to Dr. H. W. Singer, for his essay, "The Process of Unemployment, Statistically Analysed."

In last year's Report it was announced that the Council was co-operating with the Royal Society "in the preparation of lists of Fellows qualified to undertake scientific research of a statistical nature and to carry out other statistical work." The Council can now report that, in addition to those who are members of the Civil Service, a considerable number of Fellows, both members of the Council and others, are engaged in various forms of "war work" in many Departments of the Government.

The Council are confident that all fellows of the Society will learn with pleasure that the retiring Honorary Secretary, Mr. H. W. Macrosty, O.B.E., has accepted the unanimous invitation of his colleagues on the Council to allow his name to be put in nomination for the office of President. Mr. Macrosty was elected into the Society thirty-six years ago, first served on the Council in 1916–17 and has been an honorary secretary since 1928. He has contributed a number of valuable papers to the Society's *Journal* and was the author of the history of the Society published in connection with the centenary. Since his retirement from the public service, Mr. Macrosty has devoted himself to the service of the Society with an energy which his colleagues cannot too gratefully acknowledge, and it is largely due to his efforts that, in these recent difficult years, the Council has been able to carry on the work of the Society.

The abstract of the Treasurer's Accounts, viz., a Statement of Income and Expenditure for the year 1939 and the Balance Sheet as at December 31st, 1939, together with the report of the Auditors thereon, are given in Appendices A and B respectively.

There was an excess of Income over Expenditure for the year 1939 amounting to £241, as compared with an excess of £128 in 1938. Ordinary Income, at £3,261 fell by £242; this was mainly due to a fall of £202 in the income from the sale of the Society's publications, which was to be expected in prevailing conditions, and it is satisfactory to note that Fellows' subscriptions decreased by only £21. Ordinary Expenditure, at £3,020, fell by £356, mainly owing to decreases of £65 in Salaries and Wages and of £187 in Publication and Distribution Expenses. The Accumulated Fund of the Society amounted to £7,441 at the end of 1939.

The Fellows named below (nominated in accordance with Bye-law 14) are recommended for election as President, Council, and Officers of the Society for the Session 1940–41 :—

President

Henry William Macrosty, O.B.E., B.A.

Council

*R. G. D. Allen	Noel F. Hall.
Sir Percy Ashley, K.B.E., C.B.	David Heron, D.Sc.
M. S. Bartlett, D.Sc.,	A. Bradford Hill, D.Sc.
W. A. Basham, O.B.E.	J. O. Irwin, Sc.D., D.Sc.
*M. S. Birkett, O.B.E.	Leon Isserlis, D.Sc.
*Lt.-Col. William Butler, M.B.	Professor J. H. Jones.
A. M. Carr-Saunders.	*M. G. Kendall.
*Iris Douglas.	H. Leak.
Major P. Granville Edge, O.B.E.	George Rae, D.Sc.
Sir W. Palin Elderton, C.B.E.,F.I.A.	E. C. Rhodes, D.Sc.
*G. W. S. Epps, C.B., C.B.E., F.I.A.	E. C. Snow, C.B.E., D.Sc.
Dorothy P. Etlinger.	Percy Stocks, M.D.
C. O. George, Ph.D.	Sir Sylvanus Vivian, C.B.
R. F. George.	A. D. Webb, C.B.E.
Sir Gwilym Gibbon, C.B., D.Sc.	John Wishart, D.Sc.

Those marked * were not Members of Council during the preceding Session.

Honorary Treasurer

David Heron, D.Sc.

Honorary Secretaries

E. C. Snow, C.B.E., D.Sc. Leon Isserlis, D.Sc.
A. Bradford Hill, D.Sc., Ph.D.

Honorary Foreign Secretary

E. C. Snow, C.B.E., D.Sc.

Signed on behalf of the Council,
ARTHUR L. BOWLEY,
President.

H. W. MACROSTY,
E. C. SNOW,
L. ISSERLIS,
Hon. Secretaries.

June 18*th*, 1940.

APPENDICES

APPEN

STATEMENT OF INCOME AND EXPENDITURE

EXPENDITURE.

1938 £ s. d.	*1938* £ s. d.		1939. £ s. d.	1939. £ s. d.
	380 0 0	Rent		380 0 0
	132 13 4	House Expenses		137 1 10
	924 8 11	Salaries and Wages (including contribution to Staff Superannuation Scheme)		859 3 9
	217 12 1	Pension and Allowance ...		199 5 0
		Meetings :—		
128 17 4		Ordinary and General ...	96 19 9	
49 13 2		Research Section	30 11 7	
3 14 5		Study Group	1 4 3	
	182 4 11			128 15 7
		Publication and Distribution Expenses :—		
971 17 4		Journal and Reprints ...	764 7 6	
170 13 11		Supplement	190 7 8	
	1,142 11 3			954 15 2
		Library :—		
52 7 5		Books	46 6 6	
100 12 6		Binding	100 1 4	
	152 19 11			146 7 10
	15 13 1	Furniture and Office Equipment		15 0 0
	20 13 7	Insurance		20 10 3
	129 9 8	Stationery and Miscellaneous Printing		89 4 4
	69 2 6	Postage and Telephone ...		62 3 2
	4 5 0	Guy Medal		2 15 0
	4 7 10	Miscellaneous Items		25 4 2
	3,376 2 1			3,020 6 1
	128 0 3	Balance carried to Accumulated Fund: Excess of Income over Expenditure for the year		241 6 3
	3,504 2 4			3,261 12 4
	105 0 0	Amount carried to Life Composition Fund		57 15 0
	£3,609 2 4			3,319 7 4

DIX A.

FOR THE YEAR ENDED 31ST DECEMBER, 1939.

INCOME.

1938. £	s.	d.	£	s.	d.		1939. £	s.	d.	£	s.	d.
						Annual Subscriptions :—						
1,614	*18*	*0*				In respect of current year ...	1,591	16	0			
107	*2*	*0*				Arrears	109	4	0			
			1,722	*0*	*0*					1,701	0	0
			3	*0*	*0*	Study Group Subscriptions ...				3	10	0
			20	*10*	*0*	Special Subscription and Donation				11	11	0
			1,013	*11*	*10*	Sales of Journal...				911	15	3
			77	*16*	*4*	Sales of Journal Reprints ...				8	1	5
			16	*12*	*6*	Journal Advertisements ...				16	6	0
			150	*0*	*9*	Sales of Supplements				119	6	6
			2	*2*	*3*	Sales of other Publications ...				2	2	7
			50	*0*	*0*	Contribution from Royal Economic Society				50	0	0
			447	*14*	*11*	Dividends and Interest (gross)				436	2	2
				13	*9*	Miscellaneous Items				1	17	5
			3,504	*2*	*4*					3,261	12	4
			105	*0*	*0*	Life Compositions				57	15	0
			£3,609	*2*	*4*					£3,319	7	4

APPEN

BALANCE SHEET AT

LIABILITIES.

1938. £ *s. d.*	*1938.* £ *s. d.*		1939. £ *s. d.*	1939. £ *s. d.*
	69 6 0	Annual Subscriptions received in advance		46 4 0
	131 19 2	Payments for Journal and Supplement received in advance		48 3 1
	237 7 2	Sundry Creditors		139 9 2
		Life Composition Fund :—		
2,058 0 0		Balance at December 31st, 1938...	2,163 0 0	
105 0 0		*Add :* Life Composition Fees received in 1939	57 15 0	
	2,163 0 0			2,220 15 0
		Accumulated Fund of the Society :—		
7,071 19 8		Balance at December 31st, 1938, per last Accounts ...	7,199 19 11	
128 0 3		*Add :* Excess of Income over Expenditure for the year, per annexed Statement ...	241 6 3	
	7,199 19 11			7,441 6 2
	9,801 12 3			9,895 17 5
		Building Fund (per contra) :—		
819 18 2		Balance at December 31st, 1938...	850 5 8	
30 7 6		*Add :* Income for year 1939	33 1 2	
	850 5 8			883 6 10
		Frances Wood Memorial Fund (per contra) :—		
331 15 2		Balance at December 31st, 1938...	356 9 10	
24 14 8		*Add :* Income for year 1939	25 13 7	
	356 9 10			382 3 5
	£11,008 7 9			**£11,161 7 8**

–No value is placed in the Accounts on (1) Journals and other Publications in stock, (2) Books in Library, and (3) Pictures, Furniture and Equipment.

DIX B.

31st DECEMBER, 1939.

ASSETS.

1938. £ s. d.	*1938.* £ s. d.		1939. £ s. d.	1939. £ s. d.
		Investments, at cost or under :—		
5,580 0 0		£10,527 12*s.* 3*d.* 2½% Consols (Guy Bequest)	5,580 0 0	
1,185 0 0		£2,236 11*s.* 3*d.* 2½% Consols	1,185 0 0	
1,299 0 0		£1,841 3½% Conversion Loan	1,299 0 0	
490 0 0		£500 3½% War Loan ...	490 0 0	
800 0 0		£1,169 17*s.* 6*d.* 3% Local Loans Stock	800 0 0	
100 0 0		£666 4% 2nd Prefd. Stock, London and North-Eastern Railway	100 0 0	
25 0 0		£266 5% Prefd. Ord. Stock, London and North-Eastern Railway	25 0 0	
	9,479 0 0			9,479 0 0
		(Market value, December 31st, 1939, less Interest accrued, £11,966)		
	101 19 9	Interest accrued on Investments (gross)		101 19 9
	49 2 11	Cash at Bank and in hand ...		160 5 1
	84 0 0	Arrears of Subscriptions recoverable (estimated) ...		84 0 0
	87 9 7	Sundry Debtors		70 12 7
	9,801 12 3			9,895 17 5
		Building Fund :—		
	850 5 8	£972 4*s.* 5*d.* 3½% Conversion Loan		883 6 10
		(Market value, December 31st, 1939, £913)		
		Frances Wood Memorial Fund :—		
300 0 0		£600 4% Preference Stock, London, Midland and Scottish Railway	300 0 0	
		(Market value, December 31st, 1939, £357)		
56 9 10		Post Office Savings Bank Deposit	82 3 5	
	356 9 10			382 3 5
	£11,008 7 9			£11,161 7 8

REPORT OF THE AUDITORS.

We have examined the foregoing Statement of Income and Expenditure and Balance Sheet with the Books and Records of the Society. We have verified the Investments and Cash appearing in the Balance Sheet. We report that the above Balance Sheet is, in our opinion, properly drawn up so as to exhibit a true and correct view of the state of the affairs of the Society, according to the best of our information and the explanations given to us, and as shown by the Books and Records.

PLENDER *Chartered Accountant.* Auditor.
C. OSWALD GEORGE } Honorary
HERBERT W. ROBINSON } Auditors.

PROCEEDINGS OF THE ONE HUNDRED AND SIXTH ANNUAL GENERAL MEETING OF THE ROYAL STATISTICAL SOCIETY, HELD IN THE HALL OF THE ROYAL SOCIETY OF ARTS ON TUESDAY, JUNE 18TH, 1940.

The Chair was taken by the President, Professor A. L. BOWLEY, C.B.E., at 5.0 p.m.

The HONORARY SECRETARY read the notice convening the Meeting, and drew attention to the Report of the Council for the financial year 1939 and the Session 1939–40, proofs of which were distributed to those present. He explained that owing to difficulties caused by the war it had not been possible to circulate the Report to all Fellows this year as intended.

The CHAIRMAN moved that the Report of the Council be adopted and printed in the *Journal*; the motion was seconded by THE RT. HON. LORD PLENDER, and, on being put to the vote, was carried unanimously.

The HONORARY SECRETARY announced that in accordance with Bye Law No. 9, the Council had ordered the names of fourteen persons to be erased from the Roll of Fellows of the Society.

A ballot was taken for the election of the President, Council and Officers for the Session 1939–40, Mr. Dudley Walton and Mr. J. H. Hamilton being appointed scrutineers. As a result it was announced that all those nominated had been elected to the several offices.*

A cordial vote of thanks to the President, Honorary Officers and Council for their services during the past Session was proposed by Mr. G. L. Schwartz, seconded by Mr. Verdier and carried unanimously.

The Meeting then adjourned for the Ordinary Meeting.

* See list on p. 566.

REVIEWS OF STATISTICAL AND ECONOMIC BOOKS

CONTENTS

1.—*Statistical Dictionary of Terms and Symbols.* By A. K. Kurtz and H. A. Edgerton. New York: John Wiley, 1939. (London: Chapman and Hall.) 7″ × 4¾″. xiii + 191 pp. *12s.*

Statisticians are often reproached for using a language of their own, unintelligible to the non-professional reader. Although they are somehow inclined to revel in the creation of new terms, statisticians should not be blamed for having developed a specialized terminology. Every branch of science is bound to have professional terms of its own. That those of the statistician have been particularly noted by other scientists is simply due to statistics being used and read by a large number of non-statisticians who apply its results.

"The primary purpose of the 'Statistical Dictionary' is to provide the user with clear and accurate definitions of each of the various meanings of the statistical terms which he encounters in his reading of scientific literature." The first publication of its kind, it will be welcome to statisticians and non-statisticians alike.

The task of compiling such a dictionary is by no means an easy one. Completeness is but an unattainable ideal simply because the terminology of science is in a state of continuous development. Great efforts have been made to achieve its completeness at the date of publication, and an Advisory Council of experts has been consulted. Nevertheless we have to mention here a number of omissions which ought to be remedied in a revised edition. Statistical terms of modern field experimentation, now an essential part of the terminology of the design of experiments in general, have been omitted. No mention is made of terms such as "Latin Squares" and "Randomized Blocks," let alone more specialized designs such as "the split plot lay-out" and "factorial designs" or more modern developments such as "the missing plot technique," "confounding," "partial confounding," etc. Other sections appear to be fairly complete with only occasional omissions, of which we may mention the following: "stochastic variable," "fiducial limit," "discriminant function," "random sampling numbers," "sampling units," "probit," "linkage," "amount of information."

The definitions are as clear and concise as they can possibly be. In a dictionary of this kind it is, of course, difficult, if not impossible,

to draw a line between unknown terms to be defined and known terms which may be used in the definitions. At times the user will find that in the definition of an unknown term other terms, also unknown to him, have been used. These, in turn, have to be looked up in the dictionary, and he may not improbably be led back to his beginning, as, for instance, in the original edition of a celebrated musical dictionary, where the enquirer found, under *Guzla*, see *Rebab*; and under *Rebab*, see *Guzla*. H. O. H.

2.—*The Real Cost of the War*. By J. Keith Horsefield. England: Penguin Books. $7\frac{1}{4} \times 4\frac{1}{2}$. 148 pp. *6d.*

In the medical profession there are few who would tell a patient, about to undergo a major and costly operation, every unpleasant and painful detail of what he has to go through before he can once more hope to return to happy normality. The author under review would probably see little analogy between surgery and public finance. After a rather hurried diagnosis, he tells the patient (taxpayer) about every necessary cut and how terribly deep they will have to be. Other surgeons or consultants, such as Sir Kingsley Wood or Mr. Keynes, who would break the news a little more gently, are denounced as purveyors of dangerous delusions.

Working throughout on 1939 values, and after allowing for air-raid damage (£300 mns.), Mr. Horsefield estimates our 1940 national output at £5,200 mns., of which £1,700 mns. is assumed to be convertible to government use. By adding £650 mns. for possible additions to home-produced national output from longer working hours, more intensive work, and increased employment, £450 mns. for indirect government output, and £800 mns. from the realization of a substantial part of our capital resources, he obtains the £3,600 mns. which, he tells his readers, is the minimum annual rate of expenditure the Government must command by the end of 1940, if we are not to lose the war. This would mean that, after selling over one-third of our capital resources, and in spite of increases in workers' hours and workers, the average consumption per civilian head must be cut down from 36*s.* per week to about 23*s.*

This section of the book, devoted to the effects of the war on the consumer, shows much careful and useful work, but its value lies not so much in the actual estimates—modestly designated the roundest of round figures—as in the analysis of the difficulties and possible misconceptions lying in the path. Perhaps a little more might profitably have been said about the increasing difficulties of applying the money measure in war-time. And some readers, who have been led to believe that the successful prosecution of the war demands the maximization of "convertible output," may be a little puzzled to read that the value of a soldier's services is measured by his pay and allowances, and that as these are usually lower than his civilian earnings, "Convertible Output will be less for each man who joins up than the Output he previously produced."

In the remaining sections of the book, which are devoted to possible and probable effects of the war on the taxpayer, on industry, and on post-war conditions, there is little to provoke serious criticism.

Here and there one finds a tendency, only too prevalent in present-day war comment, to lay too much stress on physical factors and too little on the psychological factors, which, after all, may be ultimately decisive in any war. In the author's eyes, the citizen seems sometimes to be something even lower than *homo œconomicus*—something resembling a human machine giving an unchangeable output of energy in return for a standard input of fuel in the shape of food and other goods. Nowhere does the author pay much attention to the fact that output may be very seriously affected by financial policy. And there is undue emphasis on the importance of present goods when he says that the real cost of the war must be borne here and now, and roundly declares it is useless to pretend that any fiscal shilly-shallying can pass it on to future generations. Fortunately for our war effort, such a narrow conception of "real" cost is not widely held.

And, after all, in public finance, as in every other sphere of governmental activity, the first and all-important aim at the present time is the maximization of our war effort. Occasionally Mr. Horsefield is driven to forget this by an earnest and obviously sincere affection—strangely prevalent among realistic economists—for abstract ethical principles, which have little influence on finance in peace-time, and even less in time of war.

But these lapses are few and far between. For so small a book there is a surprising amount of sound statistics, sound economics, and sound sense. Seldom is the statistician or economist offered such value for sixpence. C. O. G.

3.—*The City: the London Financial Markets.* By Barnard Ellinger, M.A. London: King, 1940. $8\frac{3}{4}'' \times 5\frac{1}{2}''$. xv + 429 pp. 20*s*.

Mr. Ellinger is well known to Fellows by his previous books, *This Money Business* and *Credit and International Trade* (reviewed in the *Journal* for 1933, Part IV, and 1935, Part II, respectively). All will therefore agree that a wise choice was made when the Enquirers' Club of Toynbee Hall, instead of simply publishing a series of lectures on "The City and its Activities" delivered before them in 1935–36, appointed him "to act as *rapporteur* of the meetings and, out of the material he collected there and could supplement by research elsewhere, make a general survey of the City's activities." The result is a solid contribution to our knowledge of the way in which "the City" functioned in the period immediately preceding the outbreak of the present war. The financing of the 1914–18 war and the following years brought great changes, and we must expect that when peace comes changes no less great will be found to be necessary. The degree to which the financial structure of trade and industry may have to be altered can only be determined if we have a clear understanding of the working of our financial organization before the present disturbing factors came into action. Mr. Ellinger has put it in our power to have that understanding, and for that service he has our gratitude.

The book is divided into four approximately equal parts. The first treats of the Bank of England, its organization and its functions

in relation to currency, the Government, bank rate, and open-market operations, and there are three chapters dealing with criticisms, and two with the history of financial crises. The following points may be selected for notice. The "estimated net circulation" of currency in 1935 was about £380 million, of which Mr. Ellinger can only identify part, "probably at the outside a quarter." He therefore rightly urges an investigation into the monetary habits of the people in order to determine the real net active circulation, so that we may be able "to arrive at wiser decisions" on monetary problems, "than would otherwise be the case." Secondly, the full explanation of open-market operations deserves commendation. Dr. Dalton's criticisms of the Bank receive trenchant treatment, and the principal socialist criticisms are shown to have little foundation at a time when the Government has taken over the determination of the main lines of monetary policy, while leaving the day-to-day application to the Bank authorities. Mr. Ellinger distinguishes three phases in the history of major financial crises. "The first embraces the early period during which the Bank seemed mainly concerned with saving its own skin. During the second phase the Bank—usually with Government assistance—in times of crisis directed its operations mainly with its eye on the public welfare. The third phase, beginning with the Baring crisis, was one in which the Bank found means of anticipating and averting panics during times of crisis." He then asks if a fourth phase will follow in which the Bank will prevent crises by "restricting industrial and commercial development neither too early nor too late, but just at the exact moment which will prevent an unhealthy speculative over-expansion from taking place," and stresses the difficulty of determining this happy conjuncture.

The second part treats of the Deposit Banks, with separate chapters on their organization, deposits, assets, advances, investments, etc.—13 chapters in all and 118 pages. After a perusal of this section the reader will feel that he is in a position to understand and appreciate the working of the British banking system, for a great amount of information is brought together which previously was scattered through many books and periodicals. Particularly valuable are the chapters on advances and the chapter which explains the relation between the open-market operations of the Bank of England and the deposits in the joint-stock banks and the development of industry. The chapter on branch banking will probably be "an eye-opener" to most of the users of banks, and the two chapters on "criticisms" lead to the conclusion that most facile criticism is based on ignorance and that some sounder criticism reflects aspirations difficult to realize.

Apart from the banks, there are other institutions which collect savings and make loans to trade and industry. Savings banks, friendly societies, building societies, industrial assurance societies, and other thrift organizations collect small savings and invest them in Government securities, in mortgages, land, and buildings, in municipal loans, and in other securities. Life and other assurance companies similarly invest their funds, but a larger proportion goes

into industrial securities. Investment trust companies aim (or should aim) at making investment safer for the small investor; funds for the purpose of buying securities are collectively subscribed and the dividend income received is distributed. Issuing Houses, "placings," and "stock exchange introductions" "act as intermediaries between savers who desire a long-term investment and borrowers on long term," and the chapters dealing with them and "the scope and limitations of new capital market" will repay study. Mr. Ellinger discusses with much sobriety the allegations of the difficulty which businesses with £150,000 capital or less find in raising funds, pointing out that investors are repelled by the narrow market which small issues have, and suggesting that it is possible that the development of new ideas may best be left to large concerns with research departments of their own. The doubtful profitability of new issues is stressed, and the insufficiently recognized fact that about half the new borrowing of 1937 was estimated by the *Economist* to be under some form of official control is brought into prominence. The Labour Party advocate the formation of an Investment Control Board, which Mr. Ellinger thinks "might be desirable if it were certain that there existed a body of men sufficiently wise to direct investment into industrial channels which would provide the commodities that the public desired rather than the commodities which the Board in its wisdom considered that the public should desire." As it is, the Treasury control exercised through the Bank of England and the Kennet Committee may be sufficiently elastic to ensure the gradual gaining of experience and the adaptation of regulation to changing needs. Meanwhile it is observed that 158 issuing houses have gone out of existence since 1928, and that at present the investor has no means of gauging the repute of an issuing house with which he contemplates doing business similar to the protection he has when selecting a solicitor or an accountant.

The fourth and concluding section treats of "International Trade Finance," and includes chapters on "Foreign, Dominion, and Colonial Banks," "Merchant Banks," "The Foreign Exchange Market," and "The Exchange Equalization Account." Even well-informed persons find such topics rather obscure, but in 74 pages Mr. Ellinger has poured on them a flood of light of so high a concentration that the illumination is almost painful. The intricacy of the transactions involved under those general headings prevents one from giving a summary of the chapters in a form more succinct than our author's account. It may be noted, however, that in his view, "when the world has settled down to restored confidence," the Exchange Equalization Account "may become a permanent feature and solve the problem of divorcing as far as possible the means employed for adjusting the international values of currencies from the means used in contracting or expanding the internal basis of credit." Finally, we may refer to the chapter on "Merchant Banks" as one of peculiar interest, and share with Mr. Ellinger his regret at the possible disappearance of a prominent feature of the money market which the changes of recent years have made more likely.

H. W. M.

4.—*The Economic History of Steelmaking, 1867–1939: a study in competition.* By D. L. Burn. (Cambridge: University Press.) 1940. 8¾″ × 5½″. xi + 548 pp. (with two figures and two maps). 27*s.* 6*d.*

It is a great pity that this book is so expensive, even if the cost has been made greater by causes outside the publishers' control. To say that it fills a gap in the history of steel would give far too weak an idea of the author's achievement, for, with immense industry, he has collected a vast quantity of facts and has built them into an integrated whole. With remarkable insight, he has analysed the policies current at each successive interval of time and has shown how they were determined, or at least influenced, by circumstances that originated in the past, sometimes the remote past. He makes it very evident how difficult it is for an old and established industry to meet new and dangerous competition by adopting the weapons of its rivals. We warmly congratulate the author on having made a notable contribution to economic history.

The poor display made by the British iron and steel manufacturers at the Paris Exhibition of 1867 started a lively controversy as to the alleged "decline upon the largest possible scale" in the British industry, but the doubts were silenced to some extent by a great expansion of railroad-building during the next six years, for railway iron was the dominant component in British iron exports. There were, nevertheless, violent fluctuations, for the competition between iron and steel was great; after 1873 there was a world-wide collapse in capital expansion, and in 1878 the discovery of the basic process was to give a great impulse to Continental steel production. Germany and the United States had the advantage of large and rapidly growing home markets safeguarded from foreign competition, and their steel industries, being new, were not cumbered by obsolete plant and works badly placed. Although small improvements in practice were continuous, criticisms were raised as to the willingness of British ironmasters to adopt with sufficient speed the innovations introduced across the Atlantic. In particular the lack of scientific training on the part of managers and the want of expert chemists were especially noticeable, as they had been in the middle sixties. Even in 1902 Selby Bigge, "the pioneer of change," could write about improvements that "outside England people say, 'What is the saving?' In England the first question is 'What is the cost?'."

The growing fierceness of international competition during the eighties and nineties dethroned Britain from her position as the leading iron-producing country, and when Joseph Chamberlain brought to life again the almost inanimate doctrine of protection, the explanation of the sufferings of the British iron and steel industry was found in the policy of dumping by protected countries. Here Mr. Burn enters into an elaborate analysis of prices and wages, of the advantages of location, of "Britain and the advance of the processes," and of the progress of structural adaptation, which must leave those of us who took part in the controversy, thirty odd years ago, gasping at our temerity. Very briefly he finds that the Continent did benefit by discriminatory prices and by lower wages, and

that the gap between the efficiency of the British workman and the Continental workman had narrowed. But the main disadvantage of the British industry was that it was an old industry which had grown up on the widely scattered ore districts, that small-scale working had thus been imposed on it, that capital resources were insufficient for large-scale reconstruction, and that it was easier to move to the coast and import ores than to develop the imperfectly understood ores of the East Midlands. Long-lived capital equipment made it possible for an obsolete works to continue in competition with efficient works, and the policy of a highly diversified output of the more finished classes of products was adopted so that successively paying lines "carried" the unprofitable ones which were needed to keep the works running full. All those factors interposed difficulties in the way of either vertical or horizontal combination—as the author clearly shows by a comparison with American developments—and the line of progress seemed to be in the emergence of large individual companies which might or might not work more or less in agreement. Even such companies could only adopt a lopsided system of improvements, for their inability to call on the London money market forced them to make changes piecemeal where a wholesale policy of "scrap and build"—and building in a scientific manner—was required. Between 1905 and 1914 "the crucial feature in British structural development was the absence both of radical change in the location of production and of a lessening of the subdivision of production."

Then came the war of 1914–18, when British steelmakers were forced to make "extensive if not radical, structural and technical changes," when knowledge and resources had to be pooled, when the basic process was unreservedly adopted, and when the East Midlands ore resources were systematically explored. Mr. Burn holds that "while the building of the war and early post-war years did little to improve the distribution of iron and steel-making, it raised appreciably the general efficiency of British practice." Though after the close of the war "individualism remained, so to say, rampant," there was some extension of voluntary associations and of the substitution of national for regional associations. There followed the "black decade" of 1921–31, where the industry, crippled already by heavy interest burdens, almost collapsed under the attack of renewed competition, when exports dwindled and imports grew. Regretfully we must pass over the account of this period, with the advice that it should be carefully studied. In 1932 came the Import Duties Act and the recommendation of the Import Duties Advisory Committee that protection should be granted to the iron and steel industry conditional on reorganisation of the industry. "Briefly, they aimed at national and international Kartells free from monopoly evils, plus some degree of national planning." This "new start" gave a fresh life to the industry, which was later strengthened by armament demands, and as the power of the central body of the British Iron and Steel Federation grew under its independent chairman, Sir Andrew Duncan, so more and more of its functions were voluntarily made subject to the supervision of the Import Duties Advisory Committee. These changes did not go unchallenged.

"There has indeed been constant criticism—much of it plainly irrelevant, and some declamatory rather than analytical." Mr. Burn, too, is critical, especially of the slowness with which concentration or translation to the new ore fields proceeded, but his criticisms are fair and moderate and raise questions which we shall have to try to solve, in the circumstances, whatever these may be, in which the industry will find itself when we shall have restored peace to the world. H. W. M.

5.—*Growth of American Manufacturing Areas: A omparative Analysis, with Special Emphasis on Trends in the Pittsburgh District.* By Glenn E. McLaughlin. Bureau of Business Research, University of Pittsburgh. 1938. $9\frac{1}{4}'' \times 6''$. xxvii + 358 pp. \$3.

Although this notice is somewhat belated, attention may still be directed to this book in view of the interest recently shown in this country in problems arising out of the localization of industry. We may quote from the Introduction: "Variations in the rates of growth among different divisions of industry and among individual industries in each of the several divisions reflect large shifts in opportunities for employment and for capital investment. Industries which in terms of employment have grown more rapidly than the total population have had to attract workers from other types of activity or to secure them in increasing proportions from the ranks of new workers. Differential rates of growth have drastically affected the economic importance of most parts of the nation; and no industrial area has remained unaffected." The census of manufactures for the United States furnishes extensive data on manufacturing industries in 33 districts and the author has compiled comparable data for these areas extending back to 1869. After a short general survey of industrial growth in the United States, 167 pages are devoted to "measurement and statistical analysis" under four general headings—"Gainful Workers, 1870–1930," "Population, 1870–1930," "Manufacturing Wage-earners, 1869–1935," and "Value Added by Manufacture, 1869–1935." The data are contained in 41 tables, either in absolute figures, or in index numbers, or in percentages, and 58 charts display them in graphic form.

The remainder of the book deals with the interpretation of the preceding data. Sixty pages set out "the reasons for regional differences in the rate of industrial growth," and as the Director of the Bureau says in his Preface: "Wherever possible, the author has sought to point out the economic, geographic, historical, accidental, and other causal factors responsible for industrial growth and for variations in growth among industrial centres." These discussions, together "with an examination of the growth curves, shed light on prospective trends." And, as the author himself says, his object is "to single out the influences that have had a significant bearing on the rate of industrial growth in important industrial districts during the 60 years ending in 1929, to point out instances of the operation of particular influences, and to evaluate the relative importance of the several tendencies." The factors bearing on development are manifold; they react on one another, and it is often

difficult to determine whether there is a line of causation or only a time sequence. Differences in interpretation of the data must arise, but Mr. McLaughlin has been so careful in his treatment that his conclusions must be treated with respect. Finally, two chapters are given to " population and industrial production in the Pittsburgh district " and to " the position of the Pittsburgh district." His conclusion on the latter point is that in that district " retardation of growth in total industrial production since about 1910 has resulted in the main from the fact that the major local industries are old industries which several decades ago reached their phases of most rapid development and from the additional fact that few new, rapidly expanding industries have become established locally." An upward trend can only be restored again by some revolutionary change in the locally important coal and steel industries, but that is " not very likely to appear in the immediate future," or by the introduction of new industries, and the history of the past 30 years " is presumptive evidence that the district has not been a particularly advantageous location for most of the new forms of manufacturing." Hence, " the weight of the evidence seems to indicate that the future growth of industrial output in the Pittsburgh district will be the slow growth normally characteristic of industrial maturity."

It is a commonplace, and often not a complimentary one, to say of a " heavy " book that it is a " mine of information," but that well-used phrase may be employed here in all honesty, for Mr. McLaughlin does not give his readers unsifted " run-of-the-mine," but facts well arranged into classes and properly docketed according to their degrees of reliability and usefulness. H. W. M.

6.—*International Monetary Economics.* By Michael A. Heilperin. London: Longmans. 1939. xiv + 281 pp. 15*s*.

The book is meant to cover " an investigation of the structure of international monetary relations, an enquiry into the nature of monetary internationalism and into the conditions which make its continued existence possible, and an investigation of the consequences of its destruction and replacement by nationalistic monetary policies."

It is in times like these, when the very fundamentals of the world's political and economic structure are being questioned, that problems which used to evoke endless arguments amongst social scientists suddenly disappear altogether or become mere technicalities of very moderate importance. There is great realism nowadays with regard to fundamentals, and it is a greater handicap than ever for a pre-war book to be judged through the eyes of to-day.

Professor Heilperin has not achieved more than what he set out to do—namely, to make an investigation into monetary internationalism. He concentrated his main efforts on trying to find a solution for an international system where various countries adopt different currency standards and policies. His conclusion was the obvious need for international co-operation in a world where automatic " re-equilibrium " is not, and never has been, a *modus operandi*.

But it is rather his treatment of the subject than the conclusions

reached that is open to criticism. While dealing with various managed and semi-gold currency systems in great detail (and discussing a 100 per cent. gold currency to unbelievable length), he fails to emphasize those basic factors around which the whole problem is situated. It seems to be a pity that the great pillars of the international economic edifice, such as currency and sovereignty, international trade, international indebtedness, and—last but not least—the national-legal aspects of money and the guaranteeing of debts, should be so hidden in a maze of minor technical arguments. For only when grasping the fundamental elements of the problem and grouping various questions around these can the author avoid the critical reader's question : το τί ?

We have reached a stage in monetary management and economic thought when the operations of monetary authorities are no longer regarded as manifestations of some mysterious, natural physical law. Professor Heilperin seems to agree with this view, but his refusal to believe in the supernatural status of gold does not prevent him from embarking upon a lengthy statistical and theoretical argument with Cassel and Rist on the relationship between gold supply and prices. This argument, however brilliant, is slightly out of place and far too long for an empirical law which, on *a priori* grounds, has already been refuted in the book as incorrect.

There are a few excellent chapters on foreign exchanges, balance of payments, and parities, which compare favourably with anything written in text-books on the subject. The passages on " automatic re-equilibrium " and " induced equilibrium " are also well written, although it is rather difficult to see why exchange-control should necessarily lead to State-Socialism ? In arguing on Keynes's thesis in the *Treatise*, on the choice between maintaining stable prices or stable exchanges, Professor Heilperin holds the view that the dilemma is non-existent, since stable prices are not possible without stable exchanges and vice versa ; had he, however, considered the question of output and employment to a greater extent throughout the book, this problem as well as others would not have appeared as simple as the author seems to suggest.

The author is at pains to stress the difficulties involved in the concept of index-numbers. He does so throughout the text, and devotes the Appendix to that point. His very strong criticism of the violation of " the principle of homogeneity " is fairly sound on strictly methodological grounds ; but—" the proof of the pudding is in the eating " ; index-numbers *are* useful, and the author does not provide us with anything better instead.

The main theme of the book, however, is how to run the international monetary system when various policies coexist. The trade cycle is only mentioned *passim*, employment, public works, tariffs do not come into the picture at all. The problems discussed all centre around exchange parities, gold movements, *et hoc genus omne*. The conclusions of the book are that " the real opposition is that of organized *versus* disorganized monetary relations, not that of one method or organization *versus* another method." While one can fully agree that exchange stability *can* be maintained by inter-

national co-operation, even if—to use a happy phrase of the author—a "homogeneous distribution of confidence throughout the world" is not prevalent, the internal repercussions on employment and trade depend nevertheless very much on what the internal "method of organization" is. International co-operation in the financial field cannot be separated from co-operation in all other economic policies, and when viewed from that angle the monetary side seems to be little more than a minor technical detail. That this total economic collaboration is within the power of the individual States, many of us believe. That no international intercourse is possible without some form of co-operation, not even the staunchest autarchist could deny. *Sed quis custodiet custodes ipsos . . . ?* G. A. B.

7.—*Theory of the Just Price.* By Rudolf Kaulla. Translated from the German by Robert D. Hogg. London: Allen & Unwin. 1940. 219 pp. 7*s.* 6*d.*

A short and interesting book on Value, which approaches the problem from an unusual angle: the values attaching to material goods are only a part of the complex values attaching to *services*. An historical survey shows two competing tendencies in the remuneration of all services: payment in accordance with the social status of the occupation, or in relation to the utility of the service rendered. Hence the lower limit in the determination of value is given by the fact that "the total income of a profession cannot fall below the level required to maintain the number of persons" necessary, on a standard of living appropriate to the profession. The upper limit can only be given for "*necessaries*"; these must be within the reach of all the individuals concerned. Both limits must be modified, of course, in so far as (*a*) the primary needs of men can change as a result of education, and (*b*) people with alternative sources of income are in a position to charge less for their services. But it never seems to occur to the author (what in the case of his own country is fairly obvious) that his upper limit might be *below* his lower, when rationing prevents the satisfaction of all primary needs, while individual incomes, necessitating an unspent margin of cash, can only satisfy the first condition in terms of *money*.

The "justness" of a price is a political question, and must be regulated through the legislature. Regulations of contracts, tariffs, or subsidies are all various instruments of the State to maintain that justness—*i.e.*, to keep a profession or an industry on a level of prosperity that is socially desirable. After a not wholly unsuccessful criticism of *laisser faire*, the author comes to the conclusion that "the State . . . *is* an essential factor in the creation of economic value, and that it has not to choose *whether*, but only *how* it shall intervene in the economic life of its citizens." G. A. B.

STATISTICAL NOTES

(1) British Official Statistics

We give on p. 586 a table summarizing the oversea trade of the United Kingdom for the years ended November 1939 and 1940. The development of our trade up to July was dealt with in Part III of the *Journal*, and this note will be confined to a consideration of the subsequent developments. Comparisons with August–November 1939 are affected not only by the restriction of the area with which trade is now practicable, but by the disturbed conditions at the outbreak of war, and are accordingly of little value. It is only necessary to mention here that the effect of enemy action in Europe has been to reduce appreciably the strain on our overseas resources, while imposing an additional strain on shipping by reason of the longer voyages resulting from both more distant sources of supply and the closing of the Western Mediterranean. The course of our trade since July has been as follows :—

Months	Imports	Exports	Re-exports	Excess of imports over exports
	£ mill.	£ mill.	£ mill.	£ mill.
July	87·0	31·2	1·8	54·0
August	95·0	32·5	1·4	61·1
September	80·6	31·2	1·2	48·2
October	85·1	23·4	1·0	60·7
November	72·9	21·7	0·7	50·5

Both imports and exports have had a downward trend, which is not surprising in view of the enemy attacks on both our shipping and our ports, in particular the heavy attack on London in September, which caused a diversion of shipping from that port. As London normally handled some 40 per cent. of the imports into the United Kingdom and about 30 per cent. of the exports and re-exports, the transference of that trade to other ports was bound to cause congestion there and affect the internal distribution of goods to a marked extent.

The excess of imports over exports in the first eleven months of the year was £611½ million, being about £253 million higher than in the corresponding period of either 1938 or 1939. There have been considerable fluctuations in the adverse balance during recent months, the excess of imports being lower in September and November than in any other month this year. The reduction in

imports accounts for this, and it may be noted that imports in November were lower than in any month since October 1939. The marked reduction in exports in October was officially explained as being due in part to the abolition of pre-entry for a large proportion of the goods consigned to markets in the British Empire. This does not, however, serve to account for the reduction in November, but possibly the policy of discriminatory exports announced by the President of the Board of Trade may have had some effect on the figures. Exports are important in enabling this country, and with it the rest of the sterling area, to conserve accumulated resources in foreign exchange. To the extent to which countries in the sterling area are able to reduce their consumption of unessential imports—as has been and is being done in this country by import licensing, the Limitation of Supplies Orders and the purchase tax—exports from this country of such commodities tend to weaken our war effort by diverting to such exports resources of labour, plant and materials that could be better employed. The actual figure recorded for exports in November (£21·7 million) is the lowest this century and that for re-exports (£0·7) million the lowest on record.

Imports of food, drink and tobacco have recently represented a smaller proportion than normal of the total imports, as might be anticipated in view of the need for larger imports of materials for war production. The absence of quantitative information, or information about the relative movements of import prices of foodstuffs and raw materials, however, prevents any definite conclusion being drawn from the value figures. The wholesale prices index number is inadequate for the purpose, especially as it includes subsidised prices. Manufactures have represented a higher proportion than normal, and in this connection it may be noted that imports of iron and steel have risen rapidly during recent months, and the total for November (£7·2 million) has never hitherto been approached—in the last war our largest annual importation was under £12 million.

Exports of food, drink and tobacco were fairly well maintained at the July level until November. For the beverages group, which comprises mainly spirits, the value of exports has been about the same throughout the year, apart from November, when there was a fall of about £1 million. There was a considerable reduction in August in raw materials exported, the decline being mainly in coal, with a further drop in November. The bulk of the fall in October was in manufactured articles, which suffered a further small reduction in November. Comparing November with July, substantially greater falls were recorded for the various textile groups than for iron

and steel and machinery, while exports in the vehicles group (which may include aircraft for our allies) were larger in the later month.

Re-exports have fallen rapidly month by month, as might perhaps be expected in view of shipping difficulties, and in November were only 40 per cent. of their value in July. The main fall has been in re-exports of raw materials. These were normally much the largest class of our re-exports, but in November were only £131,000 out of the total of £723,000.

Movements and Classes	Twelve Months ended November 1939	Twelve Months ended November 1940	Increase (+) or Decrease (−)
Imports, c.i.f.—	£'000	£'000	£'000
Food, drink and tobacco	392,664	438,674	(+)* 46,010
Raw materials and articles mainly unmanufactured	238,310	342,269	(+) 103,95
Articles wholly or mainly manufactured	236,538	325,313	(+) 88,775
Other articles	7,557	6,003	(−) 1,554
Total Imports ...	875,069	1,112,259	(+) 237,190
Exports, f.o.b.—			
United Kingdom Produce and Manufactures—			
Food, drink and tobacco	34,888	35,323	(+) 435
Raw materials and articles mainly unmanufactured	54,457	39,447	(−) 15,010
Articles wholly or mainly manufacfactured ...	336,267	345,991	(+) 9,724
Other articles	12,173	8,040	(−) 4,133
Imported Merchandise—			
Food, drink and tobacco	10,913	8,584	(−) 2,329
Raw materials and articles mainly unmanufactured	25,903	13,260	(−) 12,643
Articles wholly or mainly manufactured	10,785	5,960	(−) 4,825
Other articles	607	233	(−) 374
Total Exports ...	485,993	456,838	(−) 29,155

THE Board of Trade index-number of *wholesale prices* (average for 1930 = 100) continued its advance during the three months August to October 1940, and by October had recorded an increase of nearly 45½ per cent. on the general level of wholesale prices in the three months preceding the war, viz. from 98·1 to 142·7. Food prices have advanced 58¼ per cent. since August 1939 and the prices of industrial materials and manufactures 39¼ per cent. It

has to be borne in mind, however, that there are some commodities the prices of which are lower through subsidization, others the prices of which are controlled, and others the supplies of which are rationed in some form or another in addition to price control. Consequently in the absence of a really free market for many commodities the value of the index-number is to some extent subject to limitations, as also are the market quotations of commodities where such are still available. No doubt in the case of market quotations some are almost nominal owing to the difficulties of supply. Subject to these restrictions it can be noted that over the three months August to October 1940 there was little change in the general level of prices of industrial materials and manufactures and a rise of nearly 6½ per cent. in prices of food and tobacco, chiefly due to increases in the prices of cereals. There were, in addition, advances (partly seasonal) in the prices of certain vegetables and in the prices of eggs. Coal and metal prices remained fairly steady throughout the period, and increases among textile commodities were not on any considerable scale. There was also a decline in the miscellaneous group of articles, due principally to a reduction in the price of hides. Since the end of October iron and steel prices have been advanced, and towards the end of October there were considerable increases in the prices of wood pulp, ranging from 11 to 42 per cent. Since the commencement of the war prices of basic materials (excluding fuel) have increased 54½ per cent., intermediate products nearly 50 per cent., manufactured articles 25½ per cent. and building materials 23⅓ per cent. Cereals have advanced in price over 73 per cent., meat, fish and eggs over 42 per cent. and other food and tobacco 60 per cent., the increase in the last-mentioned group being due principally to additional taxation on sugar and tobacco.

The Board of Trade index-numbers of wholesale prices for the last four months are given below, together with the corresponding numbers for August 1939 the month immediately preceding the commencement of hostilities.

(Averages for the year 1930 = 100)

Date	Total Food	Total net Food *	All Articles	Basic Materials †	Inter-mediate Products	Manu-factured Articles	Building Materials
July 1940	134·4	142·3	139·7	148·9	153·8	136·4	127·0
Aug. ,,	135·5	142·3	140·1	148·9	153·9	136·3	127·6
Sept. ,,	139·6	141·6	141·1	144·5	154·9	136·5	128·0
Oct. ,,	143·1	142·3	142·7	146·0	155·8	136·4	128·4
Oct. 1939	*109·4*	*111·4*	*110·9*	*107·8*	*116·2*	*114·2*	*107·7*
Aug. ,,	*90·4*	*102·2*	*98·1*	*94·5*	*104·0*	*108·7*	*104·1*

* Industrial materials and manufactures. † Excluding fuel.

The figures for certain other British index-numbers and the index-number of wholesale prices prepared by the United States Bureau of Labour are given below.

Date	Board of Trade (1930 = 100)	*Economist* (1927 = 100)	*Statist* (1866–77 = 100)	*The Times* (1913 = 100)	United States (Bureau of Labour) (1926 = 100)*
July 1940 ...	139·7	96·0	129·5	151·7	77·6
Aug. ,, ...	140·1	98·3	131·5	157·5	77·2
Sept. ,, ...	141·1	99·9	131·6	159·1	77·8
Oct. ,, ...	142·7	98·9	131·8	159·4	78·2
Oct. 1939 ...	*110·9*	*83·3*	*105·8*	*129·1*	*79·2*
Aug. ,, ...	*98·1*	*70·3*	*90·4*	*114·5*	*74·8*

* Mean per cent. weekly prices.

During the three months August to October 1940 the rise in the cost of living according to the Ministry of Labour's index-number of the prices of articles of working-class consumption amounts to about 3·8 per cent. The increase in food prices was about 4·8 per cent. In the prices of other items entering into a working-class budget there was little change during the period except in the cost of clothing, which advanced about 6 per cent., and of fuel rather less than 1½ per cent. Since the commencement of the war the increase in the cost of living up to November 1st, 1940, has been about 24 per cent., the increase in food prices being about 25 per cent., in articles of clothing 48 per cent., in fuel nearly 18 per cent. and in other articles (excluding rent) about 22½ per cent. The rise in the index-number for working-class rents is only 162 to 164, and represents the slight increase in local rates.

The index numbers for the last few months are given below :—

(Average prices for July 1914 = 100)

Date	Food	Rent and Rates	Clothing	Fuel and Light	Other Items	All Items
Aug. 1st, 1940 ...	164	164	290	212	219	185
Sept. 1st, ,, ...	166	164	295	212	219	187
Oct. 1st, ,, ...	169	164	300	214	219	189
Nov. 1st, ,, ...	172	164	305–310	215	220	192
Nov. 1st, 1939 ...	*154*	*162*	*235*	*185–190*	*185–190*	*169*
Sept. 1st, ,, ...	*138*	*162*	*205–210*	*180–185*	*180*	*155*

Since the commencement of the war the following are the most important increases in the prices of food: eggs 97 per cent., sugar 65 per cent., fish 56 per cent., potatoes 28 per cent., imported cheese 29 per cent., bacon 23 per cent. and milk 18 per cent. Some of

these articles (*e.g.*, meat, butter and margarine, sugar and bacon) are rationed, and the prices of others are controlled in some form or another, or subsidized, as in the case of bread. Some portion at least of the advance in the prices of sugar and tobacco is due to increased duties, and the price of the former commodity has been reduced since November 1st by 1*d.* per lb.

Judging by the numbers remaining in the registers of the Ministry of Labour Employment Offices in Great Britain at the middle of each month, there has been little general improvement in employment between the middle of July and November 11th, 1940, at which dates the numbers registered as unemployed were 827,266 and 791,180 respectively, a decrease of little more than 36,000. The decline in the number of unemployed male adults was somewhat larger—viz. 45,967, from 448,966 to 402,999, but there was an increase of 9,550 in the number of unemployed young persons. The number of women on the registers remain high—rather more than three-quarters of the number of men. The proportion ordinarily is between a third and a quarter. The high number is stated to be due to some extent to the fact that evacuated women workers out of work have now to register at the employment offices in the areas where they are now residing. Some allowance should also be made, no doubt, for the registration of those women anxious for war-work who are not ordinarily engaged in the industrial field. Compared with the numbers unemployed at November 13th, 1939, the figures for November 1940 show a very large reduction—from 1,402,588 to 791,180—the reduction being most considerable in the case of adult males, where the numbers have dropped from 897,061 to 402,999, a fall of 55 per cent. The fall, though considerable, is least among females (22 per cent.).

The following table gives the number of work-people (insured and uninsured) aged 14 to 64 on the registers of the Employment Offices of the Ministry of Labour in Great Britain.

Date	Wholly Unemployed	Temporarily Stopped	Persons normally in Casual Employment	Total
July 15th, 1940 ...	636,532	153,242	37,492	827,266
Aug. 12th, ,, ...	613,156	154,380	31,916	799,452
Sept. 16th, ,, ...	613,671	185,000	31,175	829,846
Oct. 14th, ,, ...	635,431	171,082	28,338	834,851
Nov. 11th, ,, ...	603,241	163,364	24,575	791,180
Nov. 13th, 1939 ...	*1,213,345*	*135,233*	*54,010*	*1,402,588*
Aug. 14th, ,, ...	*968,108*	*211,978*	*51,606*	*1,231,692*

The Ministry of Labour has made an investigation into the duration of unemployment of applicants for unemployment benefit or allowances at November 25th, 1940. Out of a total of 359,000 men, 154,000 (43 per cent.) had been on the registers for less than two weeks and 191,000 for less than four weeks. The number who had been on the register for twelve months or more was 54,000, or nearly half those on May 20th last. Three-quarters of these were aged 50 or over.

It has apparently been thought desirable by the Ministry of Labour to omit from the *Labour Gazette* since the September issue the usual information as to the numbers and proportion unemployed in the various trades and industries. No doubt during the last twelve months, owing both to recruiting for the forces and to transfers to war industries, the basis of the percentages has become less satisfactory, and there are probably other reasons for the omission. It would have been interesting, however, to have been able to see with what success the Ministry has been able to fit into war industries those thrown out of work in the less essential trades. Their number must have been considerable. The lack of work in coal-mining, owing to the decreased export demand, must figure largely in the unemployment problems confronting the Ministry.

The latest, and presumably the last, percentages to be published during the war of unemployment in the insured trades in Great Britain and Northern Ireland were for August 12th, 1940, 5·5 per cent. in the general scheme and 2·4 per cent. for the agricultural scheme and for September 16th 6·0 per cent. and 2·3 per cent., respectively. The corresponding percentages for September 11th, 1939 were 9·1 per cent. and 3·4 per cent.

(2) Other Statistics

On November 2nd, 1940, the *Economist* newspaper commenced the publication each week of a daily "Sensitive" price index-number "designed to reflect the market's short-term view of the probable supply and demand for primary products in general." Ten commodities included in the number are wheat, maize, sugar, cocoa, cotton, the four non-ferrous metals and rubber. The usual British "spot" prices are taken for seven of the commodities, with the "nearest Liverpool futures" for wheat and maize, and the nearest Amsterdam future for cocoa. The Government control of prices affects and will continue to affect the value of this new index-number during the war. In any case it is too early to judge of its

"sensitiveness," but the number has been calculated back to 1913, and from the chart published in the issue of the *Economist* for October 26th last its curve does not show as a rule much variation from that of the *Economist's* well-known index-number of wholesale prices.

Index-numbers prepared by the Bank of England and the Textile Association to show the value of wholesale trading in textiles indicate that over the four months July to October 1940 values both for export and home trade for the first two months were higher than in the corresponding months of 1939, but that in the last two months were somewhat lower, especially in the export trade. Owing to the considerable increase in prices since the war, it is not likely that the *volume* of trade has been maintained either in home or export markets. The home trade index-numbers for the four months of 1940 were 82, 124, 148, and 157 and the corresponding figures in 1939 were 65, 83, 159, and 170. The export numbers were 94, 74, 54, and 73 in 1940 and 77, 74, 74, and 88 in 1939. Roughly the weights assigned seem to be about 12 or 13 to 1, and the index-numbers for total trade were 83, 122, 143, and 152 for the months July to October 1940 and 83, 122, 143, and 152 for the same months in 1939 (average monthly sales in 1937=100).

The Labour Department of Jamaica has recently issued a report which embodies the results of an enquiry relating to the cost of living of the "poorer or labouring classes" of Kingston, Jamaica, at August 1939 (*Report on the Cost of Living Survey in Kingston, Jamaica*, 1940, price 3*d*.). The family income and expenditure was obtained of 486 households, covering 1682 persons, of whom 673 were children under 16, 343 male adults over 16, and 666 female adults. The limits of family income were £2 per week and presumably the enquiry related to the coloured population only, though it is nowhere so stated. Only 17½ per cent. of the 1,009 adults were in regular employment, 10½ per cent. being casually employed and nearly 24 per cent. working in their own account. There were 16¼ per cent. unemployed over 16. Particulars were obtained by means of very elaborate and detailed budgets filled in with a good deal of care by investigators of the Labour Department. The information secured proved very difficult to interpret. The average of weekly money receipts per household, for instance, was found to be 11*s*. 4*d*., whereas the expenditure worked out at 17*s*. 6¾*d*. Some of the discrepancy may, no doubt, be due to the fact that rents may be in arrears and that articles may be procured on credit during the week and charged

as expenditure of that week, but such explanations do not seem to account for so much difference between incomings and outgoings which have ultimately to be met, nor is the opinion, hazarded in the report, that much debt may never be liquidated quite convincing. It would seem that some sources of casual income must have been undisclosed, or that expenditure has been returned of commodities purchased but not necessarily all consumed in the particular week. Perhaps both causes operated. The rent returned averaged 2*s*. 9*d*. per week, but no information is given as to the number of rooms occupied by the households, though it would appear that some households occupy one-room tenements.

There was a very elaborate ascertainment from shops, stores, markets and street-sellers of the prices of about 60 articles of food, and of as many other kinds of commodities entering into working class consumption. Prices were obtained at two different dates, August 1939 and November 25th, 1939, and if further ascertainments can be made from time to time, the price-structure of a cost of living index-number can be satisfactorily established.

CURRENT NOTES

THE National Service Committee for Social, Economic and Statistical Research has issued the following statement:—

The Schedule of Reserved Occupations does not make any provision for economists or workers in the other social sciences, who are not whole-time members of University staffs, nor does the reservation of statisticians at thirty completely meet the need for retaining qualified men for work of national importance who may be under thirty. While there is no intention of including these classes in the Schedule, the Minister of Labour and National Service has provided machinery for dealing with them. At his invitation a Committee, called the National Service Committee for Social, Economic and Statistical Research, has been set up to consider the cases of men of military age who are highly qualified for research work in statistics, economics and the social sciences. The function of the Committee is to make recommendations to the Ministry of Labour in order that the calling-up of such men may be deferred. The Committee consists of representatives of the National Institute of Economic and Social Research, of the Royal Economic Society and of the Royal Statistical Society.

There are two categories in which the Committee may place cases: *Category A:* Cases of men identified as scientific research workers, either as statisticians, economists, or social science research workers. "Identification" is taken to mean "certification of scientific status," and is dependent upon the qualifications of knowledge and experience in the appropriate research techniques. Where men are placed in this category, indefinite deferment of calling-up is usually granted by the Ministry. This is conditional upon such cases passing successfully before the Committee at six-monthly intervals for review. On review, evidence of occupation must be submitted, so that the Committee may certify that the men in question are engaged whole-time in appropriate occupations of statistical, economic, or social science research work.

Category B: Cases of men who should, from the point of view of the public interest, remain at any rate for a time upon the work in which they are already engaged as statisticians, economists or social science research workers. Recommendations are made for given periods of deferment of calling-up, and these may sometimes be made for the specific purpose of allowing time for training a substitute. These two categories are mutually exclusive, and the requirements of A are more stringent than those of B.

The age limits are 25–42 years in the case of economists and social science research workers, 25–29 in the case of statisticians. The

only permissible exception would be a case of an exceptionally well-qualified and valuable statistician who is under 25.

Direct application can be made to the Committee either by the man himself on his own behalf or by the organisation for which he is working. Applicants can state for which category they wish the case to be considered. Where a man applies on his own behalf for inclusion in Category A, the Committee welcomes supporting testimonials from authorities in the scientific field in which he has worked. Where application is made for inclusion in Category B, it should as a rule be supported by a government department interested in the work. All applications should give clear and adequate details of qualifications, research and other relevant experience, and of present occupation.

All whole-time members of University Staffs engaged in social, economic and statistical research should apply in the first place to their University authorities to find out whether their cases come within the scope of the arrangements for dealing with University staff who are not covered by the Schedule.

All enquiries and requests for application forms should be sent to the Secretary, National Service Committee for Social, Economic and Statistical Research, 6 Duke of York Street, London, S.W.1.

Until recent times criminal science meant little more than a study of the law. Now it covers such subjects as personality of the delinquent, the nature and efficacy of the punishment administered, the origin of a career of crime, and the relationship between Society and criminal justice. In an article on the "Language of Criminal Science" (*Cambridge Law Journal*, vii.2.1940), L. Radzinowicz and J. W. C. Turner maintain that the pendulum has swung too far, and believe that three broad categories are sufficient to cover the field—namely, (*a*) criminology, which comprises criminal biology, or the study of the delinquent, and criminal sociology, or the study of his environment; (*b*) criminal policy, which must co-ordinate all measures of prevention, legislation and punishment; (*c*) criminal law or the rules for deciding whether what has been done amounts to a crime, whether the accused committed it and what is the assigned sanction. They prefer not to have statistics as a separate branch, for they naturally play an important part in each of the categories above. Like most statistics, they are, however, full of pitfalls for the ignorant or unwary, as Radzinowicz points out in a recent study ("English Criminal Statistics," *Law Quarterly Review*, October 1940). He is, in fact, inclined to believe that they come very high in the scale of imperfection and do not, and never can, furnish more than a rough approximation to the real volume of crime in a community.

Unreported crime is a varying quantity both in time and space, changes in methods of recording, in classification, and in the powers given to the various courts, compromise secular comparisons, while there are ever-present difficulties in distinguishing between indictable and non-indictable offences, felonies and misdemeanours. The result is a spate of indices—*e.g.*, crimes known to the police, number of persons tried, persons found guilty, persons convicted, persons imprisoned, all of which need care in their selection and interpretation for the purpose in hand. Radzinowicz pleads for more uniformity in the system of recording crimes by the various police forces of the country, but can suggest no other means of improvement.

It is to be noted that the authors of the studies mentioned in the preceding paragraph are members of a Committee recently set up by the Faculty Board of Law in the University of Cambridge in order " to consider the promotion of research and teaching in criminal science," of which we have just received notice. The other members of the Committee are Professor P. H. Winfield (Chairman) and Dr. R. W. Jackson; Mr. Turner is the Secretary.

The research projected by the Committee includes the study of the development of criminal science in England and other countries and the exchange of information with other institutions, here and abroad, concerning the methods of administering justice and the progress of penal reform.

The Committee also propose to arrange for lectures to be given in Cambridge by recognized authorities on various branches of criminal science, and to issue publications. The first of these, entitled *Penal Reform in England*, is shortly to appear as Volume I in a series of English Studies in Criminal Science. It will consist of " introductory essays on some aspects of English criminal policy " by various authors, edited by L. Radzinowicz and J. W. C. Turner, with a preface by Professor Winfield, and is stated to be " a work of collaboration by experts under a definite scheme."

The formation of the Committee and its promised activities will be welcomed by the Society, which has always included among its Fellows a number interested in criminal statistics, and has itself endeavoured in various ways and on various occasions to stimulate their study and press for their improvement.

The President has received from Sir Alfred Flux, through the Red Cross, the following message, dated 1 October, 1940 :—" Many thanks for your most welcome July and August messages. We are keeping well. Warmest greetings to both and to the circle of mutual friends."

STATISTICAL AND ECONOMIC ARTICLES IN RECENT PERIODICALS

UNITED KINGDOM—

Annals of Eugenics, September, 1940—Some fertility rates by age of mother and order of birth : *K. K. Conrad.* The analysis of covariance in quasi-factorial designs : *E. A. Cornish.* Studies on a child population. V. The resemblance in intelligence between sibs : *J. A. Fraser Roberts.* The measurement and some determinants of secondary school life : *F. Sandon.*

Annals of Mathematical Statistics, September, 1940—Reduction of a certain class of composite statistical hypotheses : *G. Brown.* The analysis of variance when experimental errors follow the Poisson or binomial laws : *W. G. Cochran.* The selection of variates for use in prediction with some comments on the problem of nuisance parameters : *H. Hotelling.* Parabolic test for linkage : *N. L. Johnson.* A study of a universe of *n* finite populations with application to moment-function adjustments for grouped data : *J. A. Pierce.* A method of minimizing the sum of absolute values of deviations : *R. Singleton.* The fitting of straight lines if both variables are subject to error : *A. Wald.*

Banker, November, 1940—Need war mean impoverishment? : *C. Graham.* Lending policy in wartime : *C. Graham.* Practical difficulties in the control of export proceeds : *W. W. Syrett.*

Economica, August, 1940—The drift towards a rational foreign exchange : *T. Balogh.* The analysis of producers' expectations : *R. H. Coase and R. F. Fowler.* British floating debt policy : *F. W. Paish.* A study of interest and capital : *T. de Scitovszky.* Advertising and the maximisation of profit : *E. K. Zingler.*

Eugenics Review, July, 1940—The health of working women : *M. L. Spring Rice.*

Institute of Bankers, Journal, October, 1940—War economics : Some truths and fallacies : *H. Parkinson.*

Manchester School of Economic and Social Studies, October, 1940—British war controls : an economic comment : *J. Stafford.* Six years of controlled milk prices : *P. Chantler.* Irish agriculture then and now : *J. Johnston.*

Manchester Statistical Society, Transactions, Session, 1939–1940 (Ordinary Meetings)—An experiment in tariff making : *Sir Percy Ashley.* The measurement of real income : *A. L. Bowley.* The taxation of excess profits in war-time : *U. K. Hicks.* Man power in agriculture : *J. P. Maxton.* Economic interferences with London transport : *F. Pick.*

(Group Meetings)—Analysis of Germany's foreign trade and the war : *H. C. Hillmann.* Statistics : a technical tool in the chemical industry : *S. Horrobin and Dr. O. L. Davies.* The regional development of public utility services (with special reference to South-East Lancashire) : *R. N. Spann.*

UNITED KINGDOM—*Contd.*

Review of Economic Studies, October, 1940—The nature of the inducement to invest: *G. L. S. Shackle.* Local rates and housing subsidies: *M. Bowley.* Capital levies in central Europe 1914–1924: *L. Rostas.* Economics by motion symbols: *M. Polanyi.*

SOUTH AFRICA—

South African Journal of Economics, June, 1940—An introduction to the housing problem: *P. H. Guenault and R. J. Randall.* Some reflections on the economics of wage fixation: *R. H. Smith.* New Zealand's war controls: *W. B. Sutch.*

UNITED STATES—

American Statistical Association, Journal, September, 1940—Factorial design and covariance in the biological assay of vitamin D: *C. I. Bliss.* Classification of hospital morbidity: *M. Fraenkel.* A measure of purchasing power inflation and deflation: *M. Shields.* Factors to be considered in measuring intercity and interregional differences in living costs: *F. M. Williams.*

Annals of the American Academy of Political and Social Science, September, 1940—Our foreign commerce in peace and war—whole number.

Econometrica, October, 1940—Recursive methods in business-cycle analysis: *M. Flood.* The inadequacy of testing dynamic theory by comparing theoretical solutions and observed cycles: *T. Haavelmo.* The service industries in relation to employment trends: *D. Weintraub and H. Magdoff.* Quantity adjustment factors in cost-of-living ratios: *R. M. Woodbury.*

Harvard Business Review, Autumn number, 1940 Wheels for defense: *R. Black.* Minerals and war: *P. M. Tyler.*

Journal of Political Economy—

June, 1940—The development of Italian cartels under fascism: *F. R. Pitigliani.* Treasury deposits and excess reserves: *E. C. Simmons.*

August, 1940—Extensive expansion and population growth: *A. H. Hansen.* Hicks and the time-period controversy: *W. Fellner and H. S. Ellis.* Adam Smith's empiricism and the law of nature. I: *H. J. Bitterman.*

October, 1940—Toward a theory of business taxation: *P. Studenski.* Foreign trading in American stock-exchange securities: *H. G. White, Jr.*

Monthly Labor Review—

July, 1940—Unit labor cost in 20 manufacturing industries, 1919 to 1939: *H. S. Hanna.*

August, 1940—Food expenditures of wage earners and clerical workers: *H. S. Hanna.*

UNITED STATES—*Contd.*

Quarterly Journal of Economics—

August, 1940. Part I—Regional differences in rates and rateable values in England and Wales, 1921–36 : *M. Daly*. A process analysis of bank credit expansion : *R. Vining*. "Full utilization" equilibrium, and the expansion of production : *A. B. Wolfe*.

August, 1940, Part II—Exchange control in Germany : *H. S. Ellis*.

Review of Economics Statistics, August, 1940—Income-expenditure relations : *E. Gilboy*. Graphical survey of economic developments : *E. Gilboy*. Estimates of unemployment in the United States : *R. A. Nixon and P. A. Samuelson*.

Social Research, September, 1940—Peace Economics : *J. Marschak*. Industrialization of young countries and the change in the international division of labor : *E. Peltzer*.

Wheat Studies of the Food Research Institute, September, 1940—World wheat survey and outlook, September, 1940 : *H. C. Farnsworth and V. P. Timoshenko*.

SWITZERLAND—

Zeitschrift für schweizerische Statistik und Volkswirtschaft—

1939, *Part IV. Jubiläumsheft*—75 Jahre Schweizerische Statistische Gesellschaft :

1940, *Part I*—Sterbeziffer und Frauenüberschuss in ihrer Abhängigkeit vom Bevölkerungswachstum : *W. Euchti*. Zur Frage der Lockerung oder Aufhebung der Einfuhrkontingentierung : *A. Willener-Schmid*. Uber die Anwendung von branchentypischen Normen im Steuerveranlagungsverfahren : *J. Viel*. De La valeur théorique de deux concepts statistiques; le pouvoir d'achat de la monnaie et l'indice des prix : *B. Caizzi*. Staatsbetrieb, Privatbetrieb und gemischter Betrieb : *E. His*.

LIST OF ADDITIONS TO THE LIBRARY

Since the issue of Part III, 1940, the Society has received the publications enumerated below:—

I.—OFFICIAL PUBLICATIONS

(a) United Kingdom

Committee of Public Accounts. First and second reports from the Committee of Public Accounts, together with the proceedings of the Committee, minutes of evidence, appendices and index. London: H.M.S.O., 1940. 9¾″ × 6″. xxxix + 284 pp. 6*s*. 6*d*.

Food, Ministry of. Report of the committee appointed by the Minister of Food to examine the cost of milk distribution 1940. London: H.M.S.O., 1940. 9¾″ × 6″. 38 pp. 6*d*.

Health, Ministry of. Ministry of Home Security. Recommendations of Lord Horder's Committee regarding the conditions in air-raid shelters with special reference to health; and a brief statement of action taken by the Government thereon. London: H.M.S.O., 1940. Cmd. 6234. 9¾″ × 6″. 7 pp. 2*d*.

Meteorological Office. A short course in elementary meteorology, by H. W. Pick. . . . 5th edition. London: H.M.S.O., 1938. (Reprinted 1940.) 9½″ × 6″. 144 pp. 2*s*. 6*d*.

Office of the Parliamentary Council. Defence Regulations (being Regulations made under the Emergency Powers (Defence) Acts, 1939 and 1940, printed as amended up to and including 4th October, 1940) . . . 6th edition. 4th Oct., 1940. London: H.M.S.O., 1940. 9¾″ × 6″. x + 336 pp. 5*s*.

Select Committee on National Expenditure. First report–Thirteenth report. London: H.M.S.O., 1940. 9¾″ × 6″. 13 parts. 4*s*.

Shipping, Ministry of. Memorandum on war-time financial arrangements between His Majesty's Government and British Shipowners. London: H.M.S.O., 1940. Cmd. 6218. 9¾″ × 6″. 19 pp. 4*d*.

Trade, Board of. Statistical abstract for the United Kingdom for each of the fifteen years 1924 to 1938. 83rd number. London: H.M.S.O., 1940. Cmd. 6232. 9¾″ × 6″. xvii + 450 pp. 7*s*.

Northern Ireland—

Registrar-General. Census of Population of Northern Ireland 1937. General summary. Belfast: H.M.S.O., 1940. 13″ × 8¼″. v + 15 pp. 2*s*.

(c) Foreign Countries

Brazil—

Instituto Brasileiro de Geografia e Estatistica—Revista brasileira de estatistica Ano I, No. 2. Abril-junho 1940. Rio de Janeiro: 1940. 10½″ × 7½″. pp. 195–416.

—— Sinopse Estatistica do Brasil. Ano IV, 1938. Statistical abstract of Brazil. Separate reprint of the Statistical Year-book of Brazil. Rio de Janeiro: 1940. 10½″ × 7¼″. lxiii + 314 pp.

Sweden—

Statistiska Centralbyrån. Pälsdjursräkningen den 30 september 1939. Stockholm: 1940. 9½″ × 6½″. 44 pp.

—— —— Undersökning angående skogsavverkningen år 1937. Stockholm: 1940. 9½″ × 6¼″. 89 pp.

Kungl. Skolöverstyrelsen. Lärarutbildningen läsåren 1928–1929 t.o.m. 1938–1939. Stockholm: 1940. 9½″ × 6½″. x + 64 + 40 pp.

(c) **Foreign Countries**—*Contd.*

U.S.A.—

Children's Bureau. Children in a democracy: general report adopted by the White House Conference on Children in a Democracy. January 19, 1940. Washington, D.C. Washington: Superintendent of Documents, 1940. 10″ × 7″. 86 pp. 20*c*.

(d) **International**

International Labour Office—

Studies and Reports. Series E No. 6. The compensation of war victims: medical aid, compensation, and war pensions. Geneva: 1940. (London: P. S. King.) 9½″ × 6¼″. 91 pp. 2*s*.

II.—AUTHORS AND MISCELLANEOUS

Actuarial Society of America, and The Association of Life Insurance Medical Directors, New York. Blood-pressure study: 1939. New York: 1940. 10¾″ × 8¼. ii + 69 pp. (From the Joint Committee on Mortality.)

Astbury (*S. J.*). Petroleum statistics. Reprinted from *Annual Review of Petroleum Technology,* Vol. 5, pp. 417–440. Birmingham: Institute of Petroleum, 1940. 9″ × 6″.

Beddy (*James P.*). Profits: theoretical and practical aspects. Dublin: Hodges, Figgis & Co., 1940. 8½″ × 5½″. xi + 420 pp. 12*s*. 6*d*.

Bekker (*Konrad*). Marx' philosophische Entwicklung, sein Verhältnis zu Hegel. Zürich: Verlag Oprecht, 1940. 8½″ × 6″. 134 pp.

British Association for the Advancement of Science. Mathematical tables, Vol. IX. Table of powers giving integral powers of integers, initiated by J. W. L. Glaisher extended by W. G. Bickley, C. E. Gwyther, J. C. P. Miller, E. J. Ternouth on behalf of the Committee for the Calculation of Mathematical Tables. Cambridge: University Press, 1940. 11″ × 8½″. xii + 132 pp. 15*s*.

Chamber of Mines of Rhodesia (Incorporated). First annual report for the year 1939. Bulawayo: 1940. 9¼″ × 7¼″. 64 pp.

Chambers (*E. G.*). Statistical calculation for beginners. Cambridge: University Press, 1940. 8½″ × 5½″. viii + 110 pp. 7*s*. 6*d*.

Ellinger (*Barnard*). The city: the London financial markets. London: P. S. King, 1940. 8½″ × 5½″. xiii + 429 pp. 20*s*.

Gibson (*A. H.*). War finance problems . . . Harrogate: Privately printed, 1940. 13″ × 8″. 39 pp. (From the author.)

Goodfellow (*David M.*). Tyneside: the social facts . . . Newcastle upon Tyne: Co-operative Printing Society, 1940. 8½″ × 5½″. 80 pp. 1*s*.

Horsefield (*J. Keith*). The real cost of the war. Harmondsworth, Middx.: Penguin Books, 1940. 7¼″ × 4¼″. 158 pp. 6*d*.

Indian Statistical Institute. Proceedings of the second session of the Indian Statistical Conference, held in Lahore, January 1939. Editor: P. C. Mahalanobis. Calcutta: Statistical Publishing Society, 1940. (London: P. S. King.) 10¾″ × 9″. 168 pp.

Institute of Petroleum. Annual reviews of petroleum technology, Vol. 5 (covering 1939). F. H. Garner, General editor. Birmingham: Institute of Petroleum, C/o the University, 1940. 9″ × 6″. ix + 457 pp. 11*s*.

Jack (*D. T.*). Studies in economic warfare. London: P. S. King, 1940. 8½ × 5½″. viii + 178 pp. 12*s*.

Lenz (*Friedrich*). Friedrich List der Mann und das Werk . . . Munich & Berlin: R. Oldenbourg, 1936. 9¾″ × 6½″. x + 441 pp. RM. 17. (From Prof. Hermann Levy.)

National Gas Association of Australia. Statistical year book of the gas industry in Australia & New Zealand 1938. Melbourne: Statistical Dept. of the N.G.A. of A., 1940. 13¼″ × 8¼″. 67 pp. (From Mr. A. N. Curphey.)

II.—Authors and Miscellaneous—*Contd.*

National Institute of Economic and Social Research. Guide to foreign trade statistics. Albania, 5 pp. Austria, 6 pp. Belgium and Luxemburg, 9 pp. Bulgaria, 6 pp. Czechoslovakia, 10 pp. Denmark 12 pp. Estonia, 4 pp. Finland, 6 pp. France, 21 pp. Germany, 13 pp. Greece, 6 pp. Iceland, 6 pp. Italy, 15 pp. Latvia, 6 pp. Lithuania, 3 pp. Netherlands, 24 pp. Norway (Appendix), 2 pp. Poland, 7 pp. Portugal, 6 pp. Roumania, 5 pp. Spain, 9 pp. Switzerland, 10 pp. Turkey, 8 pp. Yugoslavia 7 pp. Egypt, 7 pp. Netherlands East Indies, 4 pp. Thailand, 5 pp. Colombia, 5 pp. El Salvador, 4 pp. Peru, 6 pp. U.S.A., 10 pp. Venezuela, 6 pp. London: N.I.E.S.R., 1940. 13″ × 8″. 32 parts, typewritten.

Pownall (*J. F.*). New railway network principles: a project for applying them to British railways . . . Birmingham: Cotterell & Co., 1940. 9¾″ × 6 23 pp. 2*s.*

United Kingdom Alliance. The Alliance year book and temperance reformers' handbook . . . Edited by G. B. Wilson . . . 1939. 222 pp. 2*s.* 1940. 213 pp. 2*s.* London: U.K. Alliance, 1939–40. 8½″ × 5¼″. 2 vols. (From the General Secretary.)

Yule (*G. Udny*) and *Kendall* (*M. G.*). An introduction to the theory of statistics . . . 12th edition, revised. London: C. Griffin & Co., 1940. 8¾″ × 6″. xiii + 570 pp. 24*s.* (Presented by Mr. Kendall.)

PERIODICALS RECEIVED BY THE LIBRARY

ANNUAL LIST

In addition to the publications named in the bi-monthly lists, the Society has received (since September 1939) the official and other periodicals enumerated below.

(a) United Kingdom and its several Divisions

National.

United Kingdom—

Admiralty. Health of the Navy. Navy dockyard accounts.

Agriculture and Fisheries, Ministry of. Agricultural statistics. Journal of the Ministry of Agriculture.

Charity Commission. Report.

Civil Service Commission. Report.

Colonial Office. Economic survey of the Colonial Empire.

Crown Lands, Office of Commissioners of. Report.

Customs and Excise, Board of. Report of the Commissioners.

Education, Board of. Health of the school child.

Foreign Office. Suez Canal: annual return of shipping and tonnage.

Friendly Societies, Registry of. Report of the Chief Registrar. Report of the Industrial Assurance Commissioner.

General Register Office. Weekly return of births and deaths. Quarterly return of births, deaths and marriages. Registrar-General's statistical review.

Health, Ministry of. Accounts of the National Health Insurance Fund. Annual report of the Ministry of Health. Local government financial statistics. Reports on public health and medical subjects. State of the public health.

Home Office. Aliens order: return of alien passengers. Aliens (naturalization) return. Annual report of the Chief Inspector of Factories. Criminal statistics: England and Wales. Licensing statistics. Police (Counties and Boroughs), reports of H.M. Inspectors of Constabulary. Racecourse Betting Control Board, annual report and accounts. Report of the Commissioner of Police of the Metropolis. Report of the Commissioner of prisons and the Directors of convict prisons. Workmen's compensation statistics.

Imperial Institute. Mineral industry of the British Empire and foreign countries, statistical summary.

India Office. Statistical abstract for British India. Return of the budget of the Governor-General of India in Council.

Industrial Health Research Board. Reports.

Inland Revenue, Board of. Report of Commissioners.

Labour, Ministry of. Ministry of Labour gazette. Report of the Unemployment Assistance Board.

Lord Chancellor's Department. Civil judicial statistics.

Medical Research Council. Annual report. Special report series.

Mines Department. Annual report of Secretary for Mines.

Mint, Royal. Annual report of the Deputy Master and Comptroller.

National Debt Commissioners. Local loans fund, accounts, National Health insurance fund and Unemployment insurance fund, account of securities held. Post Office savings banks, accounts. Savings banks and friendly societies, annual account. Trustee savings banks, annual report of the Inspection Committee.

National Insurance Audit Department. Report.

Pensions, Ministry of. Annual report.

Post Office. P.O. Commercial accounts. British Broadcasting Corporation, annual report.

(a) United Kingdom and its several Divisions—*Contd.*

United Kingdom—*Contd.*

Public Trustee Office. General report.
Stationery Office. British imperial calendar. Government publications, consolidated list.
Trade, Board of. Accounts relating to trade and navigation [monthly]. Annual statement of the navigation and shipping. Annual statement of trade. Bankruptcy. Board of Trade journal. Companies. Gas undertakings. Nationality of carrying vessels. Return of shipping casualties and deaths. Statistical abstract for the British Empire. Statistical abstract for the United Kingdom. Survey of industrial development.
Transport, Ministry of. Road Fund, report on administration of.
Treasury. Civil appropriation accounts. Civil estimates. Consolidated fund, abstract account. Finance accounts. Financial statement. National debt return. National Radium Trust and Radium Commission annual report. Public income and expenditure. Trading accounts and balance sheets.
University Grants Committee. Returns from universities and university colleges in receipt of Treasury grant.
War Office. Army appropriation account. General annual report of the British Army. Health of the army.

Municipal and other local returns.

LONDON COUNTY COUNCIL. Annual report. L.C.C. gazette. London statistics.
METROPOLITAN BOROUGHS. Accounts: Battersea, Islington. Medical Officer of Health, reports: Paddington, Shoreditch.
MUNCIPAL BOROUGHS. Accounts: Birmingham, Carlisle, Chester, Ipswich, Leicester, Liverpool, Manchester, Nottingham, Southgate. Medical Officer of Health, reports: Birmingham, Liverpool, Manchester.

Scotland—

Agriculture, Department of. Agricultural statistics. Report of the Department of Agriculture. Report on the profitableness of farming in Scotland.
Education Department, Scottish. Annual reports, combined volume.
Fishery Board for Scotland. Annual report. Sea fisheries: statistical tables.
Health, Department of. Annual report. Report on incapacitating sickness in the insured population.
Prisons Department. Annual report. Civil judicial statistics. Criminal statistics.
Registrar-General. Annual report. Births, deaths and marriages [weekly and quarterly returns].
Scottish Home Department. Constabulary, report of H.M. Inspector. Local taxation returns.
ABERDEEN. Report of the Medical Officer of Health.
EDINBURGH. Municipal accounts.
GLASGOW. Report of the Medical Officer of Health.

Northern Ireland—

General Register Office. Registrar-General's annual report. Quarterly return of births, deaths, and marriages. Ulster year-book.

Miscellaneous Publications.

Accountant. Accountants' magazine. Agricultural Economics Society, journal of proceedings. Agricultural Economics Research Institute, Oxford: the agricultural register. Alliance news. Alliance year-book and temperance reformers' handbook. Anglo-Swedish review. Annals of eugenics. Association of International Accountants: list of members. Auctioneers' and Estate Agents' Institute, Journal.

(a) United Kingdom and its several Divisions—*Contd.*

Miscellaneous Publications—Contd.

Bankers' almanac and year-book. Banker. Bankers' magazine. Bank of England, statistical summary. Belfast, Queen's University calendar. Biometrika. Brewers' almanack and wine and spirit trade annual. British Association for the Advancement of Science, quarterly. British Library of Political and Economic Science annual report. Broomhall's corn trade year-book. Building industries survey. Building societies year-book.

Cambridge University abstract of dissertations. Chamber of Commerce journal. Chartered Surveyors' Institution, journal, list of Fellows. Co-operative Union Ltd., Report of the annual co-operative congress. Corporation of Foreign Bondholders, annual report.

Dalgety and Co., annual review.

East India Association journal. Economic history. Economic journal. Economica. Economist. Eugenics review.

Fabian Society: Fabian news. Fabian quarterly, Research series. Tract series. Faculty of Actuaries, transactions, year-book. Fireman.

Geographical journal.

Health and empire. Horse-owners' reference book. Hospitals year-book.

Incorporated Association of Rating and Valuation Officers: journal, Report of Annual Meeting, year-book. Institute of Actuaries, journal, year-book. Institute of Actuaries Students' Society, journal. Institute of Bankers, journal. Institute of Chartered Accountants, list of members. Institute of Petroleum: Annual reviews of petroleum technology. Institution of Civil Engineers, journal, list of members. Insurance directory and year-book. International Rubber Regulation Committee, Statistics bulletin. International Sugar Council, Statistical bulletin. Iron and Steel Institute, journal.

King Edward's Hospital Fund: Annual report.

Land and liberty. Liverpool Cotton Association, annual and weekly circulars. London (University of) Calendar. London and Cambridge Economic Service: bulletin, and special memoranda. London Passenger Transport Board, annual report and accounts.

Mallett's weekly wool chart. Manchester school. Manchester Statistical Society, transactions. Manchester University Calendar. Mersey Docks and Harbour Board, Accounts. Municipal year-book.

National Association for Prevention of Tuberculosis: transactions, report of Council. Nature.

Oxford Institute of Statistics: Bulletin; Oxford economic papers.

Peabody Donation Fund, report of Governors. People's year-book. Planning. Post magazine. Public administration. Publishers' circular.

Quarterly journal of mathematics. Quin's metal handbook and statistics.

Registered accountant. Review of economic studies. Rothamsted Experimental Station: annual report. Royal Agricultural Society of England, journal. Royal College of Surgeons of England, Calendar. Royal Institution, proceedings. Royal Meteorological Society, journal, phenological report. Royal Sanitary Institute, journal. Royal Society of Arts, journal. Royal Society of Edinburgh, proceedings, transactions.

Samuel Montagu and Co., monthly letter—Annual bullion letter. Scottish Chartered Accountants, official directory. Secretary. Seyd (R.E.), statistics of failures. Signal. Society of Motor Manufacturers: the motor industry of Great Britain. Society of Incorporated Accountants and Auditors, year-book. Sociological review. Statesman's year-book. Statist.

Taxation Times. Times literary supplement. Times trade and engineering supplement. Tin.

United empire.

Vacher's parliamentary companion.

Whitaker's almanack. Willing's press guide. Who's who.

Year-book of scientific and learned societies.

(b) British Empire

Australia—

Bureau of Census and Statistics. Demography. Finance. Labour report. Monthly review of business statistics. Official year-book. Oversea trade. Pocket compendium of Australian statistics. Production. Quarterly summary of Australian statistics. Transport and communication.
Commonwealth Grants Commission. Report.
Department of Health. Health.
Commonwealth Bank of Australia. Statistical bulletin.
Economic record.

NEW SOUTH WALES—

Auditor General. Report.
Bureau of Statistics and Economics. Official year-book. Statistical bulletin. Statistical register.
Department of Public Works and Local Government. Report.
Department of Railways. Report of the Commissioner.
Industrial Register. Trade unions.
Registry of Friendly Societies. Report of the Registrar.

QUEENSLAND—

Government Statistician's Office. Agricultural and dairying production. Live stock and pastoral production. Queensland year-book. Statistics of the State of Queensland.

SOUTH AUSTRALIA—

Public Library, Museum and Art Gallery. Annual report.
Statistical Office. Statesman's pocket year-book. Statistical register.

TASMANIA—

Bureau of Census and Statistics, Tasmania Branch. Pocket year-book. Statistics of the State of Tasmania.
Report on Government Railways and Ferry Service.

VICTORIA—

Office of the Government Statist. Victorian year-book. Annual report on friendly societies.

WESTERN AUSTRALIA—

Department of Mines. Report.
Government Statistician's Department. Pocket year-book. Quarterly statistical abstract. Statistical register.

British West Indies—

Tropical agriculture.

Canada—

Department of Agriculture. Report of the Veterinary Director-General.
Department of Finance. Public accounts.
Department of Justice. Annual report of the Superintendent of Penitentiaries.
Department of Labour. Labour gazette. Prices in Canada and other countries. Wages and hours of labour.
Department of Public Works. Report of the Minister of Public Works.
Dominion Bureau of Statistics. Annual survey of education. Canada: the official handbook of present conditions and recent progress. Canada year-book. Chemicals and allied products. Coal statistics. Fisheries statistics. Iron and steel. Live stock and animal products statistics. Manufactures of non-ferrous metals. Manufactures of the non-metallic minerals. Manufacturing industries of Canada. Mineral production. Monthly review of business statistics. Report on the grain trade. Statistics of dairy factories. Statistics of steam railways. Textile industries. Trade of Canada. Vital statistics.

(b) **British Empire**—*Contd.*

Canada—*Contd.*

ALBERTA. *Department of Agriculture*. Annual report.
Department of Public Health. Annual report of the Vital Statistics Branch.
ONTARIO. *Department of Agriculture*. Annual report of the Statistics Branch. Monthly crop report. Monthly Dairy report.
QUEBEC (PROVINCE). Statistical year-book.
Bank of Canada, statistical summary.
Bank of Nova Scotia, monthly review.
Canadian Bank of Commerce, monthly commercial letter; annual statement.
Royal Bank of Canada, monthly letters.

Ceylon—

Administrative reports. Blue book. Ceylon sessional papers. Papers of the State Council.

Eire—

Currency Commission. Quarterly statistical bulletin.
Department of Industry and Commerce. Census of industrial production. Irish trade journal and statistical bulletin. Statistical abstract. Trade and shipping statistics [annual].
Department of Local Government and Public Health. Annual report of the Registrar-General. Quarterly return of the marriages, births and deaths.
Irish Agricultural Organization Society Ltd. Report.
Statistical and Social Inquiry Society of Ireland, journal.

Federated Malay States—

Government gazette; Return of imports, exports and re-exports.

Gold Coast Colony—

Report on the Department of Agriculture.

India—

Department of Commercial Intelligence and Statistics. Agricultural statistics. Indian coal statistics. Monthly statistics of cotton spinning and weaving. Monthly statistics of production in certain selected industries. Review of the trade of India. Sea-borne trade, annual statement of. Sea-borne trade and navigation [monthly]. Sea-borne trade and navigation for the calendar year. Statistical abstract for British India. Statistical tables relating to banks. Trade at stations adjacent to the land frontier routes.
Department of Industries and Labour. Annual report of Chief Inspector of Mines.
Meteorological Office. Memoirs. Scientific notes.
ASSAM. Report on the Administration of Assam.
BOMBAY. Labour gazette.
PUNJAB. Memoirs of the Irrigation Research Institute. Report on the public health administration.
All Indian Institute of Hygiene and Public Health. Annual report.
Indian journal of economics.
Indian cotton textile industry annual.
Karachi cotton annual.
Punjab Board of Economic Enquiry, publications.
Sankhyā, Indian journal of statistics.

Jamaica—

Annual report of the Registrar-General's Department.

Malaya—

Department of Statistics, S.S. and F.M.S. Foreign trade of Malaya. Malayan statistics [monthly].

(b) **British Empire**—*Contd.*

Mauritius—

Blue book.

New Zealand—

Census and Statistics Office. Agricultural and pastoral production. Factory and building production. Friendly societies and trades unions. Insurance statistics. Justice statistics. Local authorities' handbook. Monthly abstract of statistics. New Zealand official year-book. Population and buildings. Statistics of incomes and income tax. Statistical report on prices, wages . . . banking, building societies, commercial afforestation, incomes and income tax. Trade and shipping. Vital statistics.
Auckland Chamber of Commerce, journal.
Canterbury Chamber of Commerce, bulletin.
Royal Society of New Zealand, transactions and proceedings.
Wellington Harbour Board, statement of accounts.

Nigeria—

Nigeria trade summary.

Palestine—

Department of Migration. Annual report.
Office of Statistics. Blue book. General monthly bulletin of current statistics.

Southern Rhodesia—

Department of Statistics. Annual statement of the trade. Economic and statistical bulletin. Report on the insurance statistics. Statistical year-book of Southern Rhodesia.
Chamber of Mines of Rhodesia : annual report, monthly returns.

Uganda Protectorate—

Blue book.

Union of South Africa—

Department of Customs and Excise. Annual statement of the trade and shipping.
Department of Mines. Annual report of the Government Mining Engineer.
Office of Census and Statistics. Census of industrial establishments. Official year-book. Report on agricultural and pastoral production. Report on the statistics of crime. Report on the vital statistics. Statistics of migration.
Cape of Good Hope. Ordinances.
South African journal of economics.
Transvaal Chamber of Mines : annual report, Monthly analysis of gold production.

(c) **Foreign Countries**

Alsace-Lorraine—

Comptes rendus statistiques.

Argentine Republic—

Banco Central de la Republica Argentina. Annual report.
Dirección General de Estadistica. Anuario del comercio exterior. El comercio exterior Argentino.
Buenos Aires. Rivista de estadistica municipal.
Banco de la Nacion Argentina, Annual report and balance sheet. Revista.

(c) **Foreign Countries**—*Contd.*

Belgium—

Ministère du Travail et de la Prevoyance Sociale. Rapport . . . relative au controle des entreprises d'assurances sur la vie. Revue du travail.

Office Central de Statistique. Annuaire statistique. Bulletin de statistique.

Bulletin d l'Institut de Recherches Économiques.

Brazil—

Banco do Brasil. Relatório do Banco do Brasil . . .

Instituto Nacional de Estatistica. Annuario estatistico do Brasil.

Ministerio da Agricultura. Revista de economia e estatistica.

Ministerio de Trabalho Industria e Commercio. Boletim.

Servico de Estatistica Economica e Financeira. Banking. Comercio exterior do Brasil . . . resumo por mercadorias. Estatisticas economicas. Rivista de economica e estatistica. Movimento bancario do Brasil. Quadros estatisticos.

Bulgaria—

Direction Générale de la Statistique. Annuaire statistique. Bulletin mensuel de statistique. Revue de la statistique générale. Statistique du commerce extérieur. Statistique criminelle. Statistique de finance des communes . . . Statistique des sociétés cooperatives. Statistique de l'enseignement.

Chile—

Dirección General de Estadistica. Estadistica Chilena [monthly].

China—

Chinese Maritime Customs. List of lighthouses, light-vessels, etc. Monthly returns of the foreign trade. Trade of China [annual].

Ministry of Communications. Report on the Chinese Post Office.

National Tariff Commission. Annual report of Shanghai commodity prices.

Chinese statistical and economic review.

Colombia—

Dirección Nacional de Estadistica. Anuario general de estadistica. Boletin mensuel. Comercio exterior de Colombia.

Cuba—

Dirección General de Estadistica. Comercio exterior.

Denmark—

Det Statistiske Departement. Statistisk Aarbog. Statistiske Efterretninger. Statistiske Meddelelser [including annual returns of agriculture and live-stock, electrical undertakings, housing and rents, Municipal finance, prices, production, taxation, etc.].

Vareomsaetningen med Udlandet [monthly].

Copenhagen. *Statistiske Kontor.* Statistisk Aarbog for København, Frederiksberg og Gjentofte Kommune.

Sundhetsvæsenet, Aarsberetning.

Bulletin hebdomadaire de statistique démographique.

Nationaløkonomisk Tidsskrift.

Egypt—

Administration des Postes d'Egypte. Rapport sur les opérations de la Caisse d'Epargne.

Département de la Statistique Générale. Annuaire statistique. Annual statement of the foreign trade. Monthly bulletin of agricultural and economic statistics. Monthly summary of the foreign trade. Return of shipping, cargo, and passenger traffic in the Egyptian ports and Suez Canal transits [annual, and quarterly]. Statistique des sociétés anonymes par actions. Statistique scolaire. Vital statistics. Weekly return of births, deaths, and infectious diseases.

L'Egypte contemporaine.

(c) **Foreign Countries**—*Contd.*

Estonia—

Bureau Central de Statistique. Commerce extérieur. Recueil mensuel du Bureau Central de Statistique.
Eesti Pank : Annual report. Estonian economic year-book.
Institute of Economic Research : Konjunktur [monthly]. Majandusteated [weekly].

Finland—

Ministère des Affaires Sociales. Revue sociale.

France—

Banque de France. Compte rendu des opérations.
Direction Générale des Douanes. Tableau général du commerce extérieur.
Ministère de l'Agriculture. Statistique agricole.
Ministère du Travail. Bulletin du travail.
Statistique Générale. Annuaire statistique.
Annuaire statistique de l'Afrique occidentale.
Journal de la Société de Statistique de Paris. Annuaire.
Journal des économistes.
Revue économique et financière.
Revue d'économie politique.
Travaux de l'Institut de Science Financière et d'Assurances, Université de Lyon.

Greece—

Statistique Générale. Annuaire statistique. Bulletin mensuel de statistique. Bulletin mensuel du commerce special.
Commission Financière Internationale. Compte rendu des opérations.

Hungary—

Office Central Royal Hongrois de Statistique. Bulletin statistique trimestriel. Commerce extérieur de la Hongrie. Revue hongroise de statistique.
BUDAPEST. *Kommunalstatistiches Amt.* Monatshefte.
Journal de la Société Hongroise de Statistique.

Iceland—

Bureau de Statistique. Statistique de l'Islande [including annual reports on agriculture, fisheries, trade, etc.].

Italy—

Istituto Centrale di Statistica. Annali di statistica. Annuario statistico italiano. Bollettino mensile di statistica. Bollettino mensile di statistica agraria e forestale. Compendio statistico italiano. Movimento della popolazione.
Ministero dell'Agricoltura e delle Foreste. Annali della sperimentazione agraria.
Annali di statistico e di economia.
Assicurazioni sociali.
Economia.
Giornale degli economisti.
Metron.
Organizzazione industriale.
Rivista di storia economica.

Japan—

Bureau de la Statistique Générale. Résumé statistique de l'Empire du Japon.
TOKYO. Statistical abstract.
Kyoto University. Economic review.
Nippon Life Assurance Co. Annual report and statement of accounts.
Oriental economist.
Report on progress in Manchuria.

(c) **Foreign Countries**—*Contd.*

Latvia—

Bureau de statistique. Annuaire statistique. Bulletin mensuel. Commerce extérieur.

Luxemburg, Grand Duchy—

Office de Statistique. Aperçu Statistique. Publikationen des Statistischen Amts. Bulletin trimestriel.

Mozambique—

Repartição de Estatistica. Anuário estatistico. Bulletin trimestriel de statistique. Estatistica do comércio e navegação.

Netherlands—

Centraal Bureau voor le Statistiek. Crimineele statistiek, Gevangenisstatistiek, Statistiek van de toepassing der kinderwetten. Jaarcijfers voor Nederland. Justiticele statistiek en Faillissementsstatistiek. Maandstatistiek van den in-, uit-, en doorvoer. Nederlandsche Conjunctuur. Maandschrift van het Centraal Bureau voor de Statistiek.

AMSTERDAM. *Bureau de Statistique.* Bulletin mensuel.

Norway—

Committee for Whaling Statistics. International whaling statistics.

Det Statistiske Sentralbyrå. Månedsopgaver over vareomsetningen med utlandet.

Norges Offisielle Statistikk [including annual reports of agriculture and live-stock, assurance, crime, finance, fisheries, industrial production, mineral industry, milk industry, postal and telegraph services, public health, railways, trade, veterinary service, etc.].

Statistisk årbok. Statistisk økonomisk oversikt. Statistiske meddelelser.

OSLO. Beretning fra Oslo helseråd. Statistisk årbog. Statistisk månedsskrift.

Peru—

Departamento de Estadistica General de Aduanas. Anuario del comercio exterior. Boletin de la Superintencia General de Aduanas.

Dirección Nacional de Estadistica. Extracto estadistico.

Banco Central de Reserva, boletin. Memoria.

Philippine Islands—

Bulletin of Philippine statistics.

Portugal—

Instituto Nacional de Estatistica. Anuário demográfico. Anuário estatistico. Anuário estatistico das contribuições e impostos. Boletim mensal. comércio externo. Estatistica judiciaria. Situação bancaria.

Rumania—

Caisse Autonomone des Monopoles. Bulletin.

Institut Central de Statistique. Anuarul statistic al României.

Ministère des Finances. Comertul exterior al României.

Siam—

Central Service of Statistics. Statistical year-book.

Spain—

Dirección General de Estadistica. Boletin de estadistica.

(c) **Foreign Countries**—*Contd.*

Spanish Morocco—

Inspección de Aduanas. Estadistica del comercio en la zona de Protectorad Español.

Sweden—

Kommerskollegium. Kommersiella meddelanden.
Riksgäldskontoret. Svenska statens tillgångar och schulder.
Socialstyrelsen. Sociala meddelanden.
Statistika Centralbyrån. Statistisk årsbok. Statistiska meddelanden [including Järnvägsstatistiska medelanden. Uppgifter om bankerna].
Sveriges officiella statistik [including annual returns relating to agriculture and live-stock, assurance, banking, co-operative societies, crime, excise, finance (state and municipal), fisheries, forests, industry, labour and wages, lunacy, mines, pensions, poor relief, population, postal services, public health, health of army and navy, railways, savings banks, trade and shipping, veterinary service, water-power, etc.].
Sveriges Riksbank. Årsbok.
STOCKHOLM. *Statistiska Kontor.* Statistisk årsbok.
Economisk tidskrift.
Skandinaviska Banken, Quarterly review.
Svenska Handelsbanken : Annual report, Index.
Swedish economic review.

Switzerland—

Banque Nationale Suisse, bulletin mensuel.
Bureau Fédéral de Statistique. Annuaire statistique. Contributions à la statistique Suisse.
Statistiques de la Suisse [including annual returns relating to : agricultural statistics, assurance societies, crime, finance, forestry, live-stock, vital statistics].
Département Fédéral de l'Économie Publique. La vie économique. Die Volkswirtschaft.
Direction Générale des Douanes Fédérales. Statistique annuelle du commerce extérieur.
ZÜRICH. Statistisches Jahrbuch der Stadt Zürich. Zürcher statistische Nachrichten.
Journal de statistique et revue économique suisse.
Société de Banque Suisse, Bulletin mensuel.
Union suisse du Commerce et de l'Industrie, Rapport sur le commerce et l'industrie de la Suisse.

Turkey—

Office Central de Statistique. Annuaire statistique.

U.S.S.R.—

Administration Centrale de Statistique. Плановое Хозяйство.

United States—

Agriculture, Department of. Agricultural statistics. Crops and markets. Year-book.
Census, Bureau of the. Birth, stillbirth and infant mortality statistics. Cotton production. Cotton production and distribution. Mortality statistics. Statistical abstract of the U.S.
Children's Bureau. Bulletins. The child.
Currency, Comptroller of the. Annual report.
Education, Office of. Biennial survey of education. Bulletin. School life.
Federal Reserve Board. Annual report. Federal Reserve bulletin.

(c) **Foreign Countries**—*Contd.*

United States—*Contd.*

Foreign and Domestic Commerce, Bureau of. Balance of international payments of the U.S. Foreign commerce and navigation for the calendar year. Foreign commerce weekly. Monthly summary of the foreign commerce of the United States. Survey of current business.

Internal Revenue, Bureau of. Annual report of the Commissioner. Statistics of income.

Labor, Department of. Annual report of Secretary of Labor.

Labor Statistics Bureau. Monthly labor review.

Library of Congress. Annual report of the Librarian.

Mines, Bureau of. Minerals year-book.

Public Health Service. Public health reports.

Treasury. Annual report of the Secretary of the Treasury on the state of the finances. Annual report of the Director of the Mint.

Women's Bureau. Bulletin.

CALIFORNIA. *Bureau of Marine Fisheries.* Fish bulletins.

CONNECTICUT. Registration report. Report of State Department of Health. Health bulletin.

MARYLAND. Annual report of State Board of Health.

NEW YORK STATE. *State Department of Health.* Annual report. Vol. 2, Division of vital statistics. Monthly vital statistics review.

State Tax Commission. Annual report. Tax bulletin.

NEW YORK CITY. New York Public Library, bulletin.

Actuarial Society of America, Transactions. American Academy of Political and Social Science, annals. American economic review. American Institute of Actuaries : The record, year-book. American Petroleum Institute, statistical bulletin. American Philosophical Society : Memoirs, Proceedings, Transactions, Year-book. American Statistical Association bulletin, journal. Annals of mathematical statistics.

Carnegie Endowment for International Peace, Year-book.

Chicago Board of Trade, Annual report.

Econometrica.

Harvard business review. Harvard University : Review of economic statistics.

Illinois University Bureau of Business Research, bulletins. International conciliation.

Johns Hopkins University School of Hygiene, Collected papers from the Department of Biology. Journal of political economy.

Milbank Memorial Fund : Annual report, Collected papers on research, Quarterly bulletin. Mineral industry.

Quarterly journal of economics.

Smithsonian Institution : Annual report, Publications. Social research. Stanford University Food Research Institute, wheat studies.

Uruguay—

Contaduria General de la Nacion. Boletin de hacienda.

Dirección General de Estadistica. Anuário estadistico.

Yugoslavia—

Statistique Générale d'État. Annuaire statistique.

(d) **International.**

Bank for International Settlements—

Annual report.

International Institute of Agriculture—

Annuaire international de législation agricole. International review of agriculture. International year-book of agricultural statistics.

(d) **International**—*Contd.*

International Labour Office—

International labour review. Studies and reports. Year-book of labour statistics.

International Statistical Institute—

Bulletin de l'Institut International de Statistique. Revue de l'Institut International de Statistique [quarterly].

League of Nations—

Official journal.

Economic and Financial Section. Balances of payments. International trade statistics. Money and banking. Monthly bulletin of statistics. Review of world trade. Statistical year-book of the League of Nations. World economic survey. World production and prices.

Health Organization. Annual epidemiological report. Epidemiological report: statistical supplement to the weekly epidemiological record [monthly].

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